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ScienceDirect
Applied Mathematics and Computation 181 (2006) 271–281
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Efficiency analysis of cross-region bank branches using fuzzy data envelopment analysis

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Abstract

In today's economy and society, performance analyses in the services industries attract more and more attention. The traditional data envelopment analysis (DEA) approach requires a consistent operating environment. However, in reality, there is a need to evaluate the units belonging to different environment. This reality challenges the traditional methods of applying DEA theory to real-world cases where benchmarking across region can be a very important undertaking. This paper introduces the fuzzy logic into DEA formulation to deal with the environmental variables so that the performance of bank branches from different regions can be assessed. The inner-province and inter-province comparison are given based on the fuzzy DEA results. These results are also compared with the results from traditional DEA analysis.

Keywords: Data envelopment analysis; Fuzzy sets; Efficiency; Bank

1. Introduction

The banking industry is of great importance to every one of us. With the availability of new technology and the Internet, more and more organizations are entering some aspect of the banking business and this results in intense competition in the financial services markets. Major domestic banks continue to pursue all the opportunities available to enhance their competitiveness. Consequently, performance analysis in the banking industry has become part of their management practices. Top bank management wants to identify and eliminate the underlying causes of inefficiencies, thus helping their firms to gain competitive advantage, or, at least, meet the challenges from others.

Traditionally, banks have focused on various profitability measures to evaluate their performance. Usually multiple ratios are selected to focus on the different aspects of the operations. However, ratio analysis provides relatively insignificant amount of information when considering the effects of economies of scale, the identification of benchmarking policies, and the estimation of overall performance measures of firms. As

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2006.01.037

alternatives to traditional bank management tools, frontier efficiency analyses allow management to objectively identify best practices in complex operational environments. Five different approaches, namely, data envelopment analysis (DEA) as [1–4] etc., free disposal hull (FDH) as in [5,6], stochastic frontier approach (SFA), also called econometric frontier approach (EFA) as in [7–9], thick frontier approach (TFA) as in [10–12], and distribution free approach (DFA) as in [13–15], have been reported in the literature as methods to evaluate bank efficiency. These approaches primarily differ in how much restriction is imposed on the specification of the best practice frontier and the assumption on random error and inefficiency. Compared to other approaches, DEA is a better way to organize and analyze data since it allows efficiency to change over time and requires no prior assumption on the specification of the best practice frontier. Thus, DEA is a leading approach for the performance analysis in banking industry in literature. However, the traditional DEA analysis requires a consistent infrastructure and operating environment in which the entities, appropriately called decision making units (DMUs), operate. In reality there is a need to compare DMUs where some units may have a different environment which the others cannot adopt; hence, the comparisons are not always fair. This reality challenges the traditional methods of applying DEA theory to real-world cases. Banker and Morey [16] introduced categorical inputs and outputs and their development rests on the assumption that there is a natural nesting or hierarchy of categories. The same authors [17] had also dealt with the relative technical and scale efficiencies of decision making units when some of the inputs or outputs are exogenously fixed and beyond the discretionary control of the DMU managers. Cooper et al. [18] introduced a method to do the cross-system comparison. They make use of mixed integer LP (linear programming) problem with binary variables to evaluate DMUs in different systems. The proposed mixed integer LP is solved by an algorithm where the units of one subsystem are evaluated relative to frontier based on the other subsystem. This is related to the super-efficiency proposed by Andersen and Petersen [19]. Similar to the hurdle of super-efficiency DEA, the infeasible problem often occurs when using BCC model to do cross-system comparison. As a result, the treatment offered by Cooper et al.'s cannot be applied if we need to estimate the production frontier using a variable returns to scale (VRS) technology and separate the scale effect from productivity changes [20]. Furthermore, Lozano-Vivas et al. [21] incorporate the environmental variables directly into the "basic" DEA model since adding variables to the DEA model raises the efficiency scores. Their method of adding each environmental factor guarantees that only the efficiency scores of DMUs with bad environmental conditions can change. This approach has a pre-requisite: they must know in advance the type of influence of each environmental variable on the efficiency scores. In other words, each uncontrolled factor must have an influence of know orientation.

This paper introduces the fuzzy logic into DEA formulation to deal with the environmental variables so that the performance of bank branches from different regions can be assessed. This approach can deal with both quantitative and qualitative or linguistic environmental variables. In our formulation the environmental variables serve as linking measures across different subsystems so that cross-region comparison can be done. To deal with evaluation among different systems, our methodology provides an alternative to build BCC model, which is a hurdle of Cooper et al. [18]. Our proposed fuzzy models are based upon the formulations of Lertworasirikul et al. [22]. However, it differs from theirs by incorporating both crisp and fuzzy variables in the models. Our fuzzy CCR model shows the powerful discriminating power, which is the main concern in Lertworasirikul et al. [22].

The rest of the paper is organized as follows. Section 2 presents the fuzzy data envelopment analysis (DEA) as well as the conceptual models. Section 3 gives the fuzzy DEA results and further discussion. Finally, our conclusions and future work are presented in Section 4.

2. Models and methodology

2.1. DEA model

DEA is used to establish a best practice group of units and to determine which units are inefficient compared to best practice groups as well as to show the magnitude of the inefficiencies present. Consider n DMUs to be evaluated, DMU_j ($j = 1, 2 \ldots, n$) consumes amounts $X_j = \{x_{ij}\}$ of inputs ($i = 1, 2, \ldots, m$) and produces amounts $Y_j = \{y_{rj}\}$ of outputs ($r = 1, \ldots, s$). The efficiency of a particular DMU_0 can be obtained from the following linear programs (input-oriented BCC model [23]).

$$\begin{aligned}
\min_{\theta,\lambda,s^+,s^-} \quad & z_0 = \theta - \varepsilon \cdot \vec{1}s^+ - \varepsilon \cdot \vec{1}s^- \\
\text{s.t.} \quad & Y\lambda - s^+ = Y_0, \\
& \theta X_0 - X\lambda - s^- = 0, \\
& \vec{1}\lambda = 1, \\
& \lambda, s^+, s^- \geqslant 0.
\end{aligned} \tag{1}$$

Performing a DEA analysis actually requires the solution of n linear programming problems of the above form, one for each DMU. The optimal variable θ is the proportional reduction to be applied to all inputs of DMU_0 to move it onto the frontier. A DMU is termed efficient if and only if the optimal value θ^* is equal to 1 and all the slack variables are zero. This model allows variable returns to scale. The dual program of the above formulation is illustrated by

$$\begin{aligned}
\max_{\mu,v} \quad & w_0 = \mu^{\mathrm{T}} Y_0 + u_0 \\
\text{s.t.} \quad & v^{\mathrm{T}} X_0 = 1, \\
& \mu^{\mathrm{T}} Y - v^{\mathrm{T}} X + u_0 \vec{1} \leqslant 0, \\
& -\mu^{\mathrm{T}} \leqslant -\varepsilon \cdot \vec{1}, \\
& -v^{\mathrm{T}} \leqslant -\varepsilon \cdot \vec{1}, \\
& u_0 \text{ free.}
\end{aligned} \tag{2}$$

If the convexity constraint ($\vec{1}\lambda = 1$) in (1) and the variable u_0 in (2) are removed, the feasible region is enlarged, which results in the reduction in the number of efficient DMUs, and all DMUs are operating at constant returns to scale. The resulting model is referred to as the CCR model. The reader is advised to consult the textbook by Cooper et al. [18] for a comprehensive treatment of DEA theory and application methodology.

2.2. Fuzzy DEA model

The inputs and outputs of the traditional DEA models are crisp number. Now fuzzy numbers are introduced so that part or all of the inputs and outputs are fuzzy numbers. The BCC model with fuzzy numbers and its dual are given below [22]:

$$\begin{aligned}
\max_{\boldsymbol{\omega},\boldsymbol{\mu},\boldsymbol{\mu}_0} \quad & \boldsymbol{\mu}^{\mathrm{T}} \tilde{\mathbf{y}}_0 - \boldsymbol{\mu}_0 \\
\text{s.t.} \quad & \boldsymbol{\omega}^{\mathrm{T}} \widetilde{\mathbf{X}} - \boldsymbol{\mu}^{\mathrm{T}} \widetilde{\mathbf{Y}} + \boldsymbol{\mu}_0 \geqslant 0, \\
& \boldsymbol{\omega}^{\mathrm{T}} \tilde{\mathbf{x}}_0 = 1, \\
& \boldsymbol{\omega}, \boldsymbol{\mu} \geqslant 0, \boldsymbol{\mu}_0 \text{ unrestricted.}
\end{aligned} \tag{3}$$

The dual program is formulated as

$$\begin{aligned}
& \min \ \theta \\
& \widetilde{\mathbf{X}}\lambda \leqslant \theta \tilde{\mathbf{x}}_0, \\
& \widetilde{\mathbf{Y}}\lambda \geqslant \tilde{\mathbf{y}}_0, \\
& \vec{1}\lambda = 1, \\
& \lambda \geqslant 0, \ \ \theta \text{ free in sign,}
\end{aligned} \tag{4}$$

where

$$\widetilde{\mathbf{Y}} = \begin{pmatrix} \tilde{y}_{11} & \tilde{y}_{12} & \cdots & \tilde{y}_{1s} \\ \tilde{y}_{21} & \tilde{y}_{22} & \cdots & \tilde{y}_{2s} \\ \dots & \cdots & \cdots & \cdots \\ \tilde{y}_{n1} & \tilde{y}_{n2} & \cdots & \tilde{y}_{ns} \end{pmatrix}, \qquad \widetilde{\mathbf{X}} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2m} \\ \dots & \cdots & \cdots & \cdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nm} \end{pmatrix}$$

$\tilde{\mathbf{x}}_{ji}$ $(i = 1,2,\ldots,m)$ and $\tilde{\mathbf{y}}_{jr}$ $(r = 1,\ldots,s)$ are fuzzy input and output of DMU$_j$ $(j = 1,2\ldots,n)$, respectively, $\boldsymbol{\omega}$ and $\boldsymbol{\mu}$ refer to the m-dimension and s-dimension input and output column weight vector associated with the fuzzy inputs and outputs, respectively. λ in model (4) represents n-dimension column vector.

The fuzzy DEA model cannot be solved like a crisp model. Lertworasirikul et al. [22] adopted a possibility approach and α-cut technique to convert the fuzzy CCR model to the standard linear programming (LP) problem.

$$\begin{aligned} \max_{\boldsymbol{\omega},\boldsymbol{\mu},\boldsymbol{\mu}_0} \quad & (\boldsymbol{\mu}^{\mathrm{T}}\tilde{\mathbf{y}}_0)^U_\beta - \boldsymbol{\mu}_0 \\ \text{s.t.} \quad & (-\boldsymbol{\omega}^{\mathrm{T}}\widetilde{\mathbf{X}} + \boldsymbol{\mu}^{\mathrm{T}}\widetilde{\mathbf{Y}})^L_\alpha - \boldsymbol{\mu}_0 \leqslant 0, \\ & (\boldsymbol{\omega}^{\mathrm{T}}\tilde{\mathbf{x}}_0)^U_{\alpha_0} \geqslant 1, \\ & (\boldsymbol{\omega}^{\mathrm{T}}\tilde{\mathbf{x}}_0)^L_{\alpha_0} \leqslant 1, \\ & \boldsymbol{\omega},\boldsymbol{\mu} \geqslant 0, \ \ \boldsymbol{\mu}_0 \text{ unrestricted,} \end{aligned} \tag{5}$$

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & (\theta\tilde{\mathbf{x}}_0 - \widetilde{\mathbf{X}}\lambda)^U_{\bar{\alpha}_1} \geqslant 0, \\ & (\widetilde{\mathbf{Y}}\lambda - \tilde{\mathbf{y}}_0)^U_{\bar{\alpha}_2} \geqslant 0, \\ & \vec{1}\lambda = 1, \\ & \lambda \geqslant 0, \ \ \theta \text{ unrestricted,} \end{aligned} \tag{6}$$

where α, α_0, $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are all α-cut levels used to convert associated fuzzy number to a crisp one. In model (5) and (6), fuzzy numbers are initially converted to an interval number with upper and lower bounds by use of α-cut level techniques [24]. The notation "T" is a transpose in (5). Based upon the interval number, linear programming (LP) models with crisp numbers are yielded in (5) and (6), where upper and lower bounds are represented by notation "U" and "L". For example, $(\boldsymbol{\mu}^{\mathrm{T}}\tilde{\mathbf{y}}_0)^U_\beta$ refers to the upper bound of fuzzy number $\boldsymbol{\mu}^{\mathrm{T}}\tilde{\mathbf{y}}_0$ at the β-cut level. Similar notations are viewed in the models (5) and (6).

Following the same logic, the fuzzy BCC model with crispy number can be formulated as

$$\begin{aligned} \max_{\boldsymbol{\omega}^1,\boldsymbol{\omega}^2,\boldsymbol{\mu}^1,\boldsymbol{\mu}^2,\boldsymbol{\mu}_0} \quad & (\boldsymbol{\mu}^{1^{\mathrm{T}}}\tilde{\mathbf{y}}_0)^U_\beta + \boldsymbol{\mu}^{2^{\mathrm{T}}}y_0 - \boldsymbol{\mu}_0 \\ \text{s.t.} \quad & (-\boldsymbol{\omega}^{1^{\mathrm{T}}}\widetilde{\mathbf{X}} + \boldsymbol{\mu}^{1^{\mathrm{T}}}\widetilde{\mathbf{Y}})^L_\alpha - \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{X} + \boldsymbol{\mu}^{2^{\mathrm{T}}}\mathbf{Y} - \boldsymbol{\mu}_0 \leqslant 0, \\ & (\boldsymbol{\omega}^{1^{\mathrm{T}}}\tilde{\mathbf{x}}_0)^U_{\alpha_0} + \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{x}_0 \geqslant 1, \\ & (\boldsymbol{\omega}^{1^{\mathrm{T}}}\tilde{\mathbf{x}}_0)^L_{\alpha_0} + \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{x}_0 \leqslant 1, \\ & \boldsymbol{\omega}^{1^{\mathrm{T}}},\boldsymbol{\omega}^{2^{\mathrm{T}}},\boldsymbol{\mu}^{1^{\mathrm{T}}},\boldsymbol{\mu}^{2^{\mathrm{T}}} \geqslant 0, \ \ \boldsymbol{\mu}_0 \text{ unrestricted,} \end{aligned} \tag{7}$$

$$\begin{aligned} \min_{\lambda,\theta} \quad & \theta \\ \text{s.t.} \quad & (\theta\tilde{\mathbf{x}}_0 - \widetilde{\mathbf{X}}\lambda)^U_{\bar{\alpha}_1} + \theta\mathbf{x}_0 - \mathbf{X}\lambda \geqslant 0, \\ & (\widetilde{\mathbf{Y}}\lambda - \tilde{\mathbf{y}}_0)^U_{\bar{\alpha}_2} + \mathbf{Y}\lambda - \mathbf{y}_0 \geqslant 0, \\ & \vec{1}\lambda = 1, \\ & \lambda \geqslant 0, \ \ \theta \text{ unrestricted.} \end{aligned} \tag{8}$$

In models (7) and (8), inputs and outputs are clearly divided into two types, i.e., fuzzy type and crisp one, with $\widetilde{\mathbf{X}}$, $\widetilde{\mathbf{Y}}$, $\mathbf{X}$ and $\mathbf{Y}$ referring to fuzzy input matrix, fuzzy output matrix, crisp input matrix and crisp output matrix, respectively. $\boldsymbol{\omega}^1$, $\boldsymbol{\mu}^1$, $\boldsymbol{\omega}^2$, $\boldsymbol{\mu}^2$ are weight vectors attached to them. All α-cut levels α, α_0, $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are those defined in models (5) and (6). The notation "T" is also a transpose in (7).

If the convexity constraint ($\vec{1}\lambda = 1$) in (7) and the variable $\boldsymbol{\mu}_0$ in (8) are removed, the feasible region is enlarged, then we yield the following reduced fuzzy CCR model (9) and (10).

$$\begin{aligned}
\max_{\omega^1,\omega^2,\mu^1,\mu^2} \quad & (\boldsymbol{\mu}^{1^{\mathrm{T}}}\tilde{\mathbf{y}}_0)^U_\beta + \boldsymbol{\mu}^{2^{\mathrm{T}}} y_0 \\
\text{s.t.} \quad & (-\boldsymbol{\omega}^{1^{\mathrm{T}}}\widetilde{\mathbf{X}} + \boldsymbol{\mu}^{1^{\mathrm{T}}}\widetilde{\mathbf{Y}})^L_\alpha - \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{X} + \boldsymbol{\mu}^{2^{\mathrm{T}}}\mathbf{Y} \leqslant 0, \\
& (\boldsymbol{\omega}^{1^{\mathrm{T}}}\tilde{\mathbf{x}}_0)^U_{\alpha_0} + \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{x}_0 \geqslant 1, \\
& (\boldsymbol{\omega}^{1^{\mathrm{T}}}\tilde{\mathbf{x}}_0)^L_{\alpha_0} + \boldsymbol{\omega}^{2^{\mathrm{T}}}\mathbf{x}_0 \leqslant 1, \\
& \boldsymbol{\omega}^{1^{\mathrm{T}}}, \boldsymbol{\omega}^{2^{\mathrm{T}}}, \boldsymbol{\mu}^{1^{\mathrm{T}}}, \boldsymbol{\mu}^{2^{\mathrm{T}}} \geqslant 0, \ \ \boldsymbol{\mu}_0 \text{ unrestricted},
\end{aligned} \tag{9}$$

$$\begin{aligned}
\min_{\lambda,\theta} \quad & \theta \\
\text{s.t.} \quad & (\theta\tilde{\mathbf{x}}_0 - \widetilde{\mathbf{X}}\lambda)^U_{\bar{\alpha}_1} + \theta\mathbf{x}_0 - \mathbf{X}\lambda \geqslant 0, \\
& (\widetilde{\mathbf{Y}}\lambda - \tilde{\mathbf{y}}_0)^U_{\bar{\alpha}_2} + \mathbf{Y}\lambda - \mathbf{y}_0 \geqslant 0, \\
& \lambda \geqslant 0, \ \ \theta \text{ unrestricted}.
\end{aligned} \tag{10}$$

The traditional DEA analysis requires a consistent operating environment in which the DMUs operate in bank branch evaluation studies. But the consistent culture does not exist when attempting a cross-region branch comparison. Branches in different regions may face completely different external environment, which exerts significant influence to the branches' performance. This research overcomes the above limitation by deeming all environmental variables as fuzzy variables. The proposed approach considers the influences by environmental variables and ensures that the branch's ability to produce financial products at the customer level is comparable for all the regions. Environmental variables such as the income level and population density can be well represented by fuzzy data and thus the developed fuzzy DEA models can evaluate the branches' performance from different provinces. Meanwhile, in this setting, branch performances from different subsystems, i.e., different provinces, are evaluated from a systematic point of view. Note that our models (7) and (8) differ from fuzzy models in [22] where only fuzzy numbers are taken into account.

2.3. Conceptual model

The input-oriented fuzzy DEA model was used to measure the branches' efficiency. The model aims to minimize the expenses (inputs) subject to attaining the desired output levels. In our DEA model, there were four crisp inputs, three fuzzy inputs and six crisp outputs. The diagram for the DEA model is provided in Fig. 1.

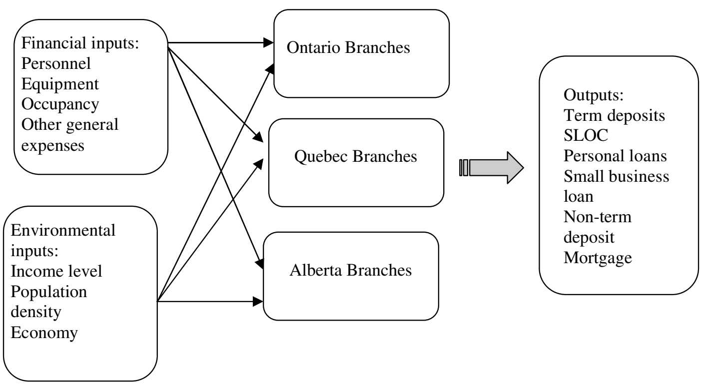

Fig. 1. Conceptual DEA model.

Table 1
Summary statistics of data

		Input				Output					
		Personnel	Equipment	Occupancy	Other expenses	Mortgage	Non-term	Per loans	SLOC	Small business loan	Term deposit
Ontario (600)	Mean	562,716	41,675	216,820	48,141	58,825,943	5,096,819	10,353,176	12,540,513	3,984,240	48,810,001
	Median	524,528	37,607	175,815	36,580	47,301,914	3,604,721	8,475,396	8,464,175	1,974,683	41,282,240
	Standard deviation	300,063	23,856	202,818	39,916	62,165,193	6,133,747	12,041,377	11,768,272	9,180,193	32,201,526
	Min	51,091	9,511	20,703	7,508	962,947	0	172,420	21,037	5,489	470,287
	Max	5,270,360	426,911	3,622,167	626,383	1,162,000,000	68,000,000	271,408,870	97,478,561	208,000,000	294,000,000
Quebec (82)	Mean	480,812	37,195	212,281	58,136	40,577,246	4,434,692	9,178,243	3,542,157	2,196,484	29,963,493
	Median	434,909	34,781	164,863	52,901	33,563,771	2,318,814	7,935,476	1,701,893	773,180	26,671,852
	Standard deviation	187,170	14,364	147,832	31,927	32,757,238	8,425,126	6,315,372	5,054,255	3,466,601	16,809,800
	Min	211,240	1,510	60,299	14,876	5,640,274	0	2,395,384	254,483	70,711	7,267,493
	Max	1,045,838	98,472	831,115	159,710	185,000,000	55,258,093	38,090,965	25,391,140	17,049,117	99,336,249
Alberta (126)	Mean	595,731	42,860	202,313	56,046	52,000,000	8,622,867	11,000,000	13,000,000	7,401,274	38,000,000
	Median	522,463	40,126	172,732	39,381	41,000,000	5,134,604	9,717,410	9,226,502	3,210,746	32,000,000
	Standard deviation	290,498	18,613	134,801	42,963	54,000,000	11,000,000	7,434,025	12,000,000	11,000,000	27,000,000
	Min	195,581	16,981	39,064	12,834	2,287,654	31,992	1,842,680	194,927	56,238	2,003,381
	Max	1,847,419	120,969	876,399	232,019	460,000,000	70,000,000	49,000,000	64,000,000	71,000,000	130,000,000

3. Results and discussion

3.1. Raw data

Summary statistics for the inputs and outputs are reported in Table 1. Eight hundred and eight branches in total are involved in this research. Among all the branches, 600 branches are from Ontario, 82 branches from Quebec and 126 branches from Alberta.

The environmental variables for three provinces are documented as shown in Table 2.

3.2. Normal DEA result

We ran the traditional BCC and CCR model based on the data set without considering the effects of the environmental variables. The results are presented in Table 3.

The DEA model identifies 75% technical efficiency. Scale efficiency can be calculated as the ratio of the CCR and BCC scores. If the frontiers of the CCR and the BCC models are very close, one can conclude that the industry operates at constant returns to scale. Otherwise, there is significant scale inefficiency. Comparison of the BCC scores with CCR scores shows that the conclusion can be drawn that most of the bank branches are operating under constant returns to scale since the scale efficiency is very close to 1. This finding is consistent with the other researchers' work such as [1,25–29]. Thus, the CCR efficiency score will be used in the following analysis unless otherwise stated.

Table 4 shows the DEA results by region.

Table 2
Environment variables of three provinces

	Income level	Population density	Economy
Ontario	High	High	Very good
Quebec	Medium	Medium	Good
Albert	High	Medium	Medium

Table 3
Normal DEA results

	BCC		CCR
Average score	0.75		0.67
Standard deviation	0.16		0.18
Maximum efficiency score	1		1
Minimum efficiency score	0.33		0.03
Number (and %) of efficient DMUs	94 (11.6%)		48 (6%)
Scale efficiency		0.90	
Returns to scale			
# Efficient DMUs exhibiting IRS	33		–
# Efficient DMUs exhibiting CRS	48		48
# Efficient DMUs exhibiting DRS	13		–

Table 4
Normal DEA results by region

	No. of branches	Average BCC score	Average CCR score
Ontario	600	0.77	0.68
Quebec	82	0.65	0.56
Alberta	126	0.76	0.69

It is noted that the inefficiency of Quebec and Alberta branches does not come from pure operational inefficiency, but a combination of inefficiency and the disadvantage imposed by the environment, which motivates our fuzzy DEA approach to deal with environmental variables.

3.3. Fuzzy DEA result

To ensure that our model can yield robust result, the user-specified parameter α, value of α-cut level, is selected from the interval $(0,1)$, each corresponding to one value of parameter a. Program is written with Matlab language as well as optimization add-in module of Matlab. The program was run on a Pentium 4 CPU computer with 480M of RAM and 2.4 GHz. After running for 2 h and 43 min, the program achieves its optimal solution. Table 4 gives the statistic results of systematic results of bank branches from three provinces. Input oriented model is applied since the bank branch management has no control of the service level that the customers request.

The very consistent trend relationship with respect to three α-cut levels in the following Table 5, i.e., $\alpha = 0.75$, 0.5 and 0.25, suggests that the fuzzy model is characterized with a good robust feature in terms of different α.

Table 6 presents the fuzzy DEA results by region.

The fuzzy DEA model (Primal form, which is model (8)) captures 31%, 29% and 35% technical and scale inefficiency for Ontario, Quebec and Alberta, respectively. This deviation from the best-practice frontier comes from purely operating inefficiency, not the combination of operating inefficiency and disadvantaged operating environment. In order to further examine our fuzzy DEA results, we compared the fuzzy DEA results with the normal DEA results in Table 4. From the comparison, we can readily observe that after we considered the environmental effects the efficiency score of Ontario branches keep almost the same, the

Table 5
Fuzzy DEA results for all branches

	Model (7)			Model (8)			Model (9)			Model (10)		
	$\alpha = 0.75$	$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.75$	$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.75$	$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.75$	$\alpha = 0.5$	$\alpha = 0.25$
Mean	0.91	0.90	0.90	0.92	0.93	0.94	0.69	0.69	0.69	0.69	0.69	0.69
Median	0.91	0.91	0.91	0.92	0.93	0.95	0.68	0.68	0.68	0.68	0.68	0.68
Standard deviation	0.09	0.13	0.10	0.07	0.06	0.05	0.18	0.18	0.18	0.18	0.18	0.18
Min	0.15	0.35	0.72	0.81	0.84	0.87	0.03	0.03	0.03	0.03	0.03	0.03
Max	1.04	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.00	1.00
No. of efficient DMUs	279	279	279	279	279	279	64	64	65	66	65	65

Table 6
Fuzzy DEA results by region

	Ontario ($\alpha = 0.5$)		Quebec ($\alpha = 0.5$)		Alberta ($\alpha = 0.5$)	
	Model (9)	Model (10)	Model (9)	Model (10)	Model (9)	Model (10)
Mean	0.69	0.70	0.71	0.70	0.65	0.63
Median	0.69	0.69	0.70	0.70	0.62	0.60
Standard deviation	0.18	0.18	0.18	0.18	0.19	0.19
Minimum	0.03	0.03	0.23	0.23	0.15	0.15
Maximum	1	1	1	1	1	1
Confidence level for mean (95.0%)	0.02	0.01	0.04	0.04	0.03	0.03
No. of efficient DMUs	45	45	10	10	9	10

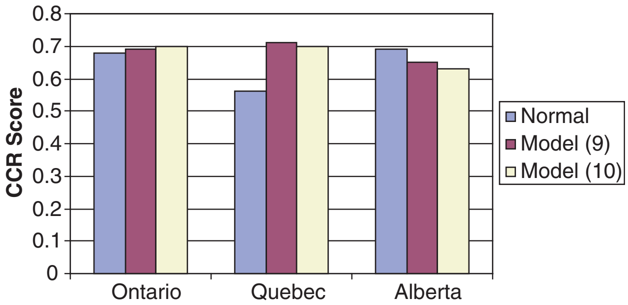

Fig. 2. DEA score comparison.

efficiency score of Quebec branches increase tremendously and the efficiency scores of Alberta branches dropped a little. Fig. 2 gives the details. This is not surprising since the bad performance of the Quebec branches partially comes from the lower income level and lower population density of Quebec and higher income level of the Alberta may help the branches to get better performance.

Table 7
Efficiency scores of 10 most inefficient branches in three provinces

Ontario		Alberta		Quebec	
DMU number	Efficiency score	DMU number	Efficiency score	DMU number	Efficiency score
DMU556	0.03	DMU601	0.15	DMU799	0.23
DMU493	0.09	DMU602	0.35	DMU728	0.33
DMU451	0.13	DMU603	0.36	DMU780	0.36
DMU557	0.16	DMU604	0.36	DMU751	0.39
DMU431	0.17	DMU605	0.36	DMU800	0.44
DMU492	0.19	DMU606	0.37	DMU764	0.46
DMU299	0.22	DMU607	0.38	DMU787	0.47
DMU438	0.23	DMU608	0.38	DMU733	0.47
DMU558	0.26	DMU609	0.40	DMU801	0.48
DMU351	0.27	DMU610	0.40	DMU763	0.51

Table 8
Efficiency scores of fifteen most efficient branches in three provinces

Ontario		Alberta		Quebec	
DMU number	Efficiency	DMU number	Efficiency	DMU number	Efficiency
DMU118	1	DMU711	0.90	DMU807	0.89
DMU281	1	DMU712	0.91	DMU771	0.90
DMU337	1	DMU713	0.91	DMU795	0.92
DMU418	1	DMU714	0.92	DMU768	0.96
DMU410	1	DMU715	0.92	DMU793	0.99
DMU482	1	DMU716	0.93	DMU737	1
DMU514	1	DMU717	0.99	DMU808	1
DMU365	1	DMU718	1	DMU788	1
DMU342	1	DMU719	1	DMU757	1
DMU36	1	DMU720	1	DMU747	1
DMU264	1	DMU721	1	DMU743	1
DMU312	1	DMU722	1	DMU784	1
DMU325	1	DMU723	1	DMU785	1
DMU450	1	DMU724	1	DMU794	1
DMU321	1.0002	DMU725	1	DMU730	1

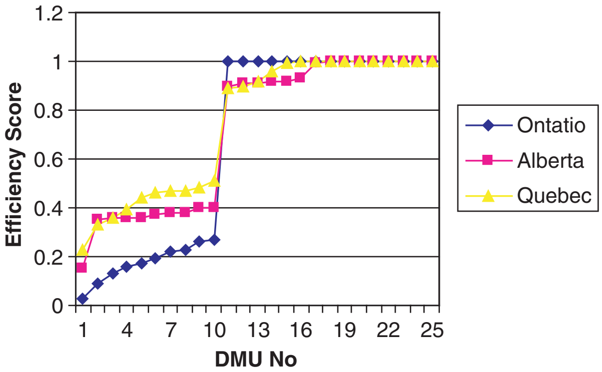

Fig. 3. Cross-province efficiency comparisons for the best and worst performers.

3.4. Inter-province efficiency

Table 7 presents 10 most inefficient branches for three provinces based on fuzzy CCR model (10) when α is equal to 0.5. Since all branches are measured against a common frontier, the branches from one province can be compared to the branches from other provinces.

As is shown in Table 7, the worst branch in the three examined provinces is from Ontario with an efficiency score of 0.03. The worst branches in Quebec and Alberta perform much better than the worst Ontario branches. By this cross-province analysis, a reference set including branches in other provinces for improvement is possible.

Similarly, Table 8 presents the efficiency scores of fifteen most efficient branches for three provinces, respectively.

From Table 8, it is easily observed that the number of best-practice performers in Ontario is much more than the number of best-practice performers in Quebec and Alberta. Fig. 3 shows the comparison of the best and worst branches in three provinces.

Due to different environment conditions for three provinces, that is, different income level, population density and economy surroundings, the number of branches in three provinces differs from each other, as well as the branch performance. The number of branches in Ontario is much more than other two provinces. In addition, its branch performance exhibits a great variation as opposed to the other two provinces.

4. Conclusions

The main objective of this paper is to apply the fuzzy DEA models to deal with the environmental variables so that the cross-region comparison is possible. In our formulation the environmental variables serve as linking measures across different subsystems in order to perform a cross-region comparison. To deal with evaluation among different systems, our methodology provides an alternative to build BCC model, which is a hurdle of previous work by Cooper et al. [18]. Although our proposed fuzzy models are built upon the formulations of Lertworasirikul et al. [22], it differs by incorporating both crisp and fuzzy variables in the models. Our fuzzy CCR model shows the powerful discriminating power, the main concern in [22]. Further consideration could be done by incorporating DEA and data mining techniques [30].

Acknowledgements

The authors would also like to thank for the National Outstanding Youth Foundation of China (70525001). Acknowledgement is given to the fund for oversea study by Chinese Academy of Sciences, special fund by Chinese Academy of Sciences for science and social work (innovation groups), University of Science & Technology of China for innovative research.

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