



Innovative Applications of O.R.

Use of DEA cross-efficiency evaluation in portfolio selection: An application to Korean stock market

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ABSTRACT

We propose a way of using DEA cross-efficiency evaluation in portfolio selection. While cross efficiency is an approach developed for peer evaluation, we improve its use in portfolio selection. In addition to (average) cross-efficiency scores, we suggest to examine the variations of cross-efficiencies, and to incorporate two statistics of cross-efficiencies into the mean-variance formulation of portfolio selection. Two benefits are attained by our proposed approach. One is selection of portfolios well-diversified in terms of their performance on multiple evaluation criteria, and the other is alleviation of the so-called “ganging together” phenomenon of DEA cross-efficiency evaluation in portfolio selection. We apply the proposed approach to stock portfolio selection in the Korean stock market, and demonstrate that the proposed approach can be a promising tool for stock portfolio selection by showing that the selected portfolio yields higher risk-adjusted returns than other benchmark portfolios for a 9-year sample period from 2002 to 2011.

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1. Introduction

Since its inception in Charnes, Cooper, and Rhodes (1978), data envelopment analysis (DEA) has been widely utilized in many application areas such education, banking, public services, and so on. In particular, multi-criteria decision-making (MCDM) is one of active fields where DEA provides a useful basis for a variety of solution approaches (Stewart, 1996). Basically, DMUs (decision making units) in DEA correspond to multiple alternatives in MCDM, input and output factors in DEA correspond to multiple performance measures in MCDM, and the notion of efficiency in DEA corresponds to that of convex efficiency of MCDM. When DEA is used as a MCDM technique, it can be called multi-factor performance measurement model. Project selection, supplier selection, and ABC inventory classification are some popular application areas where DEA is used as a multi-factor performance measurement model. Especially, portfolio selection can be considered as MCDM in the sense that multiple performance measures are typically involved in comparing alternatives (e.g., stocks, projects, products) for inclusion in a portfolio, and thus DEA can be a viable approach to it.

One key issue in MCDM is how to aggregate multiple performance measures into a single performance measure in a proper manner by choosing a set of reasonable weights on multiple measures. DEA provides a way of systematic choice of weights on multiple measures where optimal weights are determined by solving mathematical (typically linear) programs. A DEA run determines an efficiency score for a DMU, and DEA can rank DMUs according to their efficiency scores. A portfolio of DMUs can be selected based on this ranking.

However, it is well known that one of shortcomings of DEA is its too much flexibility in choosing optimal weights on input and output factors. A DMU can attain a full efficiency by choosing extremely high weights on some factors and extremely low weights on other factors. Such DMUs are referred to as *mavericks* in Green and Doyle (1994). This drawback may cause serious problems especially when DEA is used under the MCDM context since it may prevent a reasonably acceptable choice of weights for aggregating multiple criteria. This problem, in turn, leads to an unreasonable (unacceptable) ranking of DMUs. Several approaches have been proposed to address this problem. Cone-ratio model (Charnes, Cooper, Huang, & Sun, 1990) and assurance region model (Thompson, Langemeier, Lee, Lee, & Thrall, 1990) impose restrictions on weights, and super-efficiency model (Andersen & Petersen, 1993) gives a further discrimination to efficient DMUs by performing a kind of sensitivity analysis.

Cross-efficiency evaluation, on the other hand, uses a peer-evaluation mode as opposed to the self-evaluation mode of

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conventional DEA models. Under cross-efficiency evaluation, mavericks have a lower chance of attaining high appraisal. Due to this desirable property, the use of cross-efficiency evaluation has been prevalent in many DEA application areas such as project selection (Oral, Kettani, & Lang, 1991), preference voting (Green, Doyle, & Cook, 1996), supplier selection (Braglia & Petroni, 2000), ABC inventory classification (Park, Bae, & Lim, 2011), and others. Although DEA cross-efficiency evaluation has proven effective in ranking DMUs, there still exist some problems that limit its use. One such a well-known problem is the non-uniqueness of cross-efficiencies. Several approaches have been developed to alleviate this problem by introducing secondary objectives (see, e.g., Doyle & Green, 1994; Liang, Wu, Cook, & Zhu, 2008; Lim, 2012).

A traditional use of DEA cross-efficiency evaluation in portfolio selection is to rank DMUs in a decreasing order of cross-efficiency scores and choose the top k DMUs where k is the desired portfolio size. While this simple use of cross-efficiency in portfolio selection has been reported to show significant advantages over approaches based on the standard DEA (see the cited references above), we notice its two problems, which have not been studied well and thus are worthy of a careful investigation.

The first one is the *lack of portfolio diversification*³ and the second one is the “ganging-together” phenomenon (Tofallis, 1996) of cross-efficiency, which will be described in detail in Section 2. We should point out that these are not a problem of using cross-efficiency in general; rather a problem of its specific use in portfolio selection.

We attempt to address these issues by developing a mean-variance (MV) framework of portfolio selection based on DEA cross-efficiency evaluation. The basic idea is that a DMU's simple efficiency score and the variance of its cross-efficiencies are used to represent the DMU's return and risk characteristics. Subsequently, Markowitz' mean-variance formulation is used to determine the DMU's inclusion in a portfolio under consideration.

There have been several similar attempts to use information on the variability of (cross) efficiencies in addition to averages. While discussing relationships between MCDM and DEA, Stewart (1996) proposes a stochastic approach in which a probability distribution on efficiencies can be derived for each DMU, as a basis for comparison, assuming a certain probability distribution on input-output weights. Salo and Punkka (2011) develop comparative results for ratio-based efficiency analysis based on DMUs' relative efficiencies over sets of feasible weights. They develop ranking intervals and efficiency bounds, and use them for deriving dominance relations. While these two papers pay attention to the variability of (cross) efficiencies of DMUs, they do not consider it under the context of portfolio selection. Consequently, they do not take into account the effect of a DMU's inclusion in a portfolio under consideration in association with other DMUs already included in the portfolio. Chen and Zhu (2011) use the bootstrap game cross-efficiency distributions to gather information regarding efficiency variations and correlations, and they adopt the MV formulation to obtain a risk-minimizing resource allocation portfolio. They model input-output weights (referred to as shadow prices) as random variables and, as a result, treat the efficiency index for a DMU as a stochastic measure. Differently from our approach, they use the DEA game cross-efficiency model and develop a bootstrap algorithm to obtain efficiency distributions.

³ The term ‘diversification’ in finance is used to refer to reducing specifically the idiosyncratic risk of portfolios and the degree of diversification is usually measured by the level of non-idiosyncratic risk. However, note that we use the term in the general context of portfolio selections based on multiple evaluation criteria (e.g., project portfolio selection) throughout this paper. We seek to select a portfolio whose performance is well-diversified in terms of its performance on multiple evaluation criteria. We add this footnote to avoid any possible misunderstanding of readers with financial background.

As an illustration of the proposed approach, we report a case study involving 490–557 firms listed in the Korea Exchange. Using actual financial data from 2001 to 2009, the proposed approach is applied, as a means of fundamental analysis, to select stocks for inclusion in a portfolio. We demonstrate that the proposed approach can be a promising tool for stock portfolio selection by showing that the selected portfolio yields higher risk-adjusted returns than those of two stock market indices for a 9-year sample period from 2002 to 2010. We also show that the selected portfolio is superior to a portfolio purely based on cross-efficiency scores, which indicates the effectiveness of the proposed use of cross-efficiency evaluation under the MV framework.

The paper is organized as follows. Section 2 discusses the two problems arising in the simple use of cross-efficiency evaluation in portfolio selection. The development of the proposed approach is described in Section 3, followed by the case study in Section 4. Section 5 concludes.

2. DEA cross-efficiency evaluation

We assume that there are n DMUs with m inputs and s outputs. DMU k ($k = 1, 2, \dots, n$) has a vector of inputs $x_k = (x_{1k}, \dots, x_{mk})^T \in \mathfrak{R}_+^m$ and a vector of outputs $y_k = (y_{1k}, \dots, y_{sk})^T \in \mathfrak{R}_+^s$. Let us present a basic model of cross-efficiency evaluation under the following standard input-oriented constant returns to scale (CRS) DEA model (Charnes et al., 1978) in multiplier form:

$$\begin{aligned} \max \quad & \sum_{r=1}^s u_r y_{rk} \\ \text{s.t.} \quad & \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0, \quad j = 1, \dots, n \\ & \sum_{i=1}^m v_i x_{ik} = 1 \\ & v_i, u_r \geq \varepsilon \quad \forall i, r \end{aligned} \quad (1)$$

where ε is a positive non-Archimedean infinitesimal, and v_i and u_r are weights on inputs and outputs, respectively, to be determined by optimizing the model. When the above model is solved, an *efficiency score* of DMU k and cross-efficiencies of the other DMUs (evaluated by DMU k) are obtained together. Specifically, a *cross-efficiency* of DMU l is given by

$$e_{kl} = \frac{\sum_{r=1}^s u_r^* y_{rl}}{\sum_{i=1}^m v_i^* x_{il}} \quad (2)$$

where $*$ denotes an optimal solution DMU k has chosen in model (1). Collecting cross-efficiencies of all DMUs, a matrix of cross-efficiencies is obtained as $E = (e_{pq})$, $p, q = 1, \dots, n$, where the l th column $e_{\cdot l}$ is the vector of cross-efficiencies of DMU l . A *cross-efficiency score* of DMU l is also obtained by averaging $e_{\cdot l}$, denoted \bar{e}_l .

Basically, DEA allows each DMU to choose the most favorable weights on its own, by which the DMU reveals its strong and weak points (relative to its peers) that it should emphasize and/or deemphasize in order to maximize its (relative) efficiency score. The strong and weak points are manifested in *endogenously* model-determined optimal weights with higher (lower) weights on measures associated with the strong (weak) points. Under the standard DEA model, each DMU needs not consider other sets of weights possibly chosen by its competing peers since it is evaluated using only its own weights. While this mechanism is valid under the context of efficiency evaluation itself, it is not appropriate when we use DEA for portfolio selection under the context of MCDM where weights on performance measures are determined *exogenously*, may not stay the same over time, and can vary significantly according to changing environment surrounding the

evaluation context. In this regard, each DMU is exposed to the risk of change in weights, and this needs to be considered more seriously when we use DEA for portfolio selection. This consideration, in turn, justifies incorporating a peer evaluation mode into the standard DEA model, and cross-efficiency evaluation is one of such endeavors.

Under cross-efficiency evaluation, all-round players (DMUs) are highly ranked since their performance is at least moderately good on all measures. They are relatively robust to the risk of change in weights and the variances of their cross-efficiencies are relatively small. On the other hand, those DMUs are lowly ranked which perform well on only a subset of measures. They are vulnerable to the risk of change in weights in the sense that their evaluation deteriorates when lower (higher) weights are imposed on the favorable (unfavorable) subset of measures. This results in large variances of cross-efficiencies. Mavericks represent an extreme case and the variances of their cross-efficiencies are very likely to be significantly large. If a DMU has a lower variance of cross-efficiencies, it is likely to attain a higher cross-efficiency score. Here we note that the variance of a DMU's cross-efficiencies indicates the risk of change in weights involved in performance evaluation of the DMU.

Due to the reason described above, cross-efficiency evaluation helps select a portfolio where each selected DMU is relatively robust to the risk of change in weights. However, we notice two problems arising in the simple use of cross-efficiency evaluation in portfolio selection, which are (1) the lack of portfolio diversification and (2) the ganging-together phenomenon of cross-efficiency. We now demonstrate each of the two problems.

Firstly, let us see the problem of the lack of portfolio diversification. While DEA cross-efficiency evaluation is effective for preventing mavericks from being selected in a portfolio, it is likely to choose only all-round players (DMUs whose performance is at least moderately good on all measures) and exclude those DMUs whose performance is good on only a subset of measures. This leads to selection of a specialized portfolio which comprises similar DMUs and therefore lacks diversification. We illustrate this phenomenon using an example whose data set is given in Table 1 and plotted in Fig. 1. Since the output data are the same for all DMUs, the data points can be plotted in the input space. If we select a portfolio of size 7 (about 30% of the population size) based on cross-efficiency scores, 7 DMUs (DMUs 18, 14, 20, 16, 15, 10 and

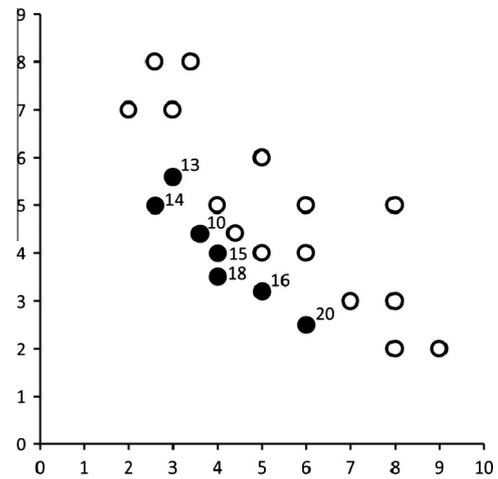


Fig. 1. Portfolio selection (data set 1).

13 in a decreasing order of cross-efficiency scores) are chosen for inclusion in the portfolio, which are indicated by black circles. As illustrated in Fig. 1, we can see that the selected DMUs are relatively similar (in the sense that they have relatively similar factor levels) and clustered around the central position, which makes the selected portfolio not well-diversified in terms its performance on the multiple input-output factors and thus vulnerable to the risk of change in weights on the two inputs.

We now turn to the problem of the ganging together phenomenon of cross-efficiency evaluation. As Tofallis (1996) demonstrated, if two DMUs have similar factor levels, they will employ similar weights and effectively raise each other's cross-efficiency score when these weights are applied to the other DMUs. If none of the remaining DMUs have similar factor levels, they will be disadvantaged because they will be isolated in the cross-efficiency evaluation. As a result, one or both of the two similar DMUs may turn out to be the winner simply because they effectively give "high votes" to each other. If a DMU's factor levels are very different from those of the other DMUs, it stands a much lower chance of winning. This phenomenon is aggravated as the distribution of DMUs' locations is skewed. It, again, leads to selection of a specialized portfolio which consists of relatively similar DMUs and in turn lacks diversification. Table 2 and Fig. 2 illustrate this phenomenon. If we select a portfolio of size 7 (about 30% of the population size) based on cross-efficiency scores, 7 DMUs (DMUs 11, 14, 18, 8, 13, 5 and 12 in a decreasing order of cross-efficiency scores) are chosen for inclusion in the portfolio, which are indicated by black circles in Fig. 2. Those DMUs that are located in the (upper left) more densely populated area employ similar weights and therefore effectively contribute to higher cross-efficiency scores of the 7 selected DMUs.

The above examples illustrate that the simple use of cross-efficiency score in portfolio selection *per se* can result in poorly-diversified portfolios in terms of their performance on multiple input-output factors. This motivates our development of a MV framework of portfolio selection based on DEA cross-efficiency evaluation, which will be detailed in the subsequent section.

3. A MV framework of portfolio selection based on cross-efficiency evaluation

As demonstrated in the previous section, while the simple use of cross-efficiency evaluation in portfolio selection effectively considers the risk of change in weights for *individual* DMUs selected in a portfolio, it fails to consider the risk for the portfolio *overall*. The overall risk of change in weights for a portfolio com-

Table 1
Data set 1: efficiency and cross-efficiency scores.

DMU	x_1	x_2	y	Efficiency score	Cross-efficiency score	Rank
1	5	4	1	0.846154	0.750776	11
2	6	5	1	0.6875	0.612769	19
3	4	5	1	0.838462	0.737136	12
4	8	5	1	0.611111	0.529195	22
5	5	6	1	0.685535	0.603653	20
6	8	3	1	0.8	0.65958	17
7	4.4	4.4	1	0.854232	0.759057	9
8	2.6	8	1	0.82	0.637516	18
9	3.4	8	1	0.706897	0.585274	21
10	3.6	4.4	1	0.942907	0.829672	6
11	2	7	1	1	0.756939	10
12	3	7	1	0.803922	0.66701	16
13	3	5.6	1	0.883306	0.771001	7
14	2.6	5	1	1	0.872922	2
15	4	4	1	0.939655	0.834963	5
16	5	3.2	1	0.964912	0.837327	4
17	6	4	1	0.785714	0.684375	15
18	4	3.5	1	1	0.896282	1
19	7	3	1	0.846154	0.714126	13
20	6	2.5	1	1	0.84287	3
21	8	2	1	1	0.767346	8
22	9	2	1	1	0.710605	14

Table 2
Data set 2: efficiency and cross-efficiency scores.

DMU	x_1	x_2	y	Efficiency score	Cross-efficiency score	Rank
1	2.6	11	1	0.769231	0.599744	18
2	3.2	11	1	0.632184	0.543619	22
3	2.4	10	1	0.833333	0.654985	12
4	3.4	10	1	0.654762	0.554932	21
5	2	9	1	1	0.754659	6
6	2.8	9	1	0.753425	0.643476	13
7	3.4	9	1	0.696203	0.585808	19
8	2	8	1	1	0.80311	4
9	2.8	8	1	0.808824	0.684006	11
10	3.4	8	1	0.743243	0.621806	15
11	2	7	1	1	0.861609	1
12	2.8	7	1	0.873016	0.732319	7
13	3	6	1	0.916667	0.763748	5
14	3	5	1	1	0.833429	2
15	4	5	1	0.863636	0.712305	9
16	5	4	1	0.846154	0.685121	10
17	6	4	1	0.785714	0.61168	16
18	4	3.5	1	1	0.826784	3
19	7	3	1	0.846154	0.611407	17
20	6	2.5	1	1	0.719933	8
21	8	2	1	1	0.633504	14
22	9	2	1	1	0.583496	20

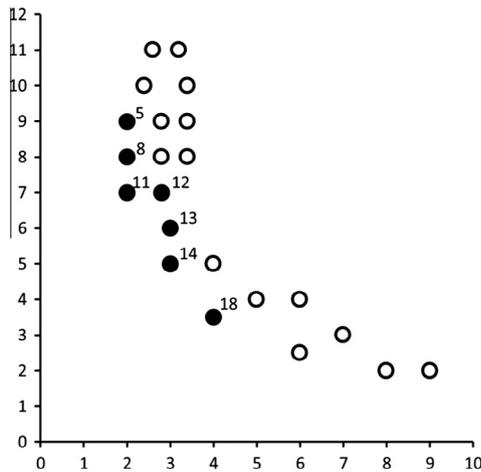


Fig. 2. Portfolio selection (data set 2).

prises two parts: *individual DMU risk* and *inter-DMUs risk*. The former is represented by the variance of cross-efficiencies of each individual DMU included in a portfolio, and the latter is represented by the covariance between each pair of DMUs. The simple use of cross-efficiency evaluation effectively reduces individual DMU risk but fails to consider inter-DMUs risk. To address this issue, we develop a MV framework of portfolio selection based on cross-efficiency evaluation as follows.

For a DMU l , the return and risk characteristics are defined as its cross-efficiency score $\bar{e}_l = \frac{1}{n} \sum_{k=1}^n e_{kl}$ and the variance of its cross-efficiencies $\sigma_l^2 = \frac{1}{n} \sum_{k=1}^n (e_{kl} - \bar{e}_l)^2$, respectively. Similarly, for a portfolio Ω with individual DMUs being weighted using a weight vector $w \in \mathcal{R}_+^n$ where $\sum_{i=1}^n w_i = 1$, the return and risk characteristics are defined as $E_\Omega = w^T \bar{e}$ and $V_\Omega = w^T \Sigma w$ where Σ is the covariance matrix of cross-efficiencies whose (k, l) th element is the covariance between DMU k 's cross-efficiencies ($e_{.k}$) and DMU l 's cross-efficiencies ($e_{.l}$) denoted σ_{kl} . An optimal portfolio Ω^* with an optimal weight vector w^* is determined by solving the following quadratic optimization model:

$$\begin{aligned} \min \quad & V_\Omega \\ \text{s.t.} \quad & E_\Omega \geq (1 - \gamma)E_\Omega^b \\ & e^T w = 1 \\ & w \geq 0, \end{aligned} \tag{3}$$

where γ is the return-risk trade-off parameter, E_Ω^b is the maximum portfolio return achievable, and e is a vector of appropriate dimension whose elements are all one. Note that E_Ω^b can be determined by maximizing E_Ω under the constraints of $e^T w = 1$ and $w \geq 0$.

Model (3) minimizes the portfolio risk (with respect to change in DEA multipliers) while imposing a lower bound on the portfolio return. Note that $V_\Omega = w^T \Sigma w = \sum_{k=1}^n w_k^2 \sigma_k^2 + \sum_{k=1}^n \sum_{l=1, l \neq k}^n w_k w_l \sigma_{kl}$, which is a weighted sum of the variance of each individual DMU's cross-efficiencies ($\sum_{k=1}^n w_k^2 \sigma_k^2$) and the covariance of each pair of DMUs' cross-efficiencies ($\sum_{k=1}^n \sum_{l=1, l \neq k}^n w_k w_l \sigma_{kl}$). As demonstrated in Section 2, the simple use of cross-efficiency evaluation for portfolio selection can effectively reduce the first part, but it fails to consider the second part. In contrast, our approach based on model (3) can collectively reduce the two parts.

Now we apply the above-described model to the two example data sets introduced in the previous section to show its effectiveness. Fig. 3 compares two portfolios selected from the data set 1; the left one is a portfolio selected based on the simple use of cross-efficiency evaluation (repeating Fig. 1), and the right one is a portfolio selected using the proposed approach with the value of the return-risk trade-off parameter (γ) set to 2%. We can visually confirm that the portfolio based on our approach is more diversified in terms of its performance on the two input factors. This can also be confirmed numerically by looking at E_Ω and V_Ω for each portfolio; the portfolio on the right have significantly smaller variance ($0.91 \rightarrow 0.64$, 29.7% reduction) with slight decrease in return ($5.89 \rightarrow 5.80$, 1.5% reduction). Similarly, Fig. 4 compares two portfolios selected from the data set 2; the left one repeats Fig. 2, and the right one is by our proposed approach with the value of γ set to 4%. It again shows that the proposed approach results in a more diversified portfolio with 59.1% reduction in variance while 4.8% reduction in return. In addition, by comparing between Figs. 3 and 4, we also observe that a relatively larger decrease in V_Ω can be achieved with a larger value of the return-risk trade-off parameter γ .

4. An application to stock portfolio selection in the Korean stock market

As an illustration of the proposed approach, we report a case study involving 490–557 firms listed in the Korea Exchange. Using actual financial data from 2001 to 2009, we apply the proposed approach, as a means of fundamental analysis, to select stocks for inclusion in a portfolio. To form the sample, we check the completeness of the financial statements data as well as monthly stock return information⁴ and include only those firms without missing values in the sample.⁵ We exclude non-December year end firms from the sample to control the timing effect of financial information on stock returns. We also leave financial institutions out since their

⁴ We obtained financial data from KIS-VALUE III that is compatible to the U.S. Compustat database and annual return data from Data Guide Pro that is compatible to the CRSP (Center for Research in Security Prices) U.S. stock database.

⁵ We exclude firms with missing values for the financial measures denoted input and output factors in Table 3. However, we include ones with missing stock return data in the sample to avoid exposing the study to both survivorship and look-ahead bias, as concerned by a reviewer of the earlier version of this paper. Although the reason for the unavailability of stock return data is various, we assume that those firms were delisted from the stock exchange and the stock returns are all -100% in the corresponding year.

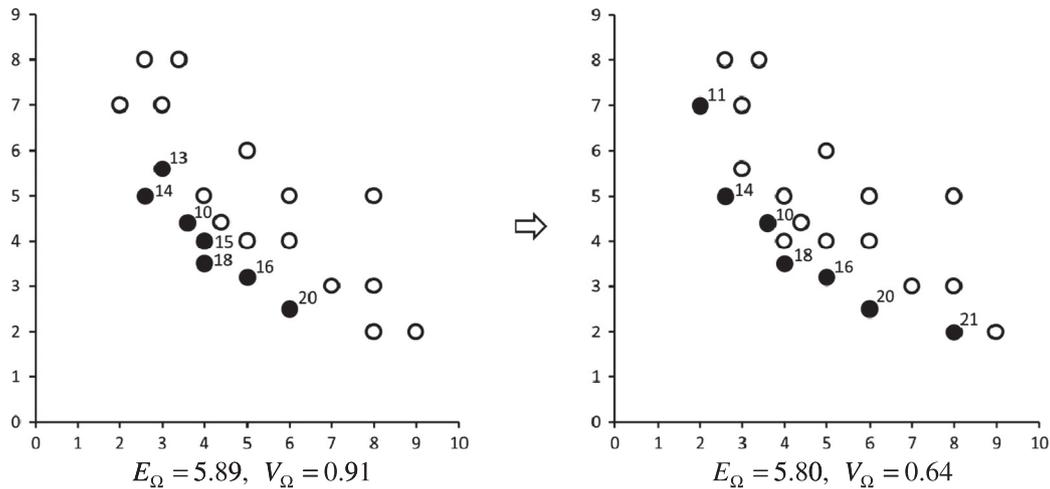


Fig. 3. MV portfolio for data set 1.

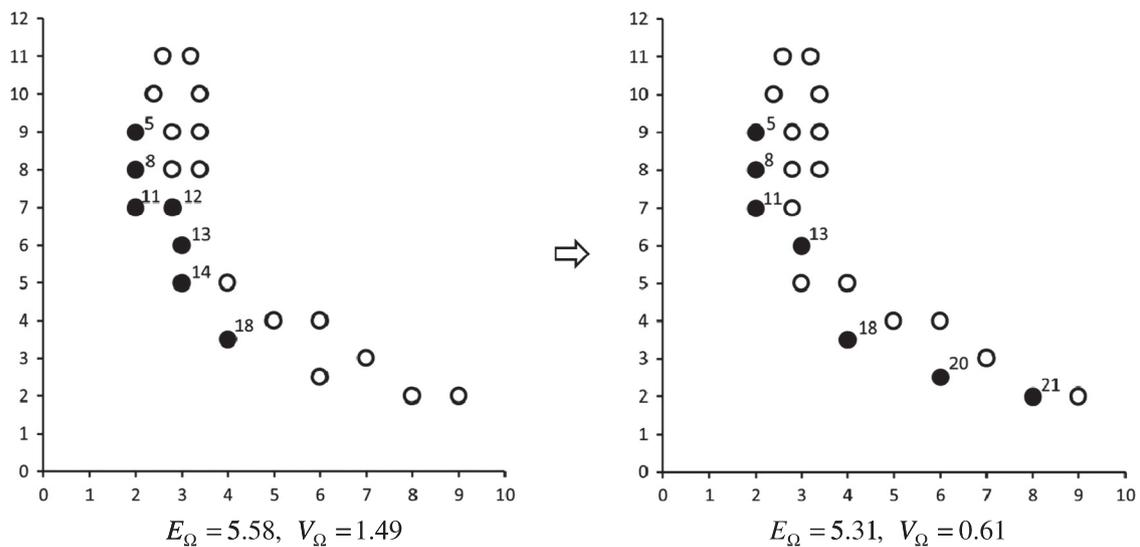


Fig. 4. MV portfolio for data set 2.

accounting standards and regulations are far different from other industries.

4.1. Input and output factors

Following [Edirisinghe and Zhang \(2007\)](#) and [Edirisinghe and Zhang \(2008\)](#), we employ a range of performance perspectives indicating profitability, asset utilization, liquidity, leverage, and growth.⁶ For each of these performance perspectives, we compute a set of financial metrics based on the raw financial data available from the annual financial statements. This selection process results in a total of 16 financial metrics for each firm, as presented in [Table 3](#).

⁶ They also include valuation perspective which is concerned with how well a firm's performance is perceived in the stock markets. However, we exclude it from our consideration since they show that it does not offer a significant predictive power for stock returns in their case study. Furthermore, from the standpoint of DEA, it is not clear if valuation perspective (unlike the other five perspectives) works as inputs or outputs in a firm's operational process. As indicated by a reviewer of the earlier version of the paper, valuation-related financial metrics might add value in predicting stock returns as evidenced by other studies (see e.g., [Pätäri, Leivo, and Honkapuro \(2012\)](#)), but we do not pursue that possibility and focus on showing how the proposed portfolio selection model performs under the input-output selection of [Edirisinghe and Zhang \(2007\)](#) and [Edirisinghe and Zhang \(2008\)](#).

See [Edirisinghe and Zhang \(2007\)](#) for a description of these financial metrics.

We categorize profitability and growth perspectives as outputs because revenue or income generation is a major objective criterion for a firm. On the other hand, we categorize asset utilization, liquidity, and leverage perspectives as inputs because they are relevant to the planning and operational strategies of a firm. We select a set of inputs and outputs from financial metrics derived from raw financial data disclosed in financial statements of listed firms in the Korea Exchange. That is, we choose financial metrics that are generally used in the accounting practice to measure financial performance as inputs and outputs of our DEA model.

4.2. DEA MV cross-efficiency model

Since both the inputs and outputs selected in our case study can have negative values, it is not appropriate to use radial DEA models and/or CRS models. For example, when CRS models are used, data translation required for ensuring positivity changes the efficient frontier. Also, when radial input (output)-oriented models are used, input (output) data translation changes efficiency scores even though the efficiency classification is persevered. Since our

Table 3
Inputs and outputs.

Type	Measure	Aspect
Inputs	Receivable turnover	Asset utilization
	Inventory turnover	
	Asset turnover	
	Current ratio	Liquidity
	Quick ratio	
	Debt to equity ratio	
Outputs	Leverage ratio	Leverage
	Solvency ratio-I	
	Solvency ratio-II	
	Return on equity	Profitability
	Return on assets	
	Net profit margin	
Earnings per Share (EPS)		
Outputs	Revenue growth rate	Growth
	Net income growth rate	
	Earnings per share growth rate	

model directly uses efficiency scores to rank DMUs, we must avoid changes in efficiency scores. As pointed out by Pastor and Ruiz (2007), additive VRS DEA models are a possible alternative for dealing with negative data in both inputs and outputs. Among various additive DEA models, we use the additive VRS DEA model with a range-adjusted inefficiency measure (Cooper, Park, & Pastor, 1999) because it has several desirable properties over the others such as inclusiveness, unit invariance, and translation invariance.

Following is the additive model with a range-adjusted measure (RAM) of inefficiency:

$$\begin{aligned}
 \min \quad & -\frac{1}{m+s}(R^-s^- + R^+s^+) \\
 \text{s.t.} \quad & X\lambda + s^- = x_k, \\
 & Y\lambda - s^+ = y_k, \\
 & e^T\lambda = 1, \\
 & \lambda, s^-, s^+ \geq 0,
 \end{aligned} \tag{4}$$

where $X = (x_{ij}) \in \mathbb{R}^{n \times m}$ and $Y = (y_{rj}) \in \mathbb{R}^{n \times s}$ denote the input and output data matrices, respectively, in which each column represents one of DMUs and each row represents the level of one of factors of the corresponding DMU. R^- and R^+ are defined as:

$$R^- = \left(\frac{1}{R_1^-}, \frac{1}{R_2^-}, \dots, \frac{1}{R_m^-} \right), \quad R^+ = \left(\frac{1}{R_1^+}, \frac{1}{R_2^+}, \dots, \frac{1}{R_s^+} \right),$$

$$R_i^- = \max_{j=1, \dots, n} \{x_{ij}\} - \min_{j=1, \dots, n} \{x_{ij}\}, \quad i = 1, \dots, m,$$

$$R_r^+ = \max_{j=1, \dots, n} \{y_{rj}\} - \min_{j=1, \dots, n} \{y_{rj}\}, \quad r = 1, \dots, s.$$

The dual program to model (4) is as follows:

$$\begin{aligned}
 \max \quad & e_k^d = py_k - qx_k + \zeta \\
 \text{s.t.} \quad & pY - qX + \zeta e \leq 0, \\
 & p \geq \frac{1}{m+s}R^+, \quad q \geq \frac{1}{m+s}R^-.
 \end{aligned} \tag{5}$$

We use the following economic interpretation of model (5). The output and input weight vectors p and q indicate the price of outputs and the cost of inputs, respectively, as discussed in Banker and Maindiratta (1988) and Scheel (2001). Based on their interpretation, we regard py_k and qx_k as revenue and costs, respectively, incurred from the operations of DMU k . Consequently, e_k^d is termed as the ζ -adjusted profit that DMU k attains when the price-cost vector (p, q) is used. The ζ -adjustment is used to make the highest profit among DMUs be equal to zero with an optimal choice of price-cost

vector for the DMU under evaluation; i.e., $\max_j p^*y_j - q^*x_j + \zeta^*e = 0$, where $*$ denotes an optimal solution to the model for the DMU under evaluation.

DMU k is efficient if and only if there exists a positive price-cost vector (p, q) such that $py^k - qx^k \geq py^l - qx^l$ for every observed DMU l . Let (p_k^*, q_k^*) and ζ_k^* be an optimal price-cost vector and an optimal adjustment value for DMU k . An optimal ζ_k^* -adjusted profit of DMU k is $p_k^*y_k - q_k^*x_k + \zeta_k^*$, which is denoted e_{kk}^* . In addition, a ζ_k^* -adjusted profit of DMU l using an optimal price-cost vector that DMU k has chosen in model (5) is $p_k^*y_l - q_k^*x_l + \zeta_k^*$, which is denoted e_{kl}^* . Notice that e_{kk}^* and e_{kl}^* correspond to the concepts of simple efficiency and cross-efficiency in conventional DEA cross-efficiency evaluation. Also note that the higher profit, the better or more efficient a DMU is. We now define the profit vector of DMU k as follows:

$$P_k = (e_{1k}^*, e_{2k}^*, \dots, e_{kk}^*, \dots, e_{nk}^*)^T$$

where e_{ik}^* is an ζ_i^* -adjusted profit of DMU k using an optimal price-cost vector that DMU l has chosen in model (5). Two statistical properties of P_k can be calculated: the average $\bar{P}_k = \frac{1}{n} \sum_{l=1}^n e_{lk}^*$ and the variance $\sigma_k^2 = \frac{1}{n} \sum_{l=1}^n (e_{lk}^* - \bar{P}_k)^2$.

Consider a portfolio Ω with individual DMUs being weighted using a weight vector w . The return and risk characteristics of the portfolio are defined as $E_\Omega = w^T \bar{P}$ and $V_\Omega = w^T \Sigma w$ where Σ is the covariance matrix whose (k, l) th element is the covariance between P_k and P_l . An optimal portfolio Ω^* with an optimal weight w^* can be determined by solving model (3) presented in Section 3. Note that the normalization constraint, $e^T w = 1$ and $w \geq 0$, can be replaced by a cardinality constraint, $e^T w = S$ and $w_i \in \{0, 1\} \forall i$, where S is the portfolio size, which is employed in our case study.

4.3. Portfolio selection strategy

We consider a buy-and-hold strategy, wherein the current year's optimal stock portfolio is selected by solving the model using the prior year's financial metrics and this portfolio is maintained during an investment horizon of 1 year. Whenever we start a new investment horizon, we revise the stock portfolio (a new set of stocks is selected) according to the model solution using the prior year's financial metrics. Each investment horizon is ranged from April of the current year to March of the next year, because all required yearly financial information of December year end firms is publicly available to the equity market in March.

For each investment horizon, we fix the portfolio size S at 30 with each stock selected being equally weighted. More specifically, at the beginning of each investment horizon, 30 stocks are selected for inclusion in a portfolio according to an optimal solution of model (3) with the normalization constraint, $e^T w = 1$ and $w \geq 0$, being replaced by a cardinality constraint, $e^T w = 30$ and $w_i \in \{0, 1\} \forall i$. Once a portfolio selected at the beginning of an investment horizon, the same dollar amount is invested on each stock in the portfolio and no further transaction is made until the end of the horizon when the portfolio is changed. This means that transaction costs are incurred only at both ends of each investment horizon. Actually, stock transaction tax law in Korea⁷ stipulates only 0.15% of stock transaction tax over trading volume which is applied only to the seller. Moreover, individual income tax law⁸ rules out taxation for capital gain from stock transaction of listed companies by individual investors (non-blockholder) with less than 3% of outstanding share of each company. Brokerage commissions levied range from 0.015% to 0.5% of trading volume. These facts imply that our findings will not be significantly altered even if we consider transaction costs. Nevertheless,

⁷ See code #5: Flexible tax rate for stock transaction.

⁸ See code #94: Range of taxation for the capital gain.

Table 4
Annual excess returns (%).

Period	KOSPI200	KOSPI50	MV Cross	Pure Cross
2002	-44.93	-44.87	-35.23	-37.08
2003	65.88	66.11	132.68	129.44
2004	3.48	0.18	63.50	52.43
2005	36.95	34.66	66.18	60.67
2006	1.63	0.48	5.62	-0.55
2007	10.79	5.40	26.12	10.95
2008	-33.13	-32.51	-28.92	-35.94
2009	37.08	38.06	34.65	39.76
2010	22.14	19.32	13.30	11.61
(Geometric) average annual excess return	5.59	4.25	21.84	16.32
Annualized volatility	22.80	22.24	28.50	27.79
Sharpe ratio	0.2452	0.1912	0.7661	0.5872

The highest buy and hold excess return of each year and the highest Sharpe ratio are indicated in bold face.

we actually take transaction costs into consideration when computing monthly returns, assuming that all stock series included in each portfolio are liquidated at the end of each investment horizon. Specifically, we apply 0.26% (average of 0.015% and 0.5%) brokerage commissions of trading volume in our test, along with 0.15% of stock transaction tax over trading volume in liquidation.

We set the risk-return trade-off parameter γ to 2% for the purpose of illustration of our proposed approach, and we do not explore other choices in this paper though it would be worthwhile as a further research to examine the sensitivity of portfolio performance with respect to the parameter choice.

4.4. Portfolio performance

We examine the 9-year performance of the portfolio (labeled 'MV Cross') and compare it to those of two market indices, KOSPI200 and KOSPI50 over the test period from 2002 to 2010. The KOSPI is the index of all common stocks traded in the Stock Market Division of the Korea Exchange. It is the representative stock market index of South Korea, compatible to the Dow Jones Industrial Average or S&P 500 in the U.S. The KOSPI200 constituents are the top 200 firms of the Stock Market Division in terms of market value regardless of industry. The KOSPI200 index is important because it is listed on futures and option markets and is one of the most actively traded indexes in the capital markets. Similarly, the KOSPI50 consists of the top 50 firms which are reselected regularly at delimitation date for KOSPI200 constituent stocks.

Besides, we compare the performance of the portfolio with that of a portfolio selected simply based on cross-efficiency scores (labeled 'Pure Cross'). The 'Pure Cross' portfolio is formed using the traditional use of DEA cross-efficiency evaluation in portfolio selection (as contrasted with the proposed approach), in which firms are ranked in a decreasing order of their cross-efficiency scores and the top 30 firms are chosen. The intent of this comparison is to show that the proposed approach (implemented in the 'MV Cross' portfolio) can possibly provide added value over Edirisinghe and Zhang (2007) and Edirisinghe and Zhang (2008)'s. Note that, in their approach, a firm is considered investment-worthy if its predicted relative financial strength indicator value (represented by its simple efficiency score) is higher than a certain pre-determined value.

Table 4 shows the annual excess return of each portfolio (or index) by each investment horizon. As a proxy for the risk-free return, we use 3-year maturity local currency denominated Korean Treasury bond rates, which range from 3.72% to 5.78% per year over the sample period. The highest buy and hold excess return of each year is indicated in bold face. What is remarkable is that, during 9 years, the 'MV Cross' portfolio yields the highest excess return for 7 years while KOSPI50, KOSPI200, and the 'Pure Cross' do for 0, 1, and 1 year, respectively. As a result, the 'MV Cross' port-

Table 5
Ledoit-Wolf test with Sharpe ratios.

Null hypothesis	Test statistic	p-value
Sharpe ratio (MV Cross) = Sharpe ratio (KOSPI50)	2.259	0.0242
Sharpe ratio (Pure Cross) = Sharpe ratio (KOSPI50)	1.617	0.1100
Sharpe ratio (MV Cross) = Sharpe ratio (Pure Cross)	1.689	0.0994

folio attains the highest (geometric) average annual excess return 21.84%, which is fairly higher than those of KOSPI50 (4.25%), KOSPI200 (5.59%) and the 'Pure Cross' portfolio (16.32%).

We calculate the Sharpe ratio to examine the risk-adjusted performance of the portfolios, which is defined as the (geometric) average annual excess return of a portfolio over the risk-free return divided by the annualized volatility of the excess return. As can be found at the bottom of Table 4, the 'MV Cross' portfolio attains the highest Sharpe ratio (0.7661), which is about 4 and 1.3 times higher than those of KOSPI50 (0.1912) and the 'Pure Cross' portfolio (0.5872), respectively.

To test whether the difference of two Sharpe ratios is statistically significant, we apply the HAC inference and the studentized circular block bootstrap based on Ledoit and Wolf (2008). The advantage of using the Ledoit-Wolf test is the consideration of skewness, kurtosis and autocorrelation effects in statistical performance comparisons based on the Sharpe ratio. Three two-sided hypotheses are formulated to be tested using the Ledoit-Wolf procedure as follows:

H_0 (MV Cross vs. KOSPI50):

Sharpe ratio (MV Cross) – Sharpe ratio (KOSPI50) = 0

H_0 (Pure Cross vs. KOSPI50):

Sharpe ratio (Pure Cross) – Sharpe ratio (KOSPI50) = 0

H_0 (MV Cross vs. Pure Cross):

Sharpe ratio (MV Cross) – Sharpe ratio (Pure Cross) = 0

The reason why we choose KOSPI50 instead of KOSPI200 for comparison is that it has a closer breadth dimension to the proposed portfolio.

We use the MATLAB implementation of Wolf (2009) for the test with the input data of pairs of monthly excess returns⁹. Table 5 summarizes the test statistics obtained from the code using the default parameter setting. As shown in the table, at the significance level of 5%, H_0 (MV Cross vs. KOSPI50) is rejected (p -value: 0.0242) indicating that the 'MV Cross' portfolio's Sharpe ratio is significantly greater than that of KOSPI50. On the other hand, H_0 (Pure Cross vs. KOSPI50) is accepted (p -value: 0.1100) indicating that the

⁹ We report only the annual excess returns in Table 4, but the monthly excess return data are available from the corresponding author upon request.

corresponding two Sharpe ratios are not significantly different. Note that while H_0 (MV Cross vs. Pure Cross) is accepted with the p -value of 0.0994, it can be rejected when we turn to one-sided hypothesis testing indicating that the Sharpe ratio of the 'MV Cross' is (marginally) significantly greater than that of the 'Pure Cross'. Taken together, these findings support our claim that the 'MV Cross' outperforms the 'Pure Cross' as well as the benchmark market index.

The results above demonstrate that the proposed approach can be a promising tool for stock portfolio selection as a means of fundamental analysis. Our results also show that the MV cross-efficiency approach is more effective than the one based on the simple use of cross-efficiency scores at least for this particular application. Overall, our findings consistently support the effectiveness of our approach.

5. Concluding remarks

The current paper has developed a new way of using DEA cross-efficiency evaluation in portfolio selection under the MV framework. This development is motivated by the observation that the traditional simple use of cross-efficiency scores in portfolio selection *per se* suffers from the problem of ill-diversification of resulting portfolios. This poor diversification problem is exacerbated due to the ganging-together phenomenon of DEA cross-efficiency evaluation. We have found that this issue arises because the simple use of cross-efficiency evaluation in portfolio selection fails to consider inter-DMUs risk involved in a portfolio with respect to change in weights (DEA multipliers) although it can effectively reduce individual DMU risk. We have addressed this issue by incorporating DEA cross-efficiency evaluation into the MV framework where these two types of risk are collectively considered.

We have illustrated the proposed approach by applying it to stock portfolio selection in the Korean stock market and showed that the selected portfolio yielded higher risk-adjusted returns over other benchmark portfolios for the 9-year sample period. While this case study empirically supports the effectiveness of the proposed approach for stock portfolio selection, it should be noted that it is only for the purpose of illustration of the proposed approach. We need to perform a more thorough investigation with a wider range of data and various choices of parameter values (S and γ) to fully justify its use for financial applications. Considering the current paper is primarily for a theoretical model development, we would like to leave this subject as a future (more application-oriented) research topic.

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