



## Financial crises in efficient markets: How fundamentalists fuel volatility

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### ABSTRACT

When a financial crisis breaks out, speculators typically get the blame whereas fundamentalists are presented as the safeguard against excessive volatility. This paper proposes an asset pricing model where two types of rational traders coexist: short-term speculators and long-term fundamentalists, both sharing the same information set. In this framework, excess volatility not only exists, but is actually fueled by fundamental trading. Consequently, efficient markets are more volatile with a few speculators than with many speculators. Regulators should therefore be aware that efforts to limit rational speculation might, surprisingly, end up increasing volatility.

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### 1. Introduction

Abreu and Brunnermeier (2003, p. 173) claim that “The *efficient markets hypothesis* (...) implies the absence of bubbles.”<sup>1</sup> Challenging this assertion, I demonstrate that, in an efficient financial market where participants have homogenous information but heterogeneous investment horizons, excess volatility is the rule, not the exception.<sup>2</sup>

The case for excess volatility under rational expectations has been made by, e.g., Blanchard (1979), Flood and Garber (1980), and LeRoy (2004). This body of literature typically assumes that all traders are short-term speculators, which is at odds with the evidence that traders with different investment horizons coexist on financial markets.<sup>3</sup> This paper lifts this restrictive assumption and shows that even a few short-term speculators may be enough to create bubbles and crashes in efficient markets.

The controversy on the (de)stabilizing effects of speculation predates modern finance (see for instance the literature review by Hart and Kreps, 1986). The modern debate on this issue likely started with Friedman's (1953) statement that rational speculation cannot destabilize exchange rate markets. Broadly speaking, this debate opposes two types of arguments. On the one hand, authors observe that trading volume and market volatility are deeply related (Andersen, 1996), so that speculation should be detrimental to price stability.<sup>4</sup> On the other hand, Friedman's (1953) argument states that speculators provide market liquidity, which should be beneficial to price stability. In an attempt to reconcile these conflicting views, Carlson and Osler (2000) propose a microstructure model that leads to the conclusion that “Rational speculation is stabilizing at low levels of speculative activity and destabilizing at high levels” (p. 231).

Considering different categories of traders has become common in asset pricing theory since at least Harrison and Kreps (1978). However, by assuming that information is homogeneous and rationally used, this paper departs from the literature that either introduces informational discrepancies (Grossman and Stiglitz, 1980; Shalen, 1993; Madrigal, 1996, etc.), or mixes rational and irrational traders. For instance, Froot et al. (1992) introduce two categories of speculators with different information sets and prove that this

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<sup>1</sup> Emphasis in the original text.

<sup>2</sup> The efficient market theory has recently been blamed for its incapacity to predict the global financial crisis. Actually, such criticisms typically arise after market crashes (see Malkiel, 2003).

<sup>3</sup> Several characteristics may explain why investors have different investment horizons: age, gender, level of risk aversion, budget constraint, and corporate governance motivations, among others. See, for instance, Bodie and Crane (1997).

<sup>4</sup> Nevertheless, the direction of causality between trading volume and stock volatility remains obscure (Darrat et al., 2003).

leads to market inefficiencies. De Long et al. (1990) and Abreu and Brunnermeier (2003) propose models where rational arbitrageurs and irrational traders coexist. In contrast, our setting assumes that all traders are rational and share the same information set, hence the market is efficient.

Rational expectations lie at the heart of the efficient market theory (Fama, 1970). Indeed, in an efficient market all investors are supposed to form expectations by using their information set in an optimal way. However, the efficient market theory remains silent on the investment horizon of traders. In my model, two types of rational traders co-exist: long-term fundamentalists who price stocks according to dividend-based fundamentals, and short-term speculators who take into account the possibility of price departure from fundamentals.

Because fundamentalists are not concerned with transitory deviations from fundamentals, they push speculators toward more aggressive trading and exacerbate market volatility. At first sight, it might look paradoxical that rational fundamentalists impact market prices similarly to irrational traders. Nevertheless, any trading behavior that restrains liquidity makes rational speculators more nervous and translates into enhanced volatility. In a nutshell, fundamentalists destabilize the market and make their own price predictions more inaccurate in the short run, despite being rational in the long run. Hence, excess volatility is fueled by the fundamentalists.

Importantly, the model leads to conclusions which might, at first sight, be perceived as in contradiction with those reached by behavioral finance models (see Hommes (2006) and LeBaron (2006) for surveys on these models). Actually, this contradiction is only apparent because efficient market theory and behavioral finance do not share the same accepted meaning of “fundamentalism”. Within the former theory, fundamentalists are passive traders who buy and hold stocks. For the latter, they are active traders who carry out belief-driven transactions (typically, they believe that stock prices will, sooner or later, revert back to their fundamental value). As a consequence, fundamentalists in an efficient market contribute to liquidity reduction while fundamentalists in behavioral models tend to stabilize stock prices (Beine et al., 2009).

The rest of the paper is organized as follows. Section 1 presents the basic efficient market pricing model where all traders have identical investment horizons. Section 2 solves the model with heterogeneous investment horizons. Section 3 concludes.

## 2. Efficient market with homogenous traders

Although challenged by behavioral finance theories, the efficient market paradigm remains economically sound and abundantly documented by the facts (see Fama, 1998; Malkiel, 2003, and many others). This section presents the well-known asset pricing model that prevails in an efficient market and emphasizes the impact of the traders' investment horizon on the nature of the equilibrium price.

Consider a market with a single risky asset, say a stock. Consider further that all traders active on that market are rational and share the same investment horizon. In this framework, I consider two possibilities: either all traders are long-term fundamentalists, or all traders are short-term speculators. I henceforth derive the equilibrium price dynamics under each of these two polar scenarios.

Importantly, all traders are rational, meaning that they form rational expectations given their common information set,  $I_t$ . Following Muth (1961), rational expectations (RE) minimize the quadratic forecast error.<sup>5</sup> The RE of a future variable, say  $x_{t+1}$ , formed at time  $t$  is given by its conditional expectation:

$$E[x_{t+1}|I_t] = \operatorname{argmin}_{\hat{x}_{t+1} \in I_t} E(\hat{x}_{t+1} - x_{t+1})^2 \quad (1)$$

All through the paper, expectations will verify condition (1). Long-term and short-term traders differ by their investment horizon, but all of them are rational given their horizon.

### 2.1. All traders are long-term rational fundamentalists

The long-term traders, referred to as “fundamentalists”, contemplate buy-and-hold strategies solely. Accordingly, they form price expectations in an infinite horizon perspective. Let  $p_t$  be the (ex-dividend) current price of the stock, and  $d_t$  be the current dividend. The pricing model underlying the fundamentalist's investment policy is based on the absence of expected arbitrage from time  $t$  onwards (see Fig. 1). Fundamentalists thus equate the current stock price to the sum of all discounted expected future dividends resulting from holding the stock. It follows that:

$$p_t = \sum_{i=1}^{\infty} \frac{\hat{d}_{t+i}}{(1+r)^i} \quad (2)$$

where  $r$  is the required rate of return for the stock under consideration, and  $\hat{d}_{t+i}$  is the dividend expected for date  $(t+i)$ .

Importantly, fundamentalists are not concerned with future stock prices because they do not intend to speculate on price differentials and sell stocks. Indeed, when buying stocks they hold them forever. In that perspective, future dividends are their sole variables of interest, and therefore the only ones they are making predictions about.

Under the RE hypothesis in (1),  $\hat{d}_{t+i} = E[d_{t+i}|I_t]$ , and Eq. (2) defines the so-called “fundamental price”, also known as the dividend-discount model:

$$p_t = \sum_{i=1}^{\infty} \frac{E[d_{t+i}|I_t]}{(1+r)^i} \quad (3)$$

Consequently, if all market participants are fundamentalists, the stock price is unique and fully determined by future expected dividends.

### 2.2. All traders are short-term rational speculators

The short-term traders, referred to as “speculators”, contemplate buying and selling stocks at any point in time. Accordingly, they form expectations on a one-period basis by postulating the absence of expected arbitrage between  $t$  and  $t+1$  (see Fig. 2). The pricing model underlying their trading strategy states that the current stock price,  $p_t$ , is equal to the discounted sum of the expected next period price and dividend. Under the rational expectation hypothesis, the stock price is thus given by:

$$p_t = \frac{1}{1+r} (E[p_{t+1}|I_t] + E[d_{t+1}|I_t]) \quad (4)$$

Technically, the presence of an endogenous expectation (namely, the expected stock price) raises mathematical issues. Because definition (1) is implicit, Eq. (4) provides no direct expression of  $E[p_{t+1}|I_t]$  in terms of observable variables. Therefore, solving Eq. (4) requires dealing simultaneously with the current price and the expectation of the future price.

Fortunately, the econometric literature puts forth several techniques that successfully apply to model (4).<sup>6</sup> In fact, the martingale representation of the general solution to (4) (see Gouriéroux et al.,

<sup>5</sup> For further theoretical formalism on rational expectations, see, e.g., Broze and Szafarz (1984, 1991).

<sup>6</sup> The general solution to model (4) may be expressed either with an arbitrary martingale (Gouriéroux et al., 1982), or with an initial condition and a martingale difference (Broze et al., 1985). See also Sims (2002) and Anderson (2008) on computational issues related to solving these models.

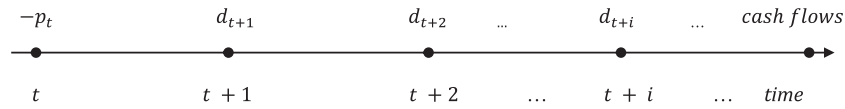


Fig. 1. The fundamentalist's time frame.



Fig. 2. The speculator's time frame.

1982) is well-adapted to financial interpretation in terms of rational bubbles (Adam and Szafarz, 1993). Indeed, Eq. (4) is equivalent to:

$$p_t = p_t^f + (1+r)^t M_t \tag{5}$$

where  $M_t$  is an arbitrary martingale<sup>7</sup> and:

$$p_t^f = \sum_{i=1}^{\infty} \frac{E[d_{t+i}|I_t]}{(1+r)^i} \tag{6}$$

As  $M_t = 0$  is a martingale, price  $p_t^f$  in (6) is a solution to model (4). Since this solution replicates the price prevailing in a market populated by fundamentalists solely (see Eq. (3)), it is called the “fundamental solution” to model (4). Therefore, the long-term traders' valuation model appears as a special case of the short-term traders' model. However, in the short-term setting, price  $p_t^f$  is but one solution within the infinite set given by Eq. (5).

Martingales are stochastic processes with constant marginal mean and non-decreasing marginal variance. Therefore, the second right-hand term in Eq. (5) is stochastic and explosive. For this reason, it is referred to as the «bubble component» of the stock price prevailing in a market where all traders are speculators. Let us denote by  $B_t$  this bubble component:

$$B_t = (1+r)^t M_t \tag{7}$$

Martingales admit multiple forms (Roll, 2002), and therefore bubble-inclusive prices may exhibit a large variety of dynamics. For this reason, they often bother model-builders. A way of getting rid of bubbles, and thus of imposing that  $M_t = 0$ , is provided by the so-called “transversality condition” stating that:

$$\lim_{i \rightarrow \infty} \frac{1}{(1+r)^i} E[d_{t+i}|I_t] = 0 \tag{8}$$

Indeed, under condition (8) the unique solution to model (4) is the fundamental solution, meaning that short-term efficient prices collapse to long-term efficient prices, and rational bubbles are no longer possible.

However, the transversality condition is a rather *ad hoc* restriction for infinite horizon equations like (4). Whether condition (8) represents a reasonable assumption in financial asset pricing is a matter of appreciation,<sup>8</sup> but it is neither a mathematical request, nor a consequence of market efficiency. Indeed, if traders contemplate transitory stock holding and/or speculate on uncertain future sales, there is no definitive motivation for favoring solution (6) amongst all solutions to (4). As a matter of fact, the fundamental

solution coincides with stock price in a financial world where all agents buy stocks with the sole perspective of keeping them forever.

### 3. Efficient market with heterogeneous traders

In Section 2, all agents were assumed to work within the same time frame. However, traders with different horizons typically interact on real financial markets. This section will therefore lift away the homogenous horizon assumption and determine the stock equilibrium price in a market where rational short- and long-term traders coexist.

#### 3.1. The model

The new model acknowledges the evidence that markets simultaneously attract short-term speculators and long-term fundamentalists. The former use one-period-ahead predictions and trade on a day-to-day basis. The latter value stocks by referring to the dividend-discount formula (3) and adopt a “buy-and-hold” strategy.

Importantly, the two categories of traders are rational and share the same information sets, but have different time frames, and hence exhibit asymmetric concerns. Indeed, the speculators care about the presence of fundamentalists (who influence prices), and the fundamentalists do not care about the presence of speculators (who do not influence dividends). The speculators are concerned with the occurrence of bubbles (that affect prices), but the fundamentalists are not (bubbles do not affect dividends). More generally, fundamentalists are insensitive to unexpected shocks affecting future prices because they solely buy and hold stocks. Thus, the speculators' and the fundamentalists' views largely differ. Nevertheless, all of them are rational, and use valuation methods derived from no-arbitrage conditions, in different time frames though.

Actually, the infinite-horizon arbitrage-free valuation illustrated by Fig. 1 holds for fundamentalists irrespectively of the presence of speculators in the market. Indeed, even when speculators are around (and rational fundamentalists may know that), buy-and-hold trading strategies are not affected by them. Therefore, rational fundamentalists do not need to take into account the speculators' expectations.

Let us assume that the pool of traders is composed in proportion  $\mu \in [0,1]$  of long-term (LT) fundamentalists, and consequently in proportion  $(1 - \mu)$  of short-term (ST) speculators.<sup>9</sup> While the fundamentalists keep valuing the stock according to formula (3), the speculators need to rationally incorporate the influence of the fundamentalists on prices. Therefore, they acknowledge the fact that the

<sup>7</sup> A martingale,  $M_t$ , is a stochastic process adapted to the information filtration,  $I_t$ , and such that its rational expectation for any future period equates its current value:  $\forall_i \geq 0: E[M_{t+i}|I_t] = M_t$ .

<sup>8</sup> The transversality condition is necessary in finite horizon optimization problems without constraint on the final state, but not in infinite horizon problems (Michel, 1990). Regarding the infinite-horizon framework, the issue is not clear-cut (see LeRoy, 2004).

<sup>9</sup> In the same spirit, Brunnermeier and Pedersen (2005) oppose strategic arbitrageurs to long-term traders. However, these authors assume that the two categories of traders have different risk aversions, and that arbitrageurs impact the price while long-term traders do not. In our model, traders from either category have a similar impact on prices and require the same rate of return.

next period price is going to be simultaneously determined by their own prediction,  $\hat{p}_{t+1}^{ST}$ , and the fundamental price prediction,<sup>10</sup>  $\hat{p}_{t+1}^{LT}$ , where:

$$\hat{p}_{t+1}^{ST} = E(p_{t+1}|I_t) \tag{9}$$

and

$$\hat{p}_{t+1}^{LT} = \sum_{i=1}^{\infty} \frac{E[d_{t+1+i}|I_t]}{(1+r)^i} \tag{10}$$

The pricing equation then reads:

$$p_t = \frac{1}{1+r} [(1-\mu)\hat{p}_{t+1}^{ST} + \mu\hat{p}_{t+1}^{LT} + \hat{d}_{t+1}] \tag{11}$$

Importantly,  $\hat{p}_{t+1}^{LT}$  relates to expected dividends solely. Fundamentalists do not take into account the possibility for bubbles. Still, they use the “true model” corresponding to their time frame and their predictions of future dividends satisfy condition (1). Indeed, the fundamentalist investment strategy does not require expectations on any variable besides dividends. They have no motivation for incorporating the presence of bubbles in their predictions. In that way, they are fully rational given their investment strategy.

Model (11) may be viewed as a generalization of model (4). In fact, (4) corresponds to the special case of (11) where  $\mu = 0$ , namely, all traders are short-term speculators. Like De Long et al. (1990), we now assume for the sake of simplicity that the required rate of return  $r$  is equal to the dividend paid by the stock, so that:  $\hat{d}_{t+1} = r$ . Consequently, the fundamental solution to model (11) is a constant,  $p_t^F = \gamma$ , and long-term traders make constant predictions:

$$\hat{p}_{t+1}^{LT} = \gamma$$

Given the expectations in (9) and (10), Eq. (11) becomes:

$$p_t = \frac{1}{1+r} [(1-\mu)E(p_{t+1}|I_t) + \mu\gamma + r] \tag{12}$$

Eq. (12) is a linear rational expectation model, to be solved similarly to Eq. (4). However, the coefficient of the rational expectation in (12),  $\frac{1-\mu}{1+r}$ , is smaller than in (4). Moreover, this coefficient decreases with the proportion of fundamentalists in the market. By using the martingale solution technique, one gets for  $\mu \neq 1$ <sup>11</sup>:

$$p_t = p_t^F + \left(\frac{1+r}{1-\mu}\right)^t M_t \tag{13}$$

where  $M_t$  is an arbitrary martingale, and the fundamental solution is:  $p_t^F = 1$ . The bubble component is thus:

$$B_t = \left(\frac{1+r}{1-\mu}\right)^t M_t \tag{14}$$

The fundamental solution is insensitive to the respective proportions of long- and short-term traders. However, the bubble intensity in (14) increases with the proportion  $\mu$  of fundamentalists in the market. Ignoring bubbles enhances them. Indeed, a market dominated by fundamentalists, but including at least some short-term traders ( $\mu \neq 1$ ), experiences stronger bubbles than a model with short-term traders only.

This outcome may be explained as follows: The presence of fundamentalists makes it more urgent to rational speculators to implement short-term arbitrage strategies as they are aware that fewer counterparts are going to be available for future trading. Indeed, by definition, fundamentalists buy and hold stocks. As a

consequence, they trade less than speculators. More precisely, when fundamentalists buy stocks they keep them forever, making them disappear from the market.<sup>12</sup>

The presence of fundamentalists is thus detrimental to market liquidity, which in turn stresses the traders who wish to settle frequent trades. Provided that some (even very few) speculators are around, fundamentalism is thus a source of market volatility. This interpretation is corroborated by the price expectation formed by the speculators. Indeed, this expectation takes into account possible bubbles and has the following expression:

$$\hat{p}_{t+1}^{ST} = 1 + \left(\frac{1+r}{1-\mu}\right)^{t+1} M_t \tag{15}$$

This expression makes it clear that the multiplier of the martingale sharply rises with the proportion of fundamentalists in the market. Moreover, the impact of this proportion grows exponentially in time. As long as a bubble is present ( $M_t \neq 0$ ), the price inflation created by the speculators' expectations is magnified by the share of long-term traders. Logically, this phenomenon leads to higher price volatility and more dramatic crashes.

In contrast, the price expectation implicit in the fundamentalists' stock valuation formula is insensitive to bubbles since these traders remain focused on the fundamentals:

$$\hat{p}_{t+1}^{LT} = 1. \tag{16}$$

Still, both expectations in (15) and (16) are rational. Indeed, on the one hand, speculators may act as buyers or sellers at any point in time. Accordingly, their expectations account for the presence of future bubbles. On the other hand, fundamentalists trade in a long-time perspective solely. Therefore, their expectations purely reflect the current expected value of their future cash-flows, namely dividends. Whether fundamentalists are aware of the possibility of future bubbles or ignore them is irrelevant because their expected cash-flows are not affected by future bubbles.

Importantly, in a rational expectation setting the argument based on liquidity risk holds for no-maturity assets, like stocks, but not for finite-maturity assets, like bonds and derivatives, of which prices cannot be subject to rational bubbles.

### 3.2. Discussion of the results

In an efficient financial market with no speculator, asset prices are governed by the fundamentals.<sup>13</sup> However, when speculators access the market the least damaging solution regarding the rise in volatility is to let as many of them as possible enter the market. Few speculators are worse than many speculators. This observation should reveal insightful to market regulators. Indeed, regulatory barriers that affect only some traders by refraining them from frequent trading introduce detrimental liquidity risk to the remaining segment of the market.

A similar conclusion is reached by Scheinkman and Xiong (2003) regarding the impact of increasing transaction costs, in a very different setting though. In their behavioral model, agents have heterogeneous beliefs generated by overconfidence, and short-selling is impossible. As a consequence, irrational bubbles

<sup>12</sup> In theory, fundamentalists can also sell stocks if they consider them as undervalued with respect to their fundamental value. However, this situation is less realistic as the corresponding investment strategy consists in shorting stocks forever. Anyway, our argument holds irrespectively of the presence of restrictions on short-selling. Indeed, strictly speaking, the equilibrium pricing model is built on the traders' expectations, and not on their actual transactions. Therefore, knowing which traders actually buy and sell stocks at each point in time is irrelevant to our argument provided that the proportions of traders of each category remain constant over time.

<sup>13</sup> Strictly speaking, such a market structure is unrealistic because fundamentalists need stock sellers as counterparts for implementing their buy-and-hold strategy, and the primary equity market is too limited to offer them stocks at any point in time.

<sup>10</sup> Actually, this prediction is never used as such by the fundamentalists. Therefore,  $\hat{p}_{t+1}^{LT}$  is better understood as the next-period fundamental value of the stock that is expected by the speculators.

<sup>11</sup> When  $\mu = 1$ , solely fundamentalists are in the market. Hence, there is a single solution to the model, namely the fundamental solution.

emerge. Scheinkman and Xiong (2003) then show that while increasing transaction costs (via Tobin’s tax, for instance) can substantially reduce speculative trading, it has only a limited impact on price volatility. Although my model is based on purely rational expectations, it leads to basically the same qualitative outcome that reducing speculation through barriers on transactions can prove to be a poor instrument to fight excess volatility.

Admittedly, the rationality assumption is instrumental to my model. In particular, the result that fundamentalists destabilize the market sharply contrasts with the typical conclusions drawn from behavioral finance models. Indeed, in behavioral finance models with heterogeneous traders (fundamentalists and speculators), fundamentalists tend to stabilize the market whereas speculators create excess volatility (Hommes, 2006; LeBaron, 2006). One can thus wonder why imposing rationality overturns this situation.

Strictly speaking, traders in an efficient market do not rely upon beliefs. Indeed, rational traders are assumed to know the true model and, accordingly, form optimal predictions. Any off-model belief would make them deviate from rationality. On the other hand, traders in behavioral models typically observe prices, but ignore the true model. To fill this gap, they form specific beliefs about future prices. These beliefs in turn drive their trading strategies. In particular, “behavioral fundamentalists” consider that future prices will tend towards their fundamental value (see, for instance, Kirman, 1991). Being concerned with short-term movements (the gap between the observed price and the fundamental value), these traders push prices to move closer to their fundamental value, so stabilizing the market. They have thus little in common with long-term rational fundamentalists who do not react to short-term price movements.

A similar reasoning applies to speculators. Schematically, in behavioral finance two broad categories of irrational speculators can be distinguished: those who trade in an erratic way – usually referred to as “noise traders” (Shleifer and Summers, 1990) or “liquidity traders” (Chowdhry and Nanda, 1991) – and those whose behavior is driven by some identified misperception of market mechanisms. In a nutshell, irrational speculators erratically and/or erroneously react to the news while rational speculators efficiently adapt to any relevant new piece of information (see Eq. (9)). The only feature common to rational and irrational speculators is short-termism. However, irrational speculators can push the stock price in any direction, and can, therefore, destabilize the market. Although rational speculators speculate on short-term deviations from the fundamental value (which creates speculative bubbles), they still acknowledge the existence of this value. As a consequence, their reactions do not alter the relevancy of the fundamentalists’ long-term strategy.<sup>14</sup>

In sum, the very meanings of “fundamentalism” and “speculation” are model-dependent. This has to do with the diverging assumptions with regard to rationality. Specifically, in behavioral models, speculators are characterized by their irrational behavior. In my model, in contrast, all traders form rational expectations – the destabilizing effect of fundamentalists needs to be understood in that context.

### 3.3. An illustration

In order to illustrate the main result of my model, let us consider an additive martingale representing a bubble starting at time 0:

<sup>14</sup> Saying that the rational speculators in my model stabilize the market would be pushing the argument too far. Actually, the rational fundamentalists destabilize the market through liquidity reduction because the market involves speculators. With or without fundamentalists in the market, speculators create rational bubbles and excess volatility. Liquidity reduction simply makes this issue more acute. Fully eradicating speculation thus remains the first-best (but likely unfeasible) solution against excess volatility.

**Table 1**  
Reaction of the expected volatility to a shock (with  $r = 10\%$  and  $\sigma = 10\%$ ).

Proportion of fundamentalists	0%	5%	25%	50%	75%	95%	100%
Expected excess volatility for $t = 1$	10%	11.5%	14.67%	22%	44%	220%	0

$$M_t = M_{t-1} + \varepsilon_t, \quad t \geq 0, \quad \text{with } M_{-1} = 0, \quad (17)$$

where  $\varepsilon_t$  is a conditionally homoskedastic white noise:

$$E[\varepsilon_t | I_{t-1}] = 0, \quad \text{and} \quad V[\varepsilon_t | I_{t-1}] = \sigma^2 \quad (18)$$

It follows that the martingale at time  $t$  may be decomposed into elementary shocks occurring between 0 and  $t$ :

$$M_t = \sum_{i=0}^t \varepsilon_i$$

The corresponding bubble in (13) is given by:

$$B_t = \left(\frac{1+r}{1-\mu}\right)^t \sum_{i=0}^t \varepsilon_i \quad (19)$$

Accordingly, the excess volatility<sup>15</sup> expected by speculators reads:

$$\sqrt{V[p_{t+1}^{ST} | I_t]} = \left(\frac{1+r}{1-\mu}\right)^{t+1} \sigma \quad (20)$$

The intensity of this expected volatility depends positively on the proportion  $\mu$  of fundamentalists in the market. Given the expression of the exponential factor in (20), it is likely that as time goes by, the impact of parameter  $\mu$  will dominate the impact of parameter  $r$  denoting the required rate of return. Therefore, the destabilizing effect of fundamentalists on bubble intensity could reveal huge.

To get a rough idea of the magnitudes at stake here, let us consider the case of an asset with a required rate of return of  $r = 10\%$ , and a shock of volatility  $\sigma = 10\%$  arising at time  $t = 0$ . Table 1 shows how such a shock affects the expected price volatility for different compositions of the pool of traders in the market.

As exhibited by Eq. (20), the proportion of fundamentalists has an exponential impact on the speculators’ expected volatility. Table 1 illustrates how spectacular this impact can be. Indeed, when fundamentalists are absent the impact of the shock on price volatility in the first period is equal to the shock standard deviation (10% in our example). With 5% of fundamentalists, a slight increase is observed (11.5%). When the market is equally split into speculators and fundamentalists, the expected excess volatility reaches the level of 22%. When the fundamentalists dominate the market, it may reach level above 100% (for instance, 220% for  $\mu = 95\%$ ). As a matter of facts, the fundamentalists’ inertia when temporary deviations from fundamentals occur is likely to be the main driver of market volatility in an efficient market.

Only when the market participants include no speculators at all is the shock not interfering with stock expected volatility. Unfortunately, this first-best situation is unrealistic, at least for stock markets. Excluding all speculators from financial markets would not only require very strong deterrence mechanisms, but also lead to perverse effects for market clearing, so that such an option is not seriously conceivable in a capitalistic economic system.

<sup>15</sup> In this simple setting, the fundamental solution is a constant so that excess volatility is equal to total volatility.

#### 4. Conclusion

The asset pricing model presented in this paper is, to my best knowledge, the first to explicitly introduce rational traders with different investment horizons. Because existing papers on rational pricing rely on homogenous investment horizon, they end up with either the dividend-discount pricing model, or prices involving rational bubbles – the intensity of which depends on the required rate of return solely. In contrast, this paper shows that long-term fundamentalists fuel price volatility. In this framework, the intensity of rational bubbles crucially depends on the composition of the pool of traders on the market.

As my model is built on the efficient market hypothesis, its outcome emphasizes that irrationality is not necessary to explain the occurrence of financial crises. Indeed, it takes only a few market participants using short-term trading strategies for the market to exhibit excess volatility. When an unpredictable shock occurs, the probability that the fundamental solution will prevail in the near future becomes negligible. As a consequence, the dividend-discount model is at best a proxy for long-term prices, but it is a hazardous benchmark for day-to-day trading.

My model also offers a consistent way to include liquidity risk pricing (Brennan and Subrahmanyam, 1996; Acharya and Pedersen, 2005) within the efficient market framework. It emphasizes that the efficient market paradigm still offers unexploited opportunities for explaining the emergence of crises. Because rationality places no restriction on the traders' investment horizon, it leaves room for considering traders with different horizons. Each market can thus be characterized not only by its price and dividend dynamics, but also by its liquidity risk parameter defined as the share of fundamentalists in the market.

As a consequence, the first extension of this paper should be empirical. The main prediction of the model is that more liquid markets, i.e. markets with fewer fundamentalists involved, should experience less dramatic crises<sup>16</sup> in terms of volatility. Nevertheless, the way to proxy the share of fundamentalists present in each market is far from obvious. I leave that issue to further work.

Theoretical extensions could investigate the consequences of heterogeneous investment horizons in other settings, including for instance more sophisticated bubbles like in Blanchard (1979) and Salge (1997), and/or multiple assets like in Sornette and Malevergne (2001). One could also endogenize the proportion of fundamentalists in the market and formalize the fundamentalists' reaction to current bubbles. Indeed, one could argue that fundamentalists who observe a current stock price being significantly higher than its fundamental value will refrain from buying the stock at stake and turn, at least temporarily, to other investment opportunities. If this is the case, then the proportion of fundamentalists becomes price-dependent. Steps in this direction have already been taken by Brock and Hommes (1998) and Lux and Marchesi (1999) for behavioral asset pricing models. Extending this logic to efficient markets with heterogeneous traders would represent a promising extension of my model. However, given the evidence of contagion across markets, disregarding bubble-inclusive assets might not always be feasible.

Lastly, the conclusions of my study can reveal useful to regulators of financial market. By proving that few rational speculators are worse than many when it comes to price stability I call for attention on the potential perverse effects of anti-speculation policies that fail to fully eradicate speculative behavior. Indeed, panic movements driven by a small number of speculators who rationally fear liquidity shortages may destabilize markets more than large-scale specula-

tive swings. Whatever the chosen policy instrument, porous barriers to speculation (for instance, taxes that do not apply to all traders to the same extent) may discourage some liquidity providers from entering the market, and therefore pave the way to excess volatility in reaction to shocks, as typically observed on thin markets.

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<sup>16</sup> See Al-Anaswah and Wilfling (2011) on the empirical identification of speculative bubbles.

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