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A 0-1 goal programming model for nurse scheduling

M.N. Azaiez[a,*], S.S. Al Sharif[b]

[a] *Industrial Engineering Department, College of Engineering, King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia*

[b] *Planning and Development Department, Riyadh Armed Forces Hospital, P.O. Box 10880 Riyadh 11443, Saudi Arabia*

Abstract

In this study, a computerized nurse-scheduling model is developed. The model is approached through a 0-1 linear goal program. It is adapted to Riyadh Al-Kharj hospital Program (in Saudi Arabia) to improve the current manual-made schedules. The developed model accounts both for hospital objectives and nurses' preferences, in addition to considering some recommended policies that are displayed in the literature. Hospital objectives include ensuring a continuous service with appropriate nursing skills and staffing size, while avoiding additional costs for unnecessary overtime. Nurses preferences, which are deduced from a survey conducted on-purpose for the sake of this study, include mainly fairness considerations, in terms of ratio of night shifts and weekends off, in addition to avoiding isolated days on and off. The model is implemented in an experimental phase of six-month period using LINGO and is considered to perform reasonably well, based both on some quality criteria displayed in the literature and on the feedback obtained from a second survey, that has been developed to assess the scheduling system performance.

1. Introduction

The objectives in nurse scheduling are multiple. These include developing a systematic procedure for allocating nurses to work shifts and workdays in a way to ensure a continuous and appropriate service of patient care, and satisfying organizational scheduling policies, such as specific work requirements while using minimum staffing to avoid wasted manpower.

The problem is further complicated by such factors as: variation in patient demand, nurse qualification and specialization, acuity of patient illnesses, organizational characteristics (e.g., minimum

* Corresponding author. Tel.: +966-1-467-6826; fax: +966-1-467-8657.
E-mail address: mazaiez@ksu.edu.sa (M.N. Azaiez).

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required coverage and days off policies), unpredictable absenteeism, and personal requests for vacations, work stretch, and work pattern. Moreover, some of these considerations may conflict with others, such as employee requests versus the need to balance workload.

Nurse scheduling is a difficult and time-consuming task. The schedule should determine the day-to-day shift assignments of each nurse for a specified horizon of time in a way that satisfies the given requirements. The schedule should also be fair enough to everyone and not disruptive to nurses' health, families, or social lives.

Nursing talents exist at a variety of levels. Some individuals are trained to handle special needs, such as intensive care and rehabilitation therapy. Depending on their training, individuals can function at different positions such as registered nurses (RNs), licensed practical nurses (LPNs) or aids (AIDs) [1]. In Riyadh-Al-Kharj Hospital (RKH), the categories of nurses in a decreasing hierarchy are nurses in charge, staff nurse 1, staff nurse 2, and nurse aid.

In RKH, nurse scheduling is performed manually. It takes approximately one workday for a head nurse to build the schedule each month. Fairness is not addressed while making the schedule. In fact, some sample schedules show very important discrepancies in the fraction of day shifts worked in some 4-week schedules. Also, the number of weekends off or consecutive days off a nurse obtains per year is very unbalanced. Moreover, nurses' preferences are never considered. This often causes nurses' frustration leading to either working under high stresses or quitting their jobs. In either case, the quality of the nursing service may highly be affected. In addition, a manual schedule is not expected to minimize overtime and utilize the nursing staff efficiently.

The current study attempts to develop a computerized nurse scheduling system for RKH that utilizes effectively the nursing personnel. The system will also rely on fairness bases among nurses and will consider nurses' preferences to maximize their satisfaction. This will help them provide a proper quality of service.

A first survey is conducted in order to gain understanding on nurses' preferences. Fairness bases are considered both from the survey results and the suggested policies in the literature. Given that satisfying all preferences while making an effective utilization of nurses seems infeasible, a number of priority levels are considered in developing the scheduling system. Required policies are formulated as model constraints. The remaining policies are modeled as soft constraints with different importance weights. After implementing the model in a 6-month trial period, a second survey was conducted to assess the model adequacy.

RKH consists of two hospitals and several clinics. The main one is in Riyadh City and the other one is in Al-Kharj City. The clinics are scattered in different places in Riyadh and belong to the main hospital. Thus, RKH has a very large nursing staff size that amounts to 1798 nurses in Riyadh only. RKH started a nurse scheduling system with three shifts per day of 8 h each. At the end of the 1980s, RKH has opted for the two-shift day system of 12 h per shift. Scheduling is performed manually through trial and error. Each schedule is made for a 4-week period. No written policies exist for this system. However, the main hospital obligates all nurses to work their contracted time (of 176 h per a 4-week period) and any additional hours-worked are considered as overtime. Policies and guidelines in the scheduling patterns are left to the nursing administration and head nurses to arrange them. Head nurses account on their experience, knowledge and department agreements for producing schedules. However, they have no official ergonomics standards to follow. RKH has more than 55 head nurses that make schedules for their units. Each unit consists of a number of specialties. Each specialty has two types of shifts.

1. One 8-h shift for clinics for 5 days (Saturday–Wednesday) from 7:30 AM to 4:00 PM and half a day (Thursday) from 7:30 AM to 12:00 PM, with half an hour break.
2. Two 12-hour shifts for hospital wards that consist of a day shift from 7:00 AM to 7:00 PM and a night shift from 7:00 PM to 7:00 AM.

For clinics, the regular load per nurse is 44 h per week. For the hospital wards, it is of 176 h per 4-week period. In case of nurses' shortage in any ward, the corresponding head nurse may borrow nurses from inside the unit or from the same specialty in the clinics. In the latter case, the borrowed nurse will work her regular time (till 4:00 PM).

The organization of the paper will be as follows. Section 2 will present some literature review. Section 3 will discuss the first survey's description and results. Section 4 will present the linear goal programming (GLP) model along with results and discussions. Section 5 will discuss the implementation of the model. Finally, Section 6 will present some concluding remarks and directions for future research.

2. Literature review

Investigations in health care systems may be classified into four interrelated nursing human-resources decisions [2].

- Staffing decisions, which specify the number of full time equivalent nursing personnel of each class of skills to be filled for each nursing unit.
- Scheduling decisions, which specify when each nurse will be on and off duty in the scheduling period and minimum number of nurses of each class of skills required for each shift on each day.
- Allocation decisions, which allocate a pool of available floating nurses to accommodate the fluctuating demand for nursing care and for absenteeism.
- Assignment decisions, which assign nurses to individual shifts.

Modeling nurse scheduling is not a new idea. Until the 1960s, scheduling tools consisted only of graphical devices such as Gantt Chart. Howell [3] outlined the steps necessary to develop a cyclical schedule. Howell's method is a step-by-step procedure for accommodating the work patterns and individual preferences of nurses. In the early 1970s, scheduling systems began to be based on heuristic models [21,22]. These models were deemed more appropriate because they could theoretically take into account of all scheduling constraints in solving the problem. Maier-Rothe and Wolfe [4] developed a cyclical scheduling procedure that assigns different types of nurses to each unit based on average patient-care requirements, hospital personnel policies, and nursing staff preferences.

Several nurse-scheduling models were based on linear programming [1,5], penalty-point algorithms and mixed-integer programming [23]. Other optimization techniques have been used in nurse scheduling particularly for the non-cyclical type. These include the assignment problem [6], integer programming [24], stochastic programming [7], non-linear programming [8], and goal programming [1,9,10], etc.

Recently, goal programming (GP) has received the most attention among optimization techniques, as it attempts to optimize a number of objectives simultaneously. These objectives include: maximizing utilization of full-time staff, minimizing understaffing and overstaffing costs, minimizing payroll costs, as well as minimizing deviations from desired staffing requirements, nurse preferences, and nurse special requests.

Trivedi [11] has developed a MIGP model for expense budgeting in a hospital nursing department. Objectives are based on cost and quality nursing care considerations. Musa and Saxena [12] have used a 0-1 goal programming formulation for nurse scheduling in one unit of a hospital. Goals with different priority levels represent hospital policies and nurses preferences. Berrada et al. [13], in their 0-1 GP model for nurse scheduling, have used for hard constraints administrative and union contract specifications, while work patterns and nurses preferences have been formulated as soft constraints. Moores et al. [14] have formulated the student nurse allocation problem using also a 0-1 GP. The problem was to produce a 3-year schedule for student nurses to comply with the minimum practical and theoretical standards while being used as part of the hospital work force.

Some authors use artificial intelligence techniques, such as knowledge-based systems [15] and declarative programming using Prolog [25].

Chen and Yeung [15] have set five rules for evaluating a good schedule, namely physiological adjustment, well being (sleep, fatigue and appetite), personal and social problems, health (gastrointestinal and nervous disorders), and performance and accidents. The authors have also recommended a number of ergonomic constraints including limiting the succession of night shifts (maximum of three night shifts is preferable), avoiding isolated working days, alternating weekends off, considering preferences on days off, preferences on shifts, requests for emergency days off, and assigning 40 h per week for full time nurses.

Oldenkamp and Simons [16] has suggested five factors for assessing a schedule quality. These factors are given below.

- *Optimality factor*: represents the degree to which nursing expertise is distributed over the different shifts.
- *Completeness factor*: represents the degree to which quantitative demands for occupation per shift are met.
- *Proportionality factor*: represents the degree to which each nurse has been given about the same amount of night shift, evening shifts and weekends off.
- *Healthiness factor*: represents the degree to which it has been taken care of the welfare and health of the nursing staff.
- *Continuity factor*: represents the degree to which there is some continuity in nursing crew during the different shifts.

3. The first survey

The nursing staff in RKH constitutes a mixture of nurses from different countries. This includes nurses from the West, Philippines, South Africa, and Arab countries, in addition to nurses from inside the Kingdom of Saudi Arabia. It is therefore expected that the nurses' preferences would widely differ. In order to gain understanding about their preferences and to incorporate these preferences in the nurse-scheduling model, a survey was conducted during summer 2000, for the purpose of the current study. This survey involved 400 questionnaires addressed to nurses working in hospital wards (having 12-h shift patterns). The questionnaires were distributed to all nurse levels. Some procedures have been used to motivate nurses in order to obtain a large response rate. The procedure was successful and 354 out of 400 forms (i.e. 88.5 %) are filled out and returned. The respondent categories are as follows:

- 33 person in charge
- 165 staff nurse 1
- 95 staff nurse 2
- 58 nurse aide
- 3 unspecified.

The form has 15 questions with multiple choice answers related mainly to work patterns, evaluation of current schedules, preferences on day and night shifts, and preferences on days and weekends off. About half the respondents (46.6%) were staff-nurses 1. The remaining were staff-nurses 2 (26.55%), nurse aids (16.81%) and persons in charge (9.1%). Married nurses were 56% while single nurses were 44% of respondents.

The main results related to nurses preferences show that about one third of the nurses negotiate their schedules. The 12-h shift is preferred to the 8-h shift to most of the nurses (77%). Also, 52% of respondents prefer day shift against 19% prefer night shift and 29% of them are indifferent. Also, 37% of the respondents prefer to decrease night shifts, while 18% prefer to have more night shifts. Concerning preferences over days off, 47% of the nurses prefer having their days off on weekends, 58% of them prefer full weekend off, while 36% are indifferent to when to have their days off.

For a 4-week schedule, 78% of the nurses prefer to have at least 2 weekends off, while 10% are undecided. Also, 47% prefer working no more than 4 consecutive days against 20% and 16% that prefer working no more than 5 and 6 consecutive days, respectively. About the same results were obtained for night shifts. Isolated days off, isolated days on, and day–night (of the following day)-off patterns are not preferred to most of the respondents.

The results suggest in particular having some balanced schedules, where the fraction of night shifts should be about the same for all nurses. Also, isolated days on and off should be avoided. In addition, fairness bases should be applied to all nurses to have them benefit from more weekends off and if possible of full weekends off. Finally, no more than 4 consecutive working days should be assigned to nurses. This last policy is consistent with the current practices of nurse scheduling at RKH.

4. The model

4.1. Nursing policies

First, nursing policies will be developed. These policies will be based mainly on current hospital practices that the head nurses consider as implicit requirements, the results of the survey, and published policies. The importance in incorporating some of the published policies relies on accounting for ergonomic considerations. In fact, the human being has various physical limitations and the lack of ergonomic consideration causes frustration and reduces productivity quantitatively and qualitatively.

Because of the large number of constraints that the schedule attempts to satisfy, it is possible that no feasible solutions to such a nurse-scheduling problem would exist. For this reason, the constraints are divided into two classes: hard constraints that must be satisfied and soft constraints that may be violated. However, the model will minimize these violations by reducing the deviations in the soft constraints from their respective targets. The schedule will extend over a 4-week period. The set of all constraints combined is given as follows.

4.1.1. Present hospital constraints

- The unit is covered by two 12-h shifts (day and night shifts for 7 days a week, and 24 h a day).
- Each nurse has to work at least 176 h per schedule (4 weeks), which is equivalent to 14.67 days per schedule.
- The regular working days are between 14 and 15 days per schedule. Any additional worked hours above 176 h per schedule are considered as overtime.
- No nurse can work for more than 4 consecutive working days.
- A nurse should not work in any case two consecutive shifts.
- Minimum staff level requirements must be satisfied.
- Night shifts must constitute at least 25% of total workload for each nurse.

4.1.2. Published policies (based mainly on ergonomic considerations)

- Avoid any isolated days on (off–on–off).
- Avoid any isolated days off (on–off–on).
- Avoid working in a day shift and the night shift of the following day.
- Preference on days off should be considered.
- All nurses should have the same amount of night shifts and weekends off.

4.1.3. Survey results (after feedback and adjustments)

- Day shifts should exceed night shifts for each nurse and schedule.
- At least 2 weekends off in the schedule is preferred.
- Full weekend is preferred.
- Isolated days off (on–off–on) are to be avoided.
- Isolated days on (off–on–off) are to be avoided.
- (day shift–night shift–day off) are to be avoided.

4.2. Development of the 0-1 LGP model

4.2.1. Model hard and soft constraints

The problem consists on scheduling a number of nurses for a 4-week period in a cyclical way for a particular unit, while satisfying minimum staffing requirements, along with other constraints. The nurse-scheduling model will also attempt to satisfy several goals including reducing overstaffing (and hence overtime cost) as well as incorporating nurses' preferences and establishing fairness bases among nurses. These suggest using a zero-one linear goal programming (LGP) approach.

The scheduling problem contains a total of 11 scheduling sets of constraints. It is not expected however that a feasible solution may be obtained while satisfying all sets of constraints. Therefore, these sets are divided into two groups: one group consists of sets of hard constraints that must be satisfied. The other group consists of the remaining sets of constraints that are considered as soft constraints. The model will attempt to satisfy these soft constraints. If not possible, the model will reduce to a least the violations of these soft constraints based on the importance of each set. Classifying constraints into hard and soft constraints as well as assigning importance weights have been made through consulting the head nurses that are in charge of nurse scheduling. The sets of hard and soft constraints are given below.

4.2.1.1. Hard constraints

- The first set of constraints ensures that the daily minimum staff level is met. Because the daily requirements may differ from one day to another, the model will allow the user to insert the daily minimum requirement for each day and night shift.
- The second set of constraints assigns for each nurse and each day a day shift, a night shift or a day off. Also, this set of constraints disallows assigning two consecutive shifts (from 7:00 AM to 7:00 PM and from 7:00 PM to 7:00 AM or from 7:00 PM to 7:00 AM and from 7:00 AM to 7:00 PM). In other words, no nurse can be assigned 24 h of continuous work.
- The third set of constraints ensures that no nurse is assigned more than four consecutive days on.
- The fourth set of constraints ensures that each nurse is assigned at least 4 days off during weekends in each 4-week schedule.
- The fifth set of constraints ensures that each nurse is assigned 14 days on and at most 16 days on per 4-week schedule. Note that the 176 h considered by the hospital, as regular load is equivalent to 14.67 days. This set of constraints will reduce overtime and establish some fairness among nurses regarding monthly loads. Note that the upper bound is not needed, as the first set of soft constraints below call for 15 days per schedule for each nurse. It is however placed for convenience to reduce the search space. Our computational experience show a tremendous increase in model implementation time when deleting the upper bounds of 16 days.
- The sixth set of constraints ensures that each nurse is assigned a minimum number of night shifts, $N_{\min}$ ($N_{\min}$ will be taken as 25% of total working shifts). This will limit day shifts to some $D_{\max}$ (in the application, $D_{\max}$ will be taken as 75% of total working shifts). This set of constraints will establish some balance in the fraction of night shifts for all nurses, leading also to more fairness among nurses.

4.2.1.2. Soft constraints

- The first set of soft constraints attempts to assign to each nurse a total of 15 days as per schedule. This will create more balance in the workload of the different nurses.
- The second set of soft constraints attempts, again for fairness reasons, to assign more day shifts than night shifts to each nurse in each 4-week schedule.

- The third set of soft constraints attempts to avoid assigning a day shift followed by a night shift on the next day. This would help nurses adjust their time to sleep. This policy is also recommended in the literature [26].
- The fourth set of soft constraints attempts to avoid isolated days on (off–on–off patterns).
- The fifth set of soft constraints attempts to avoid isolated days off (on–off–on patterns).

4.2.2. Notations and assumptions

The schedule is assumed to start at the first day of a week. A working day starts from 7:00 AM to 7:00 AM of the next day (two shifts per 24 h). Thus, the origin of time is shifted from midnight to 7:00 AM for convenience. A weekend consists of two days. The length of a schedule is 28 days (4 weeks). The following notation is introduced.

n: number of days in a schedule ($n = 28$)
m: number of nurses available for the unit of interest
i: index for days, $i = 1, 2, \ldots, n$
k: index for nurses, $k = 1, 2, \ldots, m$
D_i: staff requirement for day shift of day i, $i = 1, 2, \ldots, n$
N_i: staff requirement, for night shift of day i, $i = 1, 2, \ldots, n$
Additional notation will be introduced when appropriate.

4.2.3. Decision variables

$$XD_{i,k} = 1 \text{ if nurse } k \text{ is assigned a day shift for day } i,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, m$$
$$= 0 \text{ otherwise}$$

$$XN_{i,k} = 1 \text{ if nurse } k \text{ is assigned a night shift for day } i,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, m$$
$$= 0 \text{ otherwise}$$

$$XR_{i,k} = 1 \text{ if nurse } k \text{ is assigned a day off for day } i,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, m$$
$$= 0 \text{ otherwise}$$

4.2.4. Formulating model constraints

Hard constraints:
Satisfy daily staff requirements

$$\sum_{k=1}^{m} XD_{ik} \geqslant D_i, \quad \text{for all } i = 1, n, \tag{4.1}$$

$$\sum_{k=1}^{m} XN_{ik} \geqslant N_i, \quad \text{for all } i = 1, n. \tag{4.2}$$

Avoid any two consecutive shifts

$$XD_{ik} + XN_{ik} + XR_{ik} = 1, \text{ for all } i = 1, n \text{ and } k = 1, m, \tag{4.3}$$

$$XD_{i+1,k} + XN_{ik} \leqslant 1, \text{ for all } i = 1, n-1 \text{ and } k = 1, m. \tag{4.4}$$

No more than 4 consecutive days on

$$XR_{ik} + XR_{(i+1)k} + XR_{(i+2)k} + XR_{(i+3)k} + XR_{(i+4)k} \geqslant 1, \text{ for all } i = 1, n-4 \text{ and } k = 1, m. \quad (4.5)$$

At least 4 days off in a 4-week-schedule during weekends

$$XR_{6,k} + XR_{7,k} + XR_{13,k} + XR_{14,k} + XR_{20,k} + XR_{21,k} + XR_{27,k} + XR_{28,k} \geqslant 4, \textit{ for } k = 1, m. \quad (4.6)$$

Minimum and maximum working days per four-week schedule

$$\sum_{i=1}^{n} (XD_{ik} + XN_{ik}) \geqslant 14, \text{ for all } k = 1, m, \quad (4.7)$$

$$\sum_{i=1}^{n} (XD_{ik} + XN_{ik}) \leqslant 16, \text{ for all } k = 1, m. \quad (4.8)$$

Minimum night shifts in the scheduling period (25% of the total)

$$\sum_{i=1}^{n} XN_{ik} \geqslant 4 \text{ for all } k = 1, m. \quad (4.9)$$

4.2.5. Formulating goals

In order to incorporate the soft constraints in the scheduling model, we will include the following goals, which are consistent respectively with the above soft constraints. The problem has therefore five goals as follows.

Goal 1: It minimizes the deviations between the sum of actual days on and the minimum required days on. This goal ensures that all nurses are scheduled to have 15 days as possible in the 4-week schedule.

$$\left\{ \sum_{i=1}^{n} (XD_{ik} + XN_{ik} \right\} - (d1_k^+ - d1_k^-) = 15, \text{ for all } k = 1, m. \quad (4.10)$$

Here, $d1_k^-$ (respectively $d1_k^+$) is the amount of negative (respectively positive) deviation from *goal 1* for nurse k. Only positive deviations are penalized.

Goal 2: It attempts to have in the schedule more day shifts than night shifts for all nurses.

$$\sum_{i=1}^{n} XD_{ik} - \sum_{i=1}^{n} XN_{ik} - (d2_k^+ - d2_k^-) = 1, \text{ for all } k = 1, m. \quad (4.11)$$

Here, $d2_k^-$ (respectively $d2_k^+$) is the amount of negative (respectively positive) deviation from *goal 2* for nurse k. Only negative deviations from *goal* 2 are penalized.

Goal 3: It avoids assigning a nurse to work a day shift and the night shift of the following day.

$$XD_{ik} + XN_{(i+1)k} - (d3_{ik}^+ - d3_{ik}^-) = 1, \text{ for all } i = 1, n-1 \text{ and } k = 1, m. \quad (4.12)$$

Here, $d3_{ik}^-$ (respectively $d3_{ik}^+$) is the amount of negative (respectively positive) deviation from *goal 3* for day i and nurse k. Only positive deviations are penalized.

Goal 4: It avoids off–on–off patterns. This goal attempts to have minimum isolated day or night shifts, for all nurses.

$$XR_{ik} + XD_{(i+1)k} + XN_{(i+1)k} + XR_{(i+2)k} - (d4_{ik}^+ - d4_{ik}^-) = 2, \quad i = 1, n-2, \ k = 1, m. \quad (4.13)$$

Here, $d4^-_{ik}$ (respectively $d4^+_{ik}$) is the amount of negative (respectively positive) deviation from *goal 4* related to isolated day/night shifts on, for day i and nurse k. Only positive deviations are penalized.

Goal 5: This goal consists on minimizing isolated days off.

$$XD_{ik} + XN_{ik} + XR_{(i+1)k} + XD_{(i+2)k} + XN_{(i+2)k} - (d5^+_{ik} - d5^-_{ik}) = 2, \;\; i = 1, n-2; \;\; k = 1, m. \tag{4.14}$$

Here, $d5^-_{ik}$ (respectively $d5^+_{ik}$) is the amount of negative (respectively positive) deviation from *goal* 5 related to isolated days off for the different combinations of days on preceding and following the day off, for day i and nurse k. Only positive deviations are penalized.

4.2.6. Assigning importance weights

Importance weights are assigned to each goal reflecting the relative importance of that goal compared to the others. Penalty levels for violating the corresponding goals express these importance weights. These levels are denoted respectively by P_1, P_2, P_3, P_4, and P_5. For the sake of the application of RKH Hospital, *goal* 1 has been considered by far the most important. Goals 2–5 were considered relatively comparable with highest importance to *goal* 2, then, *goal* 3, and finally *goals* 4 and 5 simultaneously. After a number of comparisons and assessments in the same spirit as analytical hierarchy process (AHP) the importance weights have been assigned so that the highest importance $P_1 = 20$, then $P_2 = 5$, $P_3 = 3$ and finally $P_4 = P_5 = 1$. Sensitivity analysis shows that these weights need not be very accurately estimated, as the solution remains unchanged for reasonably large ranges of these weights for this particular problem. The reader is referred to Romero [17] for an interesting and extended discussion on weighting issues.

4.2.7. Objective function

The objective function consists on minimizing the sum of the weighted deviations from the corresponding goals. Note that *goal* 1 minimizes in particular the unnecessary overtime and therefore the corresponding cost. The expression of the objective function is given by

$$\begin{aligned} Z = {} & P_1 \sum_{k=1}^{m} d1^+_k + P_2 \sum_{k=1}^{m} d2^-_k \\ & + P_3 \left[\sum_{i=1}^{n-1} \sum_{k=1}^{m} d3^+_{ik} \right] + P_4 \left[\sum_{i=1}^{n-2} \sum_{k=1}^{m} d4^+_{ik} \right] \\ & + P_5 \left[\sum_{i=1}^{n-2} \sum_{k=1}^{m} d5^+_{ik} \right]. \end{aligned} \tag{OBJ}$$

4.2.8. Underlying LGP model

The LGP model consists on minimizing the above objective function (OBJ) under the hard and soft constraints (4.1)–(4.9) and (4.10)–(4.14), respectively. The model has a total of $9mn$–$10m$ decision variables ($3mn$ binary variables and $6mn$–$10m$ non-negative deviation variables). For the case of a 28-day schedule, the number of binary decision variables will be $84m$ and the number of deviation variables will be $158m$. The total number of model constraints (excluding the non-negativity and the binary constraints) is $6mn + 3n - 5m$ $(3mn + 2n - m)$ hard constraints and $3mn + n - 4m$ soft constraints. This corresponds to $83m + 56$ hard constraints and $80m + 28$ soft constraints for a 28-day schedule.

4.3. Issues on model implementation

4.3.1. Introduction

The psychiatry unit is first selected for illustration. In this unit, there are a total of 15 nurses (6 from staff nurse 1, 8 from staff nurse 2, and 1 nurse aid). Table 1 shows a manual schedule made by the head nurse. In the corresponding schedule, two nurses were on vacation (one from staff nurse 1 and one from staff nurse 2). Thus, the schedule accounts only for the 13 remaining nurses. The minimum requirement of nurses per shift is 3 nurses for all day and night shifts and for all days including weekends. Among these 3 nurses at least one nurse must be from staff nurse 1 and 1 nurse must be from staff nurse 2. The problem consists of 1135 hard constraints and 1068 soft constraints. It also consists of 1092 binary decision variables and 2054 non-negative deviation variables. The size of the problem in the current application is computationally prohibitive. Therefore, some heuristic must be used for obtaining a satisfactory solution.

4.3.2. Subgrouping

Huarng [18,19] proposes for NP-hard scheduling problems the approach of subgrouping by splitting nurses and workloads into several subgroups, so that each subgroup will be of manageable size. By aggregating these subgroups, all the hard constraints must be satisfied. According to the computational experience in Huarng [18,19], the solutions obtained by such an approach are very satisfactory. However, there is no systematic way for subgrouping and the approach is model-dependent. In fact,

Table 1

ManualSchedule for Psychiatry Unit

Nurse			Week1							Week2							Week3							Week4							Day	Night	Total	% of Days	% of Nights
Level		Name	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F					
Staff Nurse 1	1	G.M	N	N	N			N	N			N	N				D	D			D	D	D			D	D	D			8	7	15	53%	47%
	2	E.H	D	D			D	D	D			D	D	D					N	N				N	N			N	N	N	8	7	15	53%	47%
	3	M.B			D	D	D			D	D	D		N	N	N		N			N	N	N			N	N				6	9	15	40%	60%
	4	M.R				N	N			N	N				D	D	N		D	D	D			D	D	D			D	D	10	5	15	67%	33%
	6	D.M	N	N			N	N	N			N	N				D	D	D			D	D			N	N	N			5	10	15	33%	67%
Staff Nurse 2	1	V.A			N	N	N			D	D	D			D	D	N	N			N	N	N			N	N				5	10	15	33%	67%
	2	E.B		D	D	D				N	N			N	N	N			N	N				N	N			N	N	N	3	12	15	20%	80%
	4	M.D		D	D			D	D		D	D			D	D	N	N			N	N	N			N	N				8	7	15	53%	47%
	5	N.T	N	N				N	N			N	N			D	D	D			D	D	D			D	D	D			9	6	15	60%	40%
	6	R.S			N	N	N			N	N			N	N	N			N	N			D	D	D				D	D	5	10	15	33%	67%
	7	C.C	N	N				N	N			D	D	D				D	D	D				N	N			N	N	N	6	9	15	40%	60%
	8	R.C			N	N				N	N			N	N	N				D	D	D			D	D		D	D	D	8	7	15	53%	47%
NA	1	M.N	D	D	D			D	D	D				D	D				D	D	D			D	D				D	D	15	0	15	100%	0%
		TDS	2	4	4	2	2	3	3	3	3	5	2	3	4	4	3	4	4	4	5	4	4	3	4	4	2	3	4	4					
		TNS	4	4	4	4	4	4	4	4	4	3	3	4	4	4	3	3	3	3	3	3	3	3	3	4	4	4	3	3					

TDS:Total Day Staff, TNS:Total Night Staff, MDS:Min. Day Staff ,MNS:Min. Night Staff
D - Day Shift, N -Night Shift, SN1-Staff Nurse 1, SN2-Staff Nurse 2, NA- Nurse Aid

Table 2

Nurse		Main Model - Subgroup 1																												Day	Night	Total
Level	Name	Days																														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28			
SN1	G.M			N	N	N	N			N	N			D	D	D	D						D	D	D	D			N	8	7	15
	E.H	D	D			N	N	N	N			D	D			N	N	N				D	D	D					D	8	7	15
	M.B			D	D			D	D	D	D				D	D			N	N	N				N	N	N	N		8	7	15
	M.R	D			N	N	N					N	N	N	N			D	D	D	D				D	D	D			8	7	15
	D.M	N	N			D	D			N	N					D	D	D				N	N	N			D	D	D	8	7	15
Day		2	1	1	1	1	1	1	1	1	1	1	1	1	2	3	2	2	1	1	1	1	2	2	2	2	2	1	2			40
Night		1	1	1	2	3	3	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			35

Table 3

Nurse		Main Model - Subgroup 2																												Day	Night	Total
Level	Name	Days																														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28			
SN2	V.A	D			N	N			N	N				N	N			D	D	D	D			D	D	D			N	8	7	15
	E.B			D	D	D	D				N	N	N			N	N			N	N					D	D	D	D	8	7	15
	M.D	D	D	D			N	N			D	D	D					N	N			D	D			N	N	N		8	7	15
	N.T	N	N	N				D	D	D	D			D	D	D	D					N	N	N	N					8	7	15
Day		2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1			32
Night		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			28

by having a large number of subgroups, the model may become unbalanced, as the larger the number of subgroups, the less flexible the allocation of workloads. In addition, as the number of subgroups increases, the deviation from optimality tends to increase. On the other hand, a small number of subgroups may result in inefficient computational time per subgroup. This approach will be used in the current study.

4.3.3. Illustration

We opted for subgrouping to solve the scheduling problem at the psychiatry unit using LINGO. The computational time required to determine the optimal solution for a schedule with more than six nurses is found to be too long using a PC/Pentium 700 MHz. The model takes more than 2 h to solve for a subgroup of size six. It takes 10 min to solve for a subgroup of size 5, and only few seconds to solve for subgroups of size of 4 or less. Therefore, we select 3 subgroups of respectively 5, 4, and 4 nurses for the 13-nurse psychiatry unit. Subgroup 1 consists of 5 nurses from staff nurse 1. Subgroup 2 consists of 4 nurses all from staff nurse 2, and subgroup 3 consists of 4 nurses, where 3 nurses are from staff nurse 2 and 1 nurse aid. At every shift, at least one nurse from each subgroup will be assigned. This guarantees that the aggregate minimum requirement is met, including the types of nurses per shift. Tables 2–4 show the nurse scheduling for subgroups 1–3 respectively. Table 5 shows the combined nurse scheduling for the whole unit.

Table 4

Nurse		Main Model - Subgroup 3																													
Level	Name	Days																											Day	Night	Total
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28			
SN2	R.S	D			N	N			N	N				N	N			D	D	D	D			D	D	D			N	8	7	15
SN2	C.C			D	D	D	D				N	N	N			N	N			N	N					D	D	D	D	8	7	15
SN2	R.C	D	D	D			N	N			D	D	D					N	N			D	D			N	N	N		8	7	15
NA	M.N	N	N	N				D	D	D	D			D	D	D	D					N	N	N	N					8	7	15
Day		2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1			32
Night		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			28

D - Day Shift, N -Night Shift, SN1-Staff Nurse 1, SN2-Staff Nurse 2, NA- Nurse Aid

Table 5

	Nurse		Main Model - Entire Unit																													
	Level	Name	Days																											Day	Night	Total
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28			
Subgroup 1	SN 1	G.M			N	N	N	N			N	N			D	D	D	D						D	D	D	D			N	8	7	15
Subgroup 1	SN 1	E.H	D	D			N	N	N	N			D	D			N	N	N				D	D	D					D	8	7	15
Subgroup 1	SN 1	M.B			D	D			D	D	D	D				D	D			N	N	N				N	N	N	N		8	7	15
Subgroup 1	SN 1	M.R	D			N	N	N					N	N	N	N			D	D	D	D				D	D	D			8	7	15
Subgroup 1	SN 1	D.M	N	N			D	D			N	N					D	D	D				N	N	N			D	D	D	8	7	15
Subgroup 2	SN2	V.A	D			N	N			N	N				N	N			D	D	D	D			D	D	D			N	8	7	15
Subgroup 2	SN2	E.B			D	D	D	D				N	N	N			N	N			N	N					D	D	D	D	8	7	15
Subgroup 2	SN2	M.D	D	D	D			N	N			D	D	D					N	N			D	D			N	N	N		8	7	15
Subgroup 2	SN2	N.T	N	N	N				D	D	D	D			D	D	D	D					N	N	N	N					8	7	15
Subgroup3	SN2	R.S	D			N	N			N	N				N	N			D	D	D	D			D	D	D			N	8	7	15
Subgroup3	SN2	C.C			D	D	D	D				N	N	N			N	N			N	N					D	D	D	D	8	7	15
Subgroup3	SN2	R.C	D	D	D			N	N			D	D	D					N	N			D	D			N	N	N		8	7	15
Subgroup3	NA	M.N	N	N	N				D	D	D	D			D	D	D	D					N	N	N	N					8	7	15
Day			6	3	5	3	3	3	3	3	3	5	3	3	3	4	5	4	4	3	3	3	3	4	4	4	6	4	3	4			104
Night			3	3	3	4	5	5	3	3	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3			91

D - Day Shift, N -Night Shift, SN1-Staff Nurse 1,SN2-Staff Nurse 2, NA- Nurse Aid

4.3.4. Comments

The solution obtained in this illustration for the entire unit is optimal since all goals are met (objective function is zero). As mentioned above, the total computational time is about 10 min. In this computerized schedule, all nurses work for 15 days, as recommended by the first soft constraint. This is also the case for the manual-made schedule. However, The manual-made schedule is very unbalanced with respect to day and night shifts. In fact, the proportion of day shifts varies among nurses between 20% and 100%. Note however that in the computerized schedule all nurses have the same proportion of day shifts. In addition, the staff requirement (considered in the model as a hard constraint) is violated in the manual schedule (Tuesday of the second week). Moreover, the soft constraint of avoiding a day shift followed by a night shift in the next day is violated 3 times in the manual schedule. Isolated days on and off are also encountered in the manual schedule. This is also the case for the computerized schedule, except that these violations occur in the computerized schedule only at the end of the schedule. In this case, these violations may be avoided simply by assigning days on/off properly at the beginning of the next schedule (see the application below).

To assess the adequacy of selecting subgroups of sizes 5−4−4, we implemented the model with sizes of 5–5–3. In this latter case, in addition to almost doubling the computational time, the subgroup of size 3 had a much larger fraction of day shift than for the other subgroups leading to an unbalanced aggregate schedule, as discussed above. This suggests avoiding other combinations, such as 4–3–3–3. Consequently, the choice made in the original model is appropriate.

A sensitivity analysis is carried out for penalty weights. A variety of ranges of penalty weights offer also optimal solutions with slightly different schedules. Therefore, these penalty weights are not considered as sensitive. However, the original values have the advantage of broadly reflecting the relative importance as seen by the decision-makers (several head-nurses) for the different goals.

5. Application: implementation for a 6-month trial period

Before engaging in a costly project of developing a friendly user software package based on the current model, the hospital opted for a 6-month trial period of the model in some 12 units of the hospital. The largest unit, the surgical unit, has 22 nurses and the smallest, the pediatric unit, has 12 nurses. One of the authors (the 2nd) has taken in charge running the model in all selected units for the 6-month period and making the necessary changes when requested for. For this pilot experiment, head nurses were very reluctant to approve late requests of days off and other requested changes in the schedules in order not to disturb much the user of the model. Thus, this test-period was characterized by less flexibility of head-nurses than usual. This has been pointed out by several respondents to the second survey.

Before running the model, a computer code has been developed by the authors and has interfaced with LINGO to avoid violations when linking two schedules. The serious types of violations that may occur are to assign to a given nurse a night-shift at the last day of one schedule and a day-shift on the first day of the next schedule leading to a 24-h shift. A second type of violation would be to assign for a nurse say the last 3 days of a schedule and the first 3 days of the following one resulting into 6 consecutive days on, which violates the *no-more than* 4 *consecutive days on* rule. The computer code essentially keeps record of the last days of the schedule for each nurse and adds additional constraints (hard and soft when appropriate) for the following run to avoid (or reduce) violations.

Out-of the 72 (6 *times* 12) runs, optimality (i.e., zero objective value) has been obtained in 64 occasions. In the remaining cases, soft constraints violations have occurred only for the lowest importance weights ($P_i = 1$). The largest running time was of about 20 min obtained for the surgical unit (having 22 nurses), for which subgrouping was of the sizes 5–5−4−4−4.

Surprisingly, no data was found regarding the nursing overtime cost per unit or per category of nurses. However, for the last few years, total nursing overtime cost at the entire hospital level was around 720,000 US $ per year, leading to an average over time cost of around 400 $ per nurse (independently of the nurse category) per year, given that the hospital accounts for 1798 nurses. The results of the pilot experiment has shown slightly more than 14% overtime reduction in the average based on a conservative estimation. If this result will hold true when applying the model at the hospital level, then a potential annual saving of about 100,000 US $ would occur, indicating that the developed model is promising in this regard.

To evaluate the effect of this computerized nursing system on nurses preferences, a second survey was produced and distributed to 100 nurses among those that had the pilot experiment. A response rate of 74% has been obtained. Among the respondents, about 70% (52 nurses) were fully satisfied with the new system. Also, 65 nurses (i.e., 88%) believe that the new system is significantly better than the manual one with respect to fairness bases and 61 nurses (i.e., 82%) believe that it is significantly better with respect to nurses preferences. Two drawbacks were recorded. In fact, 39 nurses (52%) pointed out that the current system is less flexible than the old one. Also, 22 nurses complained from the fact that the new system offer them less overtime than usual and therefore deprive them from a good source of income. It should be noted however that inflexibility is only temporary and as soon as the head nurses would manipulate by themselves the system (in its user-friendly version, see [20]), then they would be able to make modifications at will. The overtime problem conflicts with the hospital objectives and it is even considered (for the hospital administration) as an advantage of this computerized system.

6. Conclusions and directions for future research

In this study, a 0-1 linear goal-programming model is developed for nurse scheduling in RKH hospital. Currently, the head nurse at each unit/department makes a manual schedule through a trial-and-error approach. This pencil-and-paper approach is not only costly, but also inefficient in producing satisfactory schedules. These manual schedules do not satisfy a number of important criteria for efficient scheduling. These include balanced schedules, fairness considerations, and nurses' preferences, in addition to ergonomic considerations, and staffing requirements both in quality and size. The developed model provides important improvements in this regard besides the fact that it offers a practical computerized tool.

The developed model considers nurses' preferences by relying on a survey's results reflecting these preferences. This survey is produced for the sake of this study. Other sources used in building scheduling policies are the current applied policies in the hospital, as well as recommended policies displayed in the literature and that account for ergonomic factors. Satisfying simultaneously all the suggested policies need not be feasible. Consequently, some of these policies are taken as hard constraints that must be satisfied. Hard constraints are selected based on feedback from head nurses and other specialized staff in the hospital. The rest of the constraints are taken as soft constraints with different importance weights. Also, these importance weights are assessed based on the judgement of several head nurses.

For measuring a schedule quality, Oldenkamp and Simons [16] develop five factors as specified below.

1. *Optimality*: *represents the degree in which nursing expertise is distributed over the different shifts*. The way subgrouping is made in the developed 0-1 goal program accounts for this factor.
2. *Completeness*: *represents the degree in which the quantitative demands for occupation per shift are met*. In the 0-1 model, this is formulated as hard constraint and is always satisfied.

3. *Proportionality*: *represents the degree in which each nurse has been given about the same amount of day shifts and night shifts*. This is amply satisfied in the developed 0-1 model as illustrated by the example above and during the trial-period.
4. *Healthiness*: *represents the degree in which it has been taken care of the welfare and health of the nurses*. This is also largely considered in the model by incorporating several related factors as hard and soft constraints.
5. *Continuity*: *represents the degree in which there is continuity in the nursing crew during the different shifts*. This is also satisfied in the developed model.

Therefore, the developed model performs quite well based on the quality criteria of Oldenkamp and Simons [16]. This conclusion was also supported by the results on the feedback obtained from the second survey that has been designed to assess nurses satisfaction upon the 6-month trial period of the model implementation in 12 units of the hospital. The model has been found not only to satisfy hospital's objectives but also, and to some large extent, nurses' preferences (proportionality, weekends off, isolated days on and off, etc.). The implementation also shows a 14% reduction in overtime over the tested units. This proves some potential for important savings in nurses costs if the model is to be implemented at the entire hospital level.

The solution has been obtained through subgrouping as the computational time is prohibitive for an entire unit. Subgroups sizes are selected not to exceed 5 nurses. The implementation time is within 10 min per subgroup. The solution is found most often optimal (all goals are satisfied). Suboptimality has been encountered only in few occasions, where violations of soft constraints occurred only for those with low-importance weights.

Future work may focus on building a friendly-user computer package. Work in progress by the authors [20] extend the current model to account for other important scheduling aspects, namely vacations and days off requests. For requests of few days off, the model accepts them by adding additional constraints on days off, as requested. If the number of requests becomes fairly high by several nurses, then some rules are developed (such as priority for assigning days off, first requested first accepted with a maximum number of requested days) to keep the schedule feasible. Long vacations are incorporated by taking out the corresponding nurses from the schedules or from a number of weeks in the schedules. However, other rules are under considerations to ensure that the nurses, after taking vacations, would have a balanced remaining schedule.

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