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A unified approach for the solution of power flows in electric power systems including wind farms

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ABSTRACT

This paper proposes a solution approach of the power flow problem to assess the steady-state condition of power systems with wind farms in a single frame of reference, in which the state variables of the wind generators are combined with the nodal voltage magnitudes and angles of the entire network for a unified iterative solution through the Newton–Raphson method. Different wind energy conversion systems (WECS) are mathematically derived from the steady-state representation of the induction generator. Suitable strategies for initializing the state variables of the wind generators are also proposed in this paper. Lastly, three numerical examples are presented to numerically illustrate the applicability of the proposed approach.

1. Introduction

Over the years, the use of wind energy conversion systems (WECS) has become the most rapidly growing renewable energy source utilized in electricity generation all over the world, such that the power generated by wind turbines accounts for a considerable percentage of the total power production in a number of European countries [1,2]. By the end of 2009, the worldwide wind farm capacity reached almost 160,000 MW and was expected to reach nearly 200,000 MW by the end of 2010 [1]. In the case of México, a total wind capacity of 590 MW will be exceeded within the year 2012 representing barely 1% over the total generation capacity [3]. However, recent studies have shown that Mexico's wind energy potential could reach more than 7 GW [4].

Even though wind energy is beneficial from the environmental standpoint, it makes the already complex task of achieving system controllability even more demanding. Consequently, the quantification of the effects that large-scale integration of wind generation will cause on the network is a very important matter that requires special attention when planning and operating an electrical power system. Arguably, power flow analysis is the most frequently performed computational calculation in a power system's planning and operation, and this study has been selected in this paper to quantify the electrical response of wind generators.

Mathematical models of several types of wind generators have been developed in which their active and reactive power outputs are obtained based on the steady-state equivalent representation of the induction machine [5–8]. The power injection method is then used to include these models into the power flow formulation, which is solved by using a sequential approach to obtain an operating point of the power system. In this approach, only the network's state variables are calculated through a conventional power flow algorithm, while a subproblem is formulated for updating the state variables of wind generators as well as their power injections at the end of each power flow's iteration.

In this context, an injection power flow model of fixed-speed wind generators is proposed in [5–7]. In [5], the active power is obtained from the power curve and the reactive power output is estimated by using an approximated quadratic function dependent on the active power and the voltage magnitude measured at the wind generator's terminals. This model is referred to as the PQ model. On the other hand, the active and reactive power outputs of the models proposed in [6,7] are completely expressed in terms of the generators' variables and parameters without using approximations.

The power flow analysis of variable-speed wind turbines based on doubly-fed induction generators has also been an object of study. A doubly-fed induction generator (DFIG) model for power flow studies was suggested in [6] in which the total rotor current magnitude is checked at each iteration. If there is a limit violation of

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this current, the wind generator gives priority to the reactive over the active power in order to support the bus voltage magnitude at which the wind farm is connected. From the power flow point of view, this model is treated as a PQ bus with a fixed unity power factor, i.e. the active power output is obtained from the wind generator power curve, and the reactive power is set to zero.

Instead of using the power injection concept, another way of representing a fixed-speed wind generator is by means of an equivalent variable impedance expressed in terms of the slip of the generator and its rotor and stator winding parameters [5,8]. This impedance is included in the system's admittance matrix, and the network nodal voltages are computed through the power flow analysis. Based on these voltages, the air-gap power of the wind generator is calculated, and the value of the slip of the induction generator is then computed iteratively to match the air-gap power and mechanical power extracted from the wind [5]. Following the same modeling idea, the iterative process to compute the slip can be avoided by assuming that the mechanical power is known [8].

In general terms, all the methods discussed above share the characteristic of using a sequential approach to calculate the state variables of the wind generators. This sequential iterative approach is rather attractive because it is straightforward to implement in existing power flow programs, but caution has to be exercised because it will yield no quadratic convergence [9,10], and an additional set of nonlinear algebraic equations have to be solved to obtain the values of the wind generator's state variables [5–8].

A fundamentally different approach for the modeling of WECS, within the context of the power flow problem, is a method that simultaneously combines the state variables corresponding to the wind generators and the network in a single frame-of-reference for a unified iterative solution through a Newton–Raphson (NR) technique. From the computational effort standpoint, i.e. the number of iterations required to obtain the power flow solution, this method is superior to the sequential one because all state variables are simultaneously adjusted during the iterative process. Furthermore, it arrives at the solution with local quadratic convergence regardless of the network size if proper initial conditions are selected and the Jacobian matrix is nonsingular at the solution point [11 pp. 310–318, 12 pp. 220–222]. Hence, the key contribution of this work is to provide a comprehensive and general approach for the analysis of power flows in electric power systems containing WECS in a unified single-frame of reference, as it was initially suggested in [13].

The proposed approach is described in detail in the rest of the paper as follows: Section 2 addresses the mathematical representation of various types of directly grid-connected wind generators such as fixed-speed wind generators (FSWG) and semi-variable speed wind generators (SSWG) as well as their inclusion in the proposed power flow solution approach. In addition, the Newton-based formulation of variable-speed wind generators based on DFIG (VSWG-DFIG) is also shown in this section, taking into account that the generators can operate in a wide range of a power factor, i.e. 0.95 leading/lagging. This aspect is of paramount importance because as WECS are gaining prominence in the power industry and grid code compliance is becoming mandatory, wind farms have to be able to supply the system with reactive power so as to contribute with voltage support [14–17]. Section 3 describes the strategies for the initialization of the state variables of wind generators. Three study cases are then presented in Section 4, and finally Section 5 points out the conclusions of this work.

2. Mathematical modeling of wind generators for power flow analysis

The unified approach proposed in this paper combines equations representing the active and reactive power balance at each node of the electrical network and at each wind generator into one set of nonlinear algebraic equations $f(X)$ with unknown variables given by $X=[Xn_{AC},X_{WF}]$, where Xn_{AC} is a vector of all nodal voltage angles and magnitudes, and X_{WF} is a vector of all state variables associated with the wind generators. The NR method is then used to provide an approximate solution to this set of equations given by $f(X)=0$, by solving for ΔX_i in the linear problem $J_i\,\Delta X_i=-f(X_i)$, where J is known as the Jacobian matrix [9]. The NR method starts from an initial guess for the X_0 and updates the solution at each iteration i, i.e. $X_{i+1}=X_i+\Delta X_i$, until a pre-defined tolerance is fulfilled. In this unified solution, the state variables X_{WF} of the wind farms are adjusted simultaneously with the AC system state variables in order to compute the steady-state operating condition of the power system. Hence, the proposed method retains Newton's quadratic convergence characteristics.

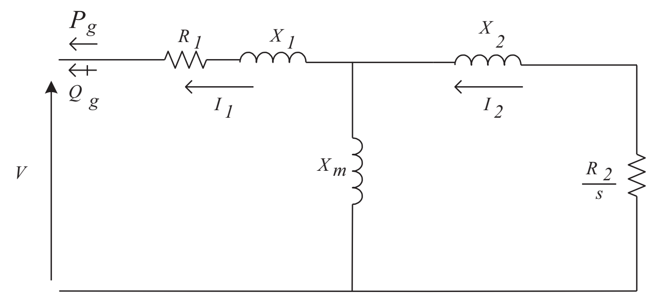

Fig. 1. Steady-state equivalent model of the induction machine.

2.1. Modeling of FSWG

The concept of this machine is based on a squirrel-cage asynchronous generator, which is driven by a wind turbine with its stator directly connected to the grid through a power transformer. Since the speed is almost fixed to the grid frequency and is not controllable, this asynchronous machine is considered a FSWG. The mathematical model suitable for power flow analysis is derived from the steady-state equivalent model of the induction machine shown in Fig. 1, where the subscripts 1 and 2 represent stator and rotor variables, respectively, and m symbolizes the magnetization branch, R is resistance and X represents a reactance, I is electric current, s is the slip of the induction generator, V is the terminal voltage, and P_g and Q_g are the active and reactive powers generated by the induction machine, respectively. Furthermore, power factor correction capacitors (fixed capacitors) are installed at each wind generator.

When the generator mode is adopted, the power converted from mechanical to electrical form (P_{conv}) can be computed as follows [18]:

$$P_{\text{conv}} = -I_2^2 R_2 \left(\frac{1-s}{s}\right) \tag{1}$$

where I_2 is the rotor current, R_2 is the rotor resistance and s is the slip of the induction generator. Furthermore, the active power (P_g), the reactive power (Q_g), the squared rotor current (I_2^2) and the squared stator current (I_1^2) are dependent on the machine's slip (s) and the terminal voltage (V), and can be determined by Eqs. (2)–(5), which are derived in Appendix A:

$$P_g(V,s) = -V^2 \left\{ \frac{K+Hs+Ls^2}{[C-Ds]^2+[E+Fs]^2} \right\} \tag{2}$$

$$Q_g(V,s) = -V^2 \left\{ \frac{A+Bs^2}{[C-Ds]^2+[E+Fs]^2} \right\} \tag{3}$$

$$I_2^2(V,s) = V^2 \left\{ \frac{[M\,s + N\,s^2]^2 + [T\,s - W\,s^2]^2}{([C - D\,s]^2 + [E + F\,s]^2)^2} \right\} \quad (4)$$

$$I_1^2(V,s) = V^2 \left\{ \frac{[K + H\,s + L\,s^2]^2 + [A + B\,s^2]^2}{\left([C - D\,s]^2 + [E + F\,s]^2\right)^2} \right\} \quad (5)$$

where the variables are defined as

$$\begin{aligned} &A = R_2^2(X_1 + X_m),\ B = (X_2 + X_m)[X_2X_m + X_1(X_2 + X_m)],\ C = R_1R_2,\\ &D = X_2X_m + X_1(X_2 + X_m),\ E = R_2(X_1 + X_m),\ F = R_1(X_2 + X_m),\\ &H = R_2X_m^2,\ K = R_1R_2^2,\ L = R_1(X_2 + X_m)^2,\ M = X_mR_2(X_1 + X_m),\\ &N = X_mR_1(X_2 + X_m),\ T = R_1R_2X_m,\ W = X_m[X_2X_m + X_1(X_2 + X_m)]. \end{aligned}$$

The mechanical power captured from the wind is limited as wind speed passes the rated wind speed of the turbine. Hence, the active power mismatch equation associated with the conversion process of mechanical power into electrical power, which is required in the power flow formulation, depends on the method used to control the rotor speed. The passive stall and pitch controls are addressed in this paper.

2.1.1. Stall-regulated FSWG

A stall-regulated FSWG (SR-FSWG) has the machine rotor blades bolted onto the hubs at a fixed attack angle. The mechanical power P_m [W] extracted from the wind by this generator is given by [19]

$$P_m = 0.5\rho c_1 \left(\frac{c_2}{\lambda_i} - c_3\beta - c_4\beta^{c_5} - c_6\right) \cdot e^{-c_7/\lambda_i} A\, V_w^3 \quad (6)$$

where

$$\lambda_i = \left[\left(\frac{1}{\lambda + c_8\,\beta}\right) - \left(\frac{c_9}{\beta^3 + 1}\right)\right]^{-1} \quad (7)$$

$$\lambda = \frac{R\,\omega_T}{V_w} = \frac{R\,n_{ng}\,\omega_S(1 - s)}{V_w} \quad (8)$$

and ρ is the air density [kg/m^3], A is the swept area of the blades [m^2], V_w is the wind speed [m/s], R is the radius of the rotor [m], n_{gb} is the gearbox ratio, ω_s is the angular synchronous speed [rad/s], s is the slip of the induction generator, β is the pitch angle [degrees], ω_T is the angular speed of the turbine [rad/s], and the constants c_1 to c_9 are the parameters of design of the wind turbine. Eqs. (6)–(8) demonstrate that P_m is only a function of the slip of the induction generator, since β is omitted. Hence, the slip of the generator is considered as the state variable in the NR algorithm to find the "internal equilibrium point" of the wind generator given by the conversion process of mechanical power into electrical power.

When the SR-FSWG is connected at terminal k of the system, the set of mismatch power flow equations is

$$\Delta P_k = P_g(V,s) - P_{Lk} - P_k^{\text{cal}} = 0 \quad (9)$$

$$\Delta Q_k = Q_g(V,s) - Q_{Lk} - Q_k^{\text{cal}} = 0 \quad (10)$$

$$\Delta P_{WT1,k} = -\{P_m - P_{\text{conv}}\} = -\left\{P_m + I_2^2 R_2 \left(\frac{1-s}{s}\right)\right\} = 0 \quad (11)$$

where $P_g(V,s)$ and $Q_g(V,s)$ are given by (2) and (3), respectively, P_{LK} and Q_{LK} represent the active and reactive powers drawn by the load at bus k, respectively, and P_k^{cal} and Q_k^{cal} are active and reactive power injections given by

$$P_k^{\text{cal}} = V_k^2\ G_{kk} + V_k \sum_{m \in k} V_m[G_{km}\cos(\theta_k - \theta_m) + B_{km}\sin(\theta_k - \theta_m)] \quad (12)$$

$$Q_k^{\text{cal}} = -V_k^2\,B_{kk} + V_k \sum_{m \in k} V_m[G_{km}\sin(\theta_k - \theta_m) - B_{km}\cos(\theta_k - \theta_m)] \quad (13)$$

Once Eqs. (9)–(11) have been defined, the set of linearized power mismatch equations given by (14) must be assembled and combined with the Jacobian matrix J, the power mismatch vector $f(X)$, and the vector of incremental changes in state variables ΔX of the entire network for a unified solution of the voltage magnitudes V and angles θ associated with the network and the wind generator's slip.

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \\ \Delta P_{WT1,k} \end{bmatrix}^j = \begin{bmatrix} \dfrac{\partial P_k^{\text{cal}}}{\partial \theta_k} & \left(\dfrac{\partial P_k^{\text{cal}}}{\partial V_k} - \dfrac{\partial P_g}{\partial V_k}\right) V_k & \dfrac{\partial P_g}{\partial s} \\ \dfrac{\partial Q_k^{\text{cal}}}{\partial \theta_k} & \left(\dfrac{\partial Q_k^{\text{cal}}}{\partial V_k} - \dfrac{\partial Q_g}{\partial V_k}\right) V_k & \dfrac{\partial Q_g}{\partial s} \\ 0 & \dfrac{\partial P_{WT1,k}}{\partial V_k} V_k & \dfrac{\partial P_{WT1,k}}{\partial s} \end{bmatrix}^j \begin{bmatrix} \Delta\theta_k \\ \dfrac{\Delta V_k}{V_k} \\ \Delta s \end{bmatrix}^j \quad (14)$$

2.1.2. Pitch-regulated FSWG

A pitch-regulated FSWG (PR-FSWG) is usually operated at a fixed pitch below rated wind speed. In cases where the wind speed is above its rated value, the power extracted from the wind is limited by adjusting the pitch angle mechanism, and a rated power output is achieved for any given wind speed [20]. In other words, this wind generator has a certain capacity of active power control as opposed to a SR-FSWG. Therefore, the generated active power $P_{g,pr}$, which can be obtained for any given wind speed from the wind generator power curve provided by the manufacturer, is constant through the iterative process. On the other hand, the reactive power $Q_{g,pr} = Q_g$ needs to be calculated as in the case of the SR-FSWG.

By neglecting the core losses in the induction machine, the mechanical power for a PR-FSWG is

$$P_{m,pr} = P_{g,pr} + P_{\text{losses},s} + P_{\text{losses},r} = P_{g,pr} + 3I_1^2R_1 + 3I_2^2R_2 \quad (15)$$

where $P_{\text{losses},s}$ and $P_{\text{losses},r}$ are the three-phase stator and rotor power losses, respectively.

Based on Eqs. (4), (5) and (15), and assuming that the PR-FSWG is connected at node k, the values of the state variables that satisfy the mismatch equations (16)–(18) are obtained by superimposing the set of linear equations (19) to the entire set of the network's linearized equations and solving the resulting set of equations iteratively. In this case, Eq. (18) represents the power balance inside the induction machine

$$\Delta P_k = P_{g,pr} - P_{Lk} - P_k^{\text{cal}} = 0 \quad (16)$$

$$\Delta Q_k = Q_g(V,s) - Q_{Lk} - Q_k^{\text{cal}} = 0 \quad (17)$$

$$\Delta P_{WT2,k} = -P_{g,pr} - \left\{(3I_1^2R_1 + 3I_2^2R_2) + I_2^2R_2\left(\frac{1-s}{s}\right)\right\} = 0 \quad (18)$$

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \\ \Delta P_{WT2,k} \end{bmatrix}^j = \begin{bmatrix} \dfrac{\partial P_k^{\text{cal}}}{\partial \theta_k} & \dfrac{\partial P_k^{\text{cal}}}{\partial V_k} V_k & 0 \\ \dfrac{\partial Q_k^{\text{cal}}}{\partial \theta_k} & \left(\dfrac{\partial Q_k^{\text{cal}}}{\partial V_k} - \dfrac{\partial Q_g}{\partial V_k}\right) V_k & \dfrac{\partial Q_g}{\partial s} \\ 0 & \dfrac{\partial P_{WT2,k}}{\partial V_k} V_k & \dfrac{\partial P_{WT2,k}}{\partial s} \end{bmatrix}^j \begin{bmatrix} \Delta\theta_k \\ \dfrac{\Delta V_k}{V_k} \\ \Delta s \end{bmatrix}^j \quad (19)$$

2.2. Modeling of SSWG

This wind generator is equipped with a wound-rotor induction generator (WRIG) and a diode bridge that electronically controls an external resistance added in the rotor circuit, i.e. R_{ext} [21], in such a way that resistance adjustment allows for the maintenance of the generated active power at a specified value. Also, the turbine is constructed with a pitch angle mechanism that enables the system not only to boost the energy captured from the wind, but to achieve an efficient power regulation above the rated one, thus mitigating the mechanical stress and transient loads of the mechanical components of the wind generator.

In this case, s cannot be regarded as the single state variable because the resistor connected to the rotor circuit is also adjusted. In order to overcome this inconvenience, the total resistance of the rotor circuit, $(R_2+R_{\text{ext}})/s$, can be considered a single unknown variable, R_x [6]. Hence, stator and rotor currents as well as active and reactive powers will be dependent functions on R_x, and can be determined by Eqs. (20)–(23), whose derivation is similar to that of Eqs. (2)–(5) but considering R_x instead of R_2/s.

$$Q_{g,ss}(V,R_x) = -V^2\left\{\frac{A'R_x^2+B}{[C'R_x-D]^2+[E'R_x+F]^2}\right\} \tag{20}$$

$$P_{g,ss}(V,R_x) = -V^2\left\{\frac{K'{R_x}^2+H'R_x+L}{[C'R_x-D]^2+[E'R_x+F]^2}\right\} \tag{21}$$

$$I_{1,ss}^2(V,R_x) = V^2\left\{\frac{[K'R_x^2+H'R_x+L]^2+[A'R_x^2+B]^2}{([C'R_x-D]^2+[E'R_x+F]^2)^2}\right\} \tag{22}$$

$$I_{2,ss}^2(V,R_x) = V^2\left\{\frac{[M'R_x+N]^2+[T'R_x-W]^2}{([C'R_x-D]^2+[E'R_x+F]^2)^2}\right\} \tag{23}$$

where $A'=(X_1+X_m)$, $C'=R_1$, $E'=(X_1+X_m)$, $H'=X_m^2$, $K'=R_1$, $M'=X_m(X_1+X_m)$, $T'=R_1X_m$.

For the purpose of power flow calculation, the reactive power output $Q_{g,ss}$ of the SSWG is computed iteratively, while its active power $P_{g,ss}$, is set to a fixed value obtained from the wind generator power curve. Consequently, assuming no core losses in the induction machine, the mechanical power $P_{m,ss}$, is computed by

$$P_{m,ss} = P_{g,ss}^{sp} + P_{\text{losses},s} + P_{\text{losses},r} = P_{g,ss}^{sp} + 3I_{1,ss}^2R_1 + 3I_{2,ss}^2R_2 \tag{24}$$

where $P_{\text{losses},s}$ and $P_{\text{losses},r}$ are the three-phase stator and rotor power losses, respectively. The slip's value approaches zero as the generator is closer to its rated operation; hence, the converted power P_{conv}, and the active power mismatch associated with the internal energy balance, ΔP_{WT3}, is

$$P_{\text{conv}} = -I_{2,ss}^2R_2\left(\frac{1-s}{s}\right) \approx -I_{2,ss}^2R_2\left(\frac{1}{s}\right) = -I_{2,ss}^2R_x \tag{25}$$

$$\begin{aligned}\Delta P_{WT3} &= -P_{g,ss}^{sp} - \{(3I_{1,ss}^2R_1+3I_{2,ss}^2R_2)+I_{2,ss}^2R_x\}\\ &\approx -P_{g,ss}^{sp} - \{3I_{1,ss}^2R_1+I_{2,ss}^2R_x\}\end{aligned} \tag{26}$$

Based on Eqs. (25) and (26) and assuming that the wind generator is connected at node k of the power system, the Newton-based power flow formulation is given by

$$\Delta P_k = P_{g,ss}^{sp} - P_{Lk} - P_k^{\text{cal}} = 0 \tag{27}$$

$$\Delta Q_k = Q_{g,ss}(V,R_x) - Q_{Lk} - Q_k^{\text{cal}} = 0 \tag{28}$$

$$\Delta P_{WT3,k} = -P_{g,ss}^{sp} - \{3I_{1,ss}^2R_1+I_{2,ss}^2R_x\} = 0 \tag{29}$$

$$\begin{bmatrix}\Delta P_k\\ \Delta Q_k\\ \Delta P_{WT3,k}\end{bmatrix}^j = \begin{bmatrix}\dfrac{\partial P_k^{\text{cal}}}{\partial\theta_k} & \dfrac{\partial P_k^{\text{cal}}}{\partial V_k}V_k & 0\\ \dfrac{\partial Q_k^{\text{cal}}}{\partial\theta_k} & \left(\dfrac{\partial Q_k^{\text{cal}}}{\partial V_k}-\dfrac{\partial Q_{g,ss}}{\partial V_k}\right)V_k & \dfrac{\partial Q_{g,ss}}{\partial R_x}\\ 0 & \dfrac{\partial P_{WT3,k}}{\partial V_k}V_k & \dfrac{\partial P_{WT3,k}}{\partial R_x}\end{bmatrix}^j \begin{bmatrix}\Delta\theta_k\\ \dfrac{\Delta V_k}{V_k}\\ \Delta R_x\end{bmatrix}^j \tag{30}$$

Accordingly, the state variable R_x is updated according to $R_x^{j+1} = R_x^j + \Delta R_x^j$ after each iteration.

Table 1
Initializations for PR-FSWG and SSWG.

PR-FSWG (Eq. (34))	SSWG (Eq. (35))
$s^{(0)} = \min\left\lvert\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right\rvert$	$R_x^{(0)} = \min\left\lvert\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right\rvert$
$a=(D^2+F^2)P_g+LV^2$	$a=(C'^2+E'^2)P_g+K'V^2$
$b=-2CDP_g+2EFP_g+HV^2$	$b=-2C'DP_g+2E'FP_g+H'V^2$
$c=(C^2+E^2)P_g+KV^2$	$c=(D^2+F^2)P_g+LV^2$

2.3. Modeling of VSWG-DFIG

Currently, this is the most popular scheme of wind turbines used for wind power extraction, where the stator is directly connected to the grid through a power transformer, while a back-to-back (B-B) converter is used to connect the rotor of the generator to the power transformer and grid. An advantage of this system is that the converter enables a decoupling between the electrical frequency of the system and the mechanical frequency of the rotor. This decoupled control permits the separate handling of the active and reactive powers in the DFIG.

In the case of active power control, the decoupled d–q vector control is used to ensure a maximum energy capture when the wind speed is below the rated value, which is referred to as maximum power tracking. On the other hand, if the wind speed is above nominal speed, a blade-pitch angle control acts to limit the amount of active power injected into the network, preventing the wind turbine from suffering mechanical damages. Regarding the reactive power control, the B-B converter is operated according to the reactive power control mode set in the DFIG, which is usually operated at a fixed power factor.

Based on the mentioned above control operation, this wind generator can be treated in the power flow formulation as a PQ node with a fixed power factor [6,7], where P_g is the generated active power obtained from the power curve, and Q_g is the reactive power output computed as $Q_g=P_g\tan(\varphi)$, where φ is the power factor angle. Therefore, if the VSWG-DFIG is connected at node k, the linearized set of power mismatch equations that must be included in the entire set of linearized equations to be solved iteratively are given by

$$\Delta P_k = P_g - P_{Lk} - P_k^{\text{cal}} = 0 \tag{31}$$

$$\Delta Q_k = Q_g - Q_{Lk} - Q_k^{\text{cal}} = 0 \tag{32}$$

$$\begin{bmatrix}\Delta P_k\\ \Delta Q_k\end{bmatrix}^j = \begin{bmatrix}\dfrac{\partial P_k^{\text{cal}}}{\partial\theta_k} & \dfrac{\partial P_k^{\text{cal}}}{\partial V_k}V_k\\ \dfrac{\partial Q_k^{\text{cal}}}{\partial\theta_k} & \dfrac{\partial Q_k^{\text{cal}}}{\partial V_k}V_k\end{bmatrix}^j \begin{bmatrix}\Delta\theta_k\\ \dfrac{\Delta V_k}{V_k}\end{bmatrix}^j \tag{33}$$

3. Initialization of wind generators

As is commonly understood, Newton's quadratic convergence may be affected if inadequate initial conditions are chosen for the state variables to be solved. In this context, proposals to initialize the state variables of wind generators are described in this section in order to maintain the quadratic convergence of the Newton algorithm.

In the case of SR-FSWG, a good initial value to execute simulations is given by $s^{(0)}=s_{\text{nom}}/2$, which implies starting approximately at the middle of the interval of mechanical speed values that the wind generator could experience, i.e. from low to high wind speeds.

Regarding PR-FSWG and SSWG, the initializations for the NR algorithm may be estimated by Eqs. (34) and (35) reported in Table 1. These equations are obtained by solving (3) and (21) for

Table 2
Comparison between proposed formulation and formulation from [6].

	WECS	P_g (MW)	Q_g (MVAr)	V (p.u)	s	R_x	Iterations
Formulation from [6]	I. SR-FSWG	2.9803	−0.7380	1.05181	−0.00263	–	8
	II. PR-FSWG	1.7985	−0.0271	1.05809	−0.00272	–	
	III. SSWG	2.9545	−0.6964	1.05685	–	−164.46175	
Proposed formulation	I. SR-FSWG	2.9882	−0.7399	1.05181	−0.00264	–	4
	II. PR-FSWG	1.7985	−0.0271	1.05809	−0.00272	–	
	III. SSWG	2.9545	−0.7026	1.05684	–	−163.30541	

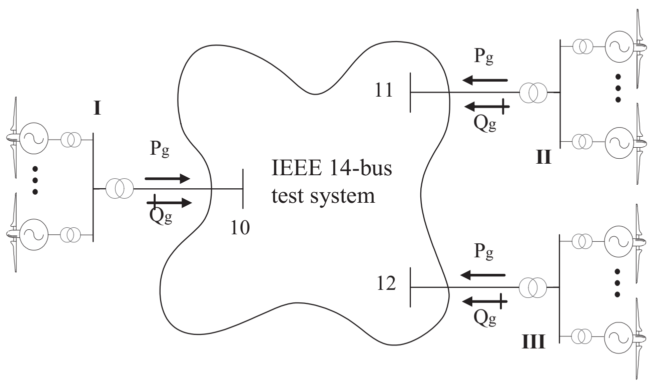

Fig. 2. Test system used for comparison.

s and R_x, respectively, assuming that the active power output can be obtained from the power curve for a given wind speed.

4. Case studies with wind farms

4.1. Example I

A typical test system is used in order to validate the proposed formulation. A comparison is carried out with the algorithms presented in [6] and the approach employed in this paper. The mathematical models of wind generators are implemented in a NR algorithm for solving power flows with a convergence criterion of 10^{-12}. The IEEE 14-bus test system [22] including three wind farms comprising five wind generators each is depicted in Fig. 2. The comparison is effectuated assuming that wind speed is 10 m/s at all wind farms. The additional information regarding parameters of wind generators can be found in Appendix B.

The computed state variables associated with wind generators are quite similar in both methods, as reported in Table 2, causing the generated active and reactive powers as well as the voltage magnitudes at the point of interconnection of the wind farms to be practically equal. Furthermore, the proposed formulation has a quadratic convergence to a very stringent tolerance, which is not the case for the sequential approach proposed in [6].

Table 3
Models of wind generators connected to IEEE 30-bus test system.

Model	Number of WG	P_{rated} (MW)	$Q_{compensation}$	WECS total capacity
I. SR-FSWG	20	0.9	30% of P_n	18 MW
II. PR-FSWG	30	0.6	30% of P_n	18 MW
III. SSWG	20	1.0	30% of P_n	20 MW
IV.VSWG-DFIG	10	2.0	±0.95 pf	20 MW

4.2. Example II

The standard IEEE 30-bus test system [22] was slightly modified to incorporate different models of wind farms connected (one at a time) at node 30 (see Table 3) through a step-up transformer (rated 25 MVA) with a reactance of 0.08 pu on its own base. In addition, the reactive power limits of the synchronous generators are not considered in the simulations. The parameters for the wind generators can be found in Appendix B.

For the WECS described in Table 3, the same wind speed is assumed for all wind farms. The convergence criterion in the NR algorithm for power flow mismatches is assumed to be 10^{-12}. The outcome of the power flow simulations for different wind conditions is shown in Table 4. The simulations indicate that the principles of the operation of wind generators are consistent, i.e. the higher the wind speed, the higher the active power generated by the wind farms. Furthermore, clearly regardless of the value of the wind speed, the algorithm retains quadratic convergence since four iterations were needed in most cases; therefore, this demonstrates that the proposed solution method efficiently determines the impact of the inclusion of WECS in a power system.

4.3. Example III

In this case, a more complex system has been used to show the performance of the proposed algorithm. The IEEE 118-bus test system [22] has been modified to consider four different kinds of wind farms connected to four buses as described in Table 5, with 80 wind

Table 4
Results with WECS connected to node 30 of the IEEE 30-bus test system.

Case	Type of WECS	P_g (MW)	Q_g (MVAR)	V (pu)	s	R_x	Iterations
1. V_w = 8 m/s	I	6.409	−1.933	0.997	−0.00158	–	4
	II	5.787	0.894	1.015	−0.00155	–	4
	III	7.887	−1.792	1.002	–	−221.768	4
	IV (1 pf)	5.556	−0.225	1.007	–	–	4
2. V_w = 10 m/s	I	11.953	−3.337	1.002	−0.00296	–	4
	II	10.791	−0.761	1.017	−0.00297	–	4
	III	11.818	−3.406	1.001	–	−143.1047	4
	IV (0.98 pf)	12.000	2.575	1.045	–	–	4
3. V_w = 12 m/s	I	16.079	−4.990	1.001	−0.00408	–	4
	II	15.304	−3.455	0.996	−0.00452	–	4
	III	17.000	−7.127	0.987	–	−88.990	5
	IV (0.96 pf)	16.888	3.871	1.061	–	–	5

Table 5
WECS connected to IEEE 118-bus test system.

Model	Number of WG	Connection bus	WECS capacity
I. SR-FSWG	20	38	18 MW
II. PR-FSWG	30	81	18 MW
III. SSWG	20	102	20 MW
IV. VSWG-DFIG (1 pf)	10	109	20 MW

Table 6
Power flow results with four wind farms connected to the IEEE 118-bus test system.

WECS		Wind speed (m/s)			
Bus no.	Results	–	11	13	15
38	P_g (MW)	0.0000	14.262	17.361	18.318
	Q_g (MVAR)	0.0000	−4.345	−5.925	−6.495
	V (pu)	0.9613	0.9609	0.9606	0.9605
81	P_g (MW)	0.0000	13.500	17.109	18.762
	Q_g (MVAR)	0.0000	−2.414	−5.175	−6.845
	V (pu)	0.9968	0.9967	0.9963	0.9961
102	P_g (MW)	0.0000	14.600	18.400	19.800
	Q_g (MVAR)	0.0000	−5.189	−8.535	−10.20
	V (pu)	0.9891	0.9882	0.9870	0.9864
109	P_g (MW)	0.0000	14.666	18.666	20.000
	Q_g (MVAR)	0.0000	−1.711	−2.795	−3.220
	V (pu)	0.9670	0.9687	0.9689	0.9689
	Iterations	5	5	5	5

generators in total. Likewise, the same assumptions described in Example II have been considered in performing the simulations whose results are reported in Table 6. The third column reports the results associated with the base case: wind farms are not connected to the system. Results obtained for wind speeds of 11 m/s, 13 m/s, and 15 m/s are detailed in columns 4, 5, and 6, respectively. The same number of iterations was required for all simulations.

5. Conclusions

This work has presented the mathematical models of several types of wind generators and their inclusion in a NR-based power flow algorithm suitable for the power flow analysis of electric power networks with a large number of wind generators. Guidelines for the initializations of the state variables associated with different types of wind generators have been provided. The algorithm's efficiency has been illustrated by numerical examples, demonstrating that the proposal retains the quadratic convergence of the Newton's method. In particular, the superiority of the unified approach over the sequential method has been clearly shown in terms of the number of iterations required to compute the power flow solution.

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Appendix A.

Based on the steady-state equivalent machine of the induction machine shown in Fig. 1, the total equivalent impedance seen from machine's terminals is $Z_{eq} = Z_a + R_1 + jX_1$, where Z_a corresponds to the equivalent impedance between the rotor and the magnetizing branch and is given by

$$Z_a = \frac{jX_m((R_2/s) + jX_2)}{(R_2/s) + j(X_2 + X_m)} \tag{A.1}$$

Table A.1
Power curve for the PR-FSWG.

Wind speed (m/s)	Active power (kW)
5	37.5
7	116.4
9	269.4
11	450.0
13	570.3
15	625.4
17	635.0

The stator current is then determined as,

$$I_1 = \frac{V}{Z_{eq}} = \frac{[(R_2/s) + j(X_2 + X_m)]V}{R_1(R_2/s) - X_m X_2 - X_1(X_2 + X_m) + j[(R_2/s)(X_1 + X_m) + R_1(X_2 + X_m)]} \tag{A.2}$$

which is squared and shown in rearranged form in Eq. (5). Having defined the stator current, we can now proceed to calculate the active and reactive powers generated by the induction generator as $P_g = -\mathrm{Re}\left\{VI_1^*\right\}$ and $Q_g = -\mathrm{Im}\left\{VI_1^*\right\}$. The resulting expressions of powers are given by Eqs. (2) and (3). Lastly, the rotor's current of the induction machine can be found by applying the Kirchhoff's current law and is given by

$$I_2 = I_1\left(\frac{jX_m}{(R_2/s) + j(X_2 + X_m)}\right) \tag{A.3}$$

which is squared and presented in Eq. (4).

Appendix B.

The parameters of each wind generator model are given next.

(1) SR-FSWG (same as [23]):

Generator data: stator impedance is 0.0027 + j0.025 Ω; rotor impedance is 0.0022 + j0.046 Ω; magnetizing reactance is j1.38 Ω; rated voltage is 690 V. Wind turbine data: rated power is 900 kW; rotor diameter 57 m; gearbox ratio is 65.27; air density is 1.225 kg/m^3. The coefficients of Eqs. (6) and (7) are: $c_1 = 0.5$; $c_2 = 116$; $c_3 = 0.4$; $c_4 = 0.0$; $c_5 = 0$; $c_6 = 5$; $c_7 = 21$; $c_8 = 0.08$; $c_9 = 0.035$; $\beta = 0$.

(2) PR-FSWG (same as [7]):

Generator data: stator impedance is 0.0 + j0.09985 Ω; rotor impedance is 0.00373 + j0.10906 Ω; magnetizing reactance is j3.54708 Ω; rated voltage is 690 V. Wind turbine data: rated power is 600 kW. The wind generator power curve is shown in Table A.1.

(3) SSWG (same as [6]):

Generator data: stator impedance is 0.00269 + j0.072605 Ω; rotor impedance is 0.002199 + j0.04599 Ω; magnetizing reactance is j1.37997 Ω; rated voltage is 690 V. Wind turbine data: rated power is 1000 kW. The wind generator power curve is shown in Fig. A.1.

(4) VSWG: Generator data [24]:

Stator impedance is 0.0010236 + 0.02073415 Ω; rotor impedance is 0.0011426 + 0.0192582 Ω; magnetizing reactance is j0.82341495 Ω; rated voltage is 690 V. Wind turbine data: rated power is 2000 kW. The wind generator power curve is shown in Fig. A.2.

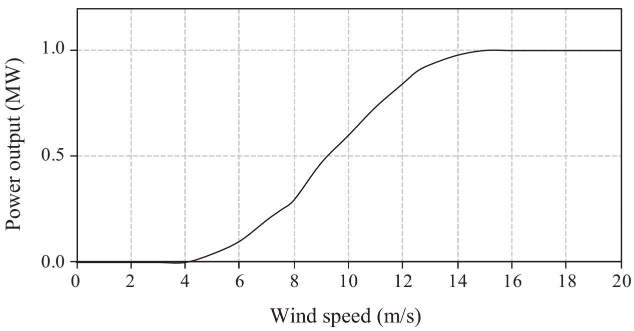

Fig. A.1. SSWG power curve.

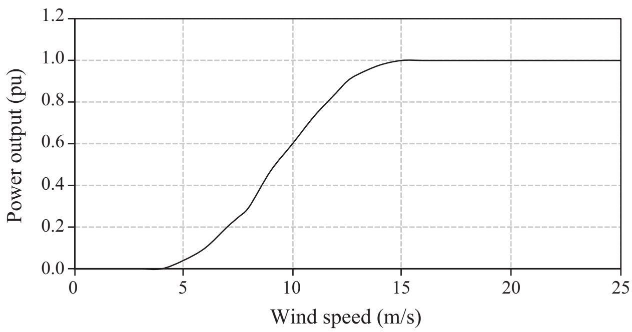

Fig. A.2. VSWG power curve [25].

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