# A simple method for acquiring the conducting angle in a multilevel cascaded inverter using step pulse waves

D.-W. Kang, H.-C. Kim, T.-J. Kim and D.-S. Hyun

**Abstract:** In recent years, the multilevel inverter synthesising the output voltage with a step pulse, has been widely used in high power and high voltage applications. To acquire the conducting angles of switches, the conventional method needs to solve simultaneous equations, corresponding to the fundamental and harmonic components, by the use of a Fourier series. Since they are calculated by an iterative method, the process is requires with a large amount of calculation and time. Moreover, they are calculated by means of an off-line operation. To overcome these drawbacks, this paper proposes a simple method of obtaining the conducting angles. This method reduces the amount of calculation needed to obtain the conducting angles and determines them through an on-line operation. It makes use of the voltage-second areas of the divided reference voltage according to the output voltage levels of the inverter. It does not solve the set of nonlinear transcendental equations, but calculates several trigonometric functions. The validity of the proposed method is demonstrated according to each modulation index through simulation and experiment results. The results indicate that the harmonic components obtained by the proposed method are similar to those from the conventional one in the high modulation index. However, in the low modulation index, the proposed method includes more harmonics than the conventional one.

## 1 Introduction

Multilevel inverters have become an effective and practical solution for increasing the power and reducing the harmonics of AC waveforms. The topologies for high-power multilevel inverters are classified into three types: the diode clamped inverter, the flying capacitor inverter and the cascaded inverter [1–4]. Among these inverters, the cascaded inverter has the advantages that the DC-link voltage is balanced, the modularised circuit layout is possible, and it has the least components per phase [4]. Therefore, the cascaded inverter has been widely applied to high-power applications including motor drives, static var compensator (SVC) stabilisers etc [5–8].

To obtain a good performance, various Pluse-width modulation (PWM) strategies have been studied [9–11]. As the number of levels increases, multilevel inverters have low harmonic components and a low switching frequency, leading to the advent of step-pulse waves that switch once per fundamental cycle. Individual devices operate at high efficiencies because they can switch at a much lower frequency than PWM-controlled inverters [7].

For output step-pulse waveforms, it is necessary to obtain the switching timing angles or the conducting angles of switching devices. The conventional method has the merit that the predominant low-order harmonics can be eliminated. However, it has to solve simultaneous equations, which are the set of nonlinear transcendental equations for the fundamental component and the harmonic ones. It is difficult to obtain the conducting angles because the conventional method needs an iterative method such as the Newton–Raphson one. Additionally, the switching angles are obtained by means of an off-line calculation to minimise the harmonics for each modulation index, which leads to increased use of look-up tables. To overcome these drawbacks, an on-line method has been - studied [12]. However, this method requires too much calculation.

This paper proposes a simple method of obtaining the conducting angles. The method makes use of the voltage-second areas of the divided reference voltage according to the output voltage levels of the inverter. It does not solve the set of nonlinear transcendental equations, but calculates several trigonometric functions. Moreover, the proposed method can obtain the switching angles through on-line operation according to the modulation index.

## 2 Structure and control of the cascaded inverter with step pulses

A cascaded multilevel inverter consists of a series of modularised H-bridge inverter units. The output phase voltage ($V_{ao}$) is synthesised with the output waveforms of several H-bridge inverters. Figure 1 shows one leg of the cascaded inverter.

The H-bridge inverter unit is made up of four switches $S_1$, $S_1'$, $S_2$ and $S_2'$. Switches $S_1'$ and $S_2'$ operate complementarily to $S_1$ and $S_2$, respectively. The combination of switches produces three different voltage outputs,



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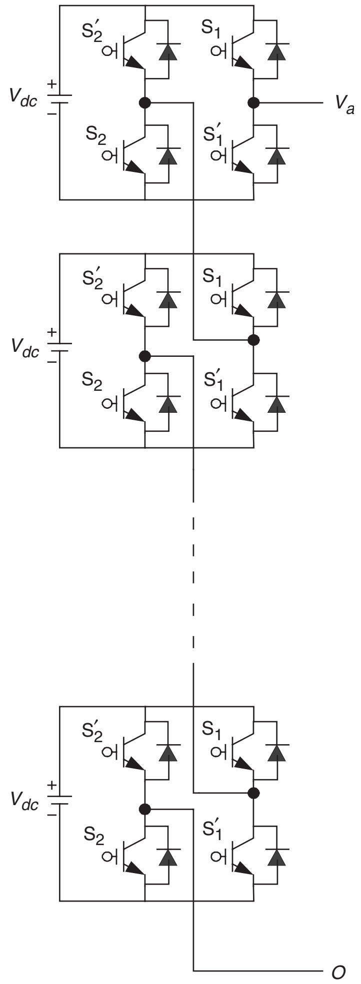


**Fig. 1** *One leg of the cascaded inverter*

$+V_{dc}$, 0 and $-V_{dc}$. To obtain $+V_{dc}$, switches $S_1$ and $S_2$ are turned on. Turning on switches $S_1'$ and $S_2'$ yields -$V_{dc}$. Turning on $S_1$ and $S_2'$ or $S_1'$ and $S_2$, the output voltage is 0.

Figure 2 shows the switching timings to generate a quasi-square waveform in an H-bridge inverter. Each switching device always conducts for 180° (or 1/2 cycle), regardless of the pulse-width of the quasi-square wave. In other words, it switches once per cycle. To obtain more voltage levels, H-bridge inverter units are connected in series. An *M*-level cascaded inverter consists of (*M*-1)/2 H-bridge inverters when it is modulated fully.

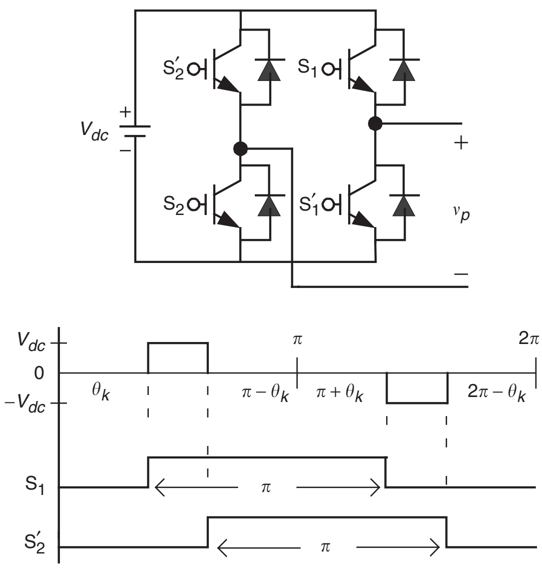


**Fig. 2** *Structure and switching timings of an H-bridge inverter*

The Fourier transform of a step-pulse waveform with $k$ steps is as follows:

$$V(\omega t) = \frac{4V_{dc}}{\pi}\sum_{n}[\cos(n\theta_1)+\cos(n\theta_2)+\cdots+\cos(n\theta_k)] \times \frac{\sin(n\omega t)}{n} \tag{1}$$

where $n = 1, 3, 5, \ldots$, and

$k$ is the number of conducting angles according to the modulation indices.

From (1), the magnitude of the Fourier coefficients when normalised with respect to $V_{dc}$ is as follows:

$$H(n) = \frac{4}{\pi n}[\cos(n\theta_1)+\cos(n\theta_2)+\cdots+\cos(n\theta_k)] \tag{2}$$

where $n = 1, 3, 5, \ldots$, and

$\theta_1$, $\theta_2$, …, and $\theta_k$ are switching timing angles. They indicate the on or off instant of switches in each H-bridge inverter unit. The modulation index, $M_i$, is defined as

$$M_i = \frac{V^*}{V_{\max}} \tag{3}$$

Here, $V^*$ is the reference voltage, that is, the fundamental wave of the output phase voltage. $V_{max}$ is the maximum attainable output phase voltage of the cascaded inverter, i.e., $s \cdot V_{dc}$, where $s$ is the number of H-bridge inverter units.

## 3 The conventional conducting-angle determination method

Each H-bridge inverter unit has a conducting angle, which is calculated to minimise the harmonic components. The conducting angles are the factors determining the amplitude of the harmonic components. The conventional method reduces the predominant low-order harmonics and maximises the fundamental wave of the output phase voltage [13]. Using a Fourier series, simultaneous equations related to the fundamental and harmonic components are built up. To eliminate the specified harmonic component, the related equation is put equal to zero. Conducting angles are determined by solving simultaneous equations. This paper deals with a five H-bridge cascaded inverter, because its 11-level output voltage can be almost sinusoidal and it has less than 5% total harmonic distortion (THD) [7]. In that case, the conventional method can eliminate the 5th, 7th, 11th, and 13th harmonics except for the fundamental wave. From (2), we can obtain the following equations.

$$\begin{aligned}
&\cos(\theta_1)+\cos(\theta_2)+\cos(\theta_3)+\cos(\theta_4)+\cos(\theta_5) = 5\cdot M_i\\
&\cos(5\theta_1)+\cos(5\theta_2)+\cos(5\theta_3)+\cos(5\theta_4)+\cos(5\theta_5)=0\\
&\cos(7\theta_1)+\cos(7\theta_2)+\cos(7\theta_3)+\cos(7\theta_4)+\cos(7\theta_5)=0\\
&\cos(11\theta_1)+\cos(11\theta_2)+\cos(11\theta_3)+\cos(11\theta_4)\\
&+\cos(11\theta_5)=0\\
&\cos(13\theta_1)+\cos(13\theta_2)+\cos(13\theta_3)+\cos(13\theta_4)\\
&+\cos(13\theta_5)=0
\end{aligned} \tag{4}$$

The conventional method has the merit of eliminating the required harmonic component, but it has some problems. First, it is difficult to solve simultaneous equations, which are a set of nonlinear transcendental equations such as (4). These equations can be solved by an iterative method such as the Newton–Raphson one. If the number of simultaneous equations increases, so does the time and the amount of calculations to obtain the conducting angles. Moreover, the method is an approximate one depending on the iteration, which leads to the inclusion of some errors. Secondly, conducting angles are calculated through an off-line operation. Therefore, they have to be arranged in the look-up table. It needs much data in order to implement switching angles with an accurate resolution. If the scale of the modulation index is divided in detail, the data of the conducting angles increases. For example, look-up tables for conducting angles become 8 for the modulation index divided into ten ($M_i = 0.1, 0.2, \ldots, 0.8$). However, in case of the modulation index divided into hundred ($M_i = 0.01, 0.02, \ldots 0.79, 0.80$), they become 80. In other words, the data of the conducting angles depends on the resolution of the modulation index. Therefore, the conventional method has a limitation in its application to an adjustable motor drive.

This paper proposes a simple method of acquiring conducting angles for a multilevel cascaded inverter. The proposed method does not solve the set of nonlinear transcendental equations, but calculates several trigonometric functions. Moreover, the proposed method can obtain the switching angles through on-line operation according to the modulation index.

## 4 The proposed conducting-angle determination method

### 4.1 Determination of the conducting angles

The proposed method obtains the conducting angles ($\theta_k$) by calculating the voltage–second area of the reference voltage waveform that is equal to that of the output voltage waveform of the cascaded inverter. As shown in Fig. 1, the cascaded inverter consists of $n$ H–bridge inverter units and the maximum of its output phase voltage ($V_{ao}$) is $nV_{dc}$. If the reference voltage has $k$ steps during the positive half cycle, it is divided into $k$ areas. Here, one step means the output phase voltage of an H–bridge inverter unit and its magnitude is $V_{dc}$. At this time, only $k$ inverters of one cascade operate and $k$ conducting angles are required. Figure 3 shows the reference voltage waveform and the synthesised step-pulse wave under the condition of the operation of $k$ H-bridge inverters among $n$ units.

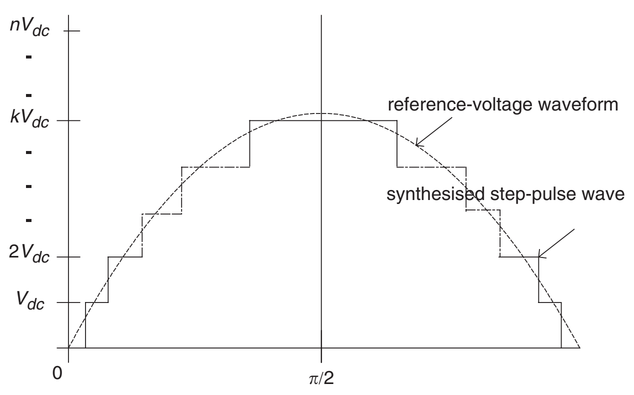


**Fig. 3** *Reference-voltage waveform and the synthesised step-pulse wave in a cascaded inverter*

In the case of five H-bridge units connected in series, the reference voltage ($V_{ref}$) can be obtained from (5).

$$V_{ref} = 5 \cdot \frac{4V_{dc}}{\pi} M_i \cdot \sin \omega t \tag{5}$$

During the positive half-cycle, an 11-level cascaded inverter can produce the following five voltage levels ($V_{output\_phase}$):

$$V_{output\_phase} = m \cdot V_{dc} \; (1 \leq m \leq 5, \text{ where } m \text{ is the integer number}) \tag{6}$$

When (5) become equal to (6), its intersections are defined as dummy angles ($\theta_k'$), which indicate the intersections of the reference voltage waveform and the output levels of the cascaded inverter.

Figure 4 shows dummy conducting angles ($\theta_k'$) in the case of $M_i = \pi/4$. The areas of $A'$, $B'$, $C'$, $D'$ and $E'$ are surrounded by the reference voltage waveform and the output voltage levels of the cascaded inverter. The cascaded inverter generates step-pulse waves to meet the same areas as $A'$, $B'$, $C'$, $D'$ and $E'$. Here, real conducting angles ($\theta_k$) are defined as the switching timing angles of step-pulse waves in the cascaded inverter. Figure 5 shows real conducting angles and the output voltage of the 11-level cascaded inverter during the positive half cycle.

Assuming that areas $A$, $B$, $C$, $D$ and $E$ made by real conducting angles in Fig. 5 are equivalent

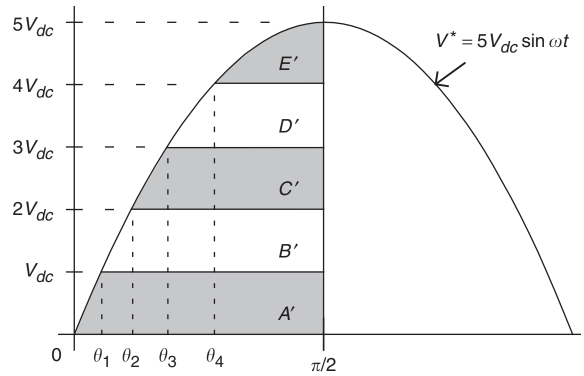


**Fig. 4** *The reference voltage and dummy conducting angles in the case of $M_i = \pi/4$*

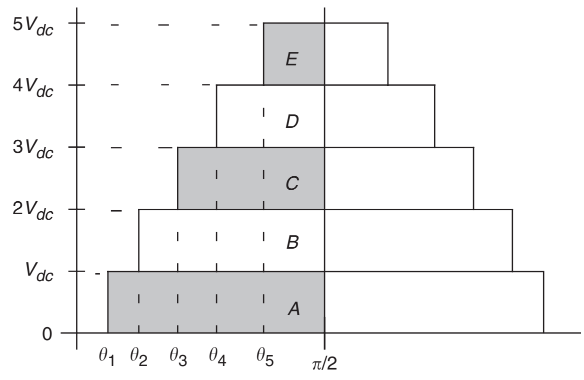


**Fig. 5** *Output voltage of an 11-level cascaded inverter during the positive half cycle in the case of $M_i = \pi/4$*

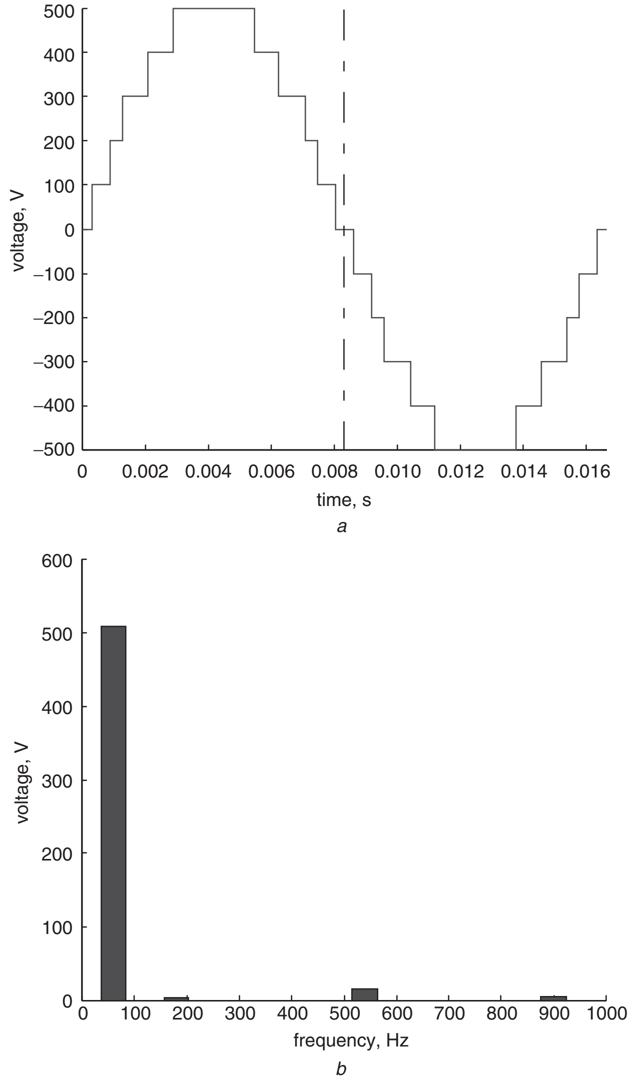


**Fig. 6** *Output step pulse and its FFT by the conventional method in the case of $M_i = 0.8$*
*a* output waveform
*b* FFT

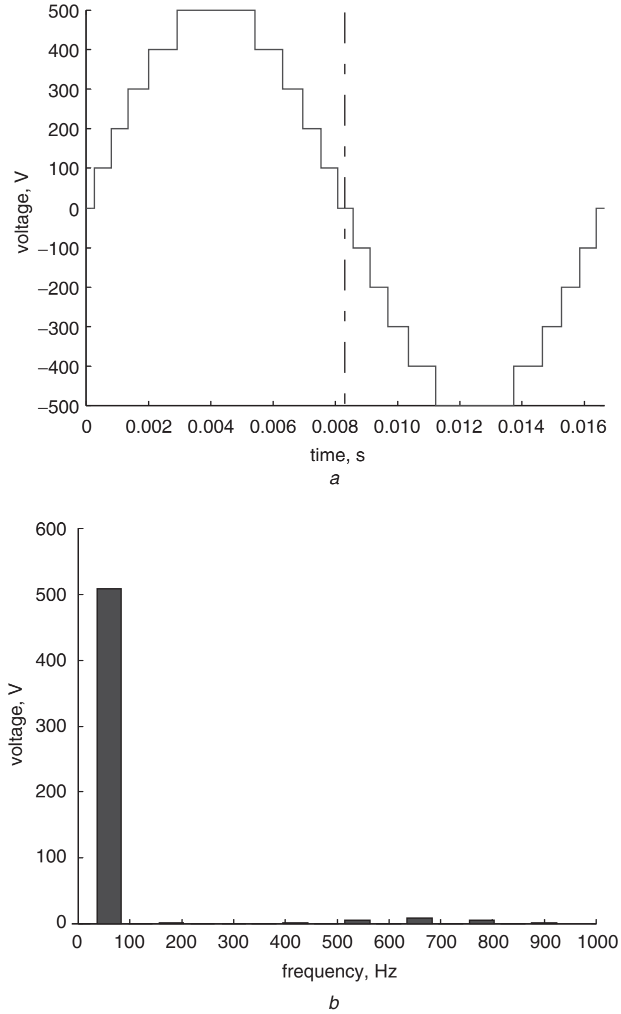


**Fig. 7** *Output step pulse and its FFT by the proposed method in the case of $M_i = 0.8$*
*a* output waveform
*b* FFT

**Table 1: The number of conducting angles according to the modulation index**

| The range of $M_i$ | $k$ |
|---|---|
| $0 < M_i < 0.1571$ | 1 |
| $0.1571 \leq M_i < 0.3142$ | 2 |
| $0.3142 \leq M_i < 0.4712$ | 3 |
| $0.4712 \leq M_i < 0.6283$ | 4 |
| $0.6283 \leq M_i < 1$ | 5 |

**Table 3: Simulation conditions**

| Parameters | Value |
|---|---|
| phase | single |
| frequency | 60 Hz |
| $V_{dc}$ | 40 V |
| load | no load |

**Table 2: Conducting angles according to the modulation indices**

| Conducting angles *deg.* | Modulation indices ($M_i$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| $\theta_1$ | 53.52 | 23.96 | 15.37 | 11.40 | 9.08 | 7.54 | 6.46 | 5.64 |
| $\theta_2$ | × | 83.09 | 55.20 | 36.52 | 28.28 | 23.21 | 19.72 | 17.16 |
| $\theta_3$ | × | × | × | 76.17 | 52.64 | 41.14 | 34.25 | 29.47 |
| $\theta_4$ | × | × | × | × | 87.62 | 69.26 | 52.18 | 43.58 |
| $\theta_5$ | × | × | × | × | × | × | 82.07 | 62.35 |

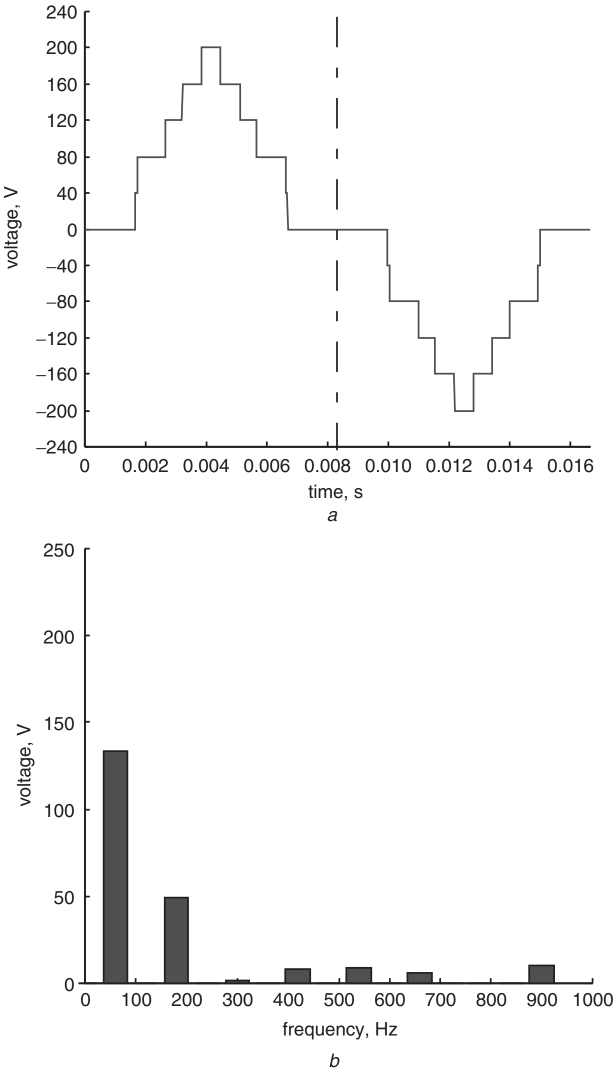


**Fig. 8** *Output step pulse and its FFT by the conventional method in the case of $M_i = 0.5$*
*a* output waveform
*b* FFT

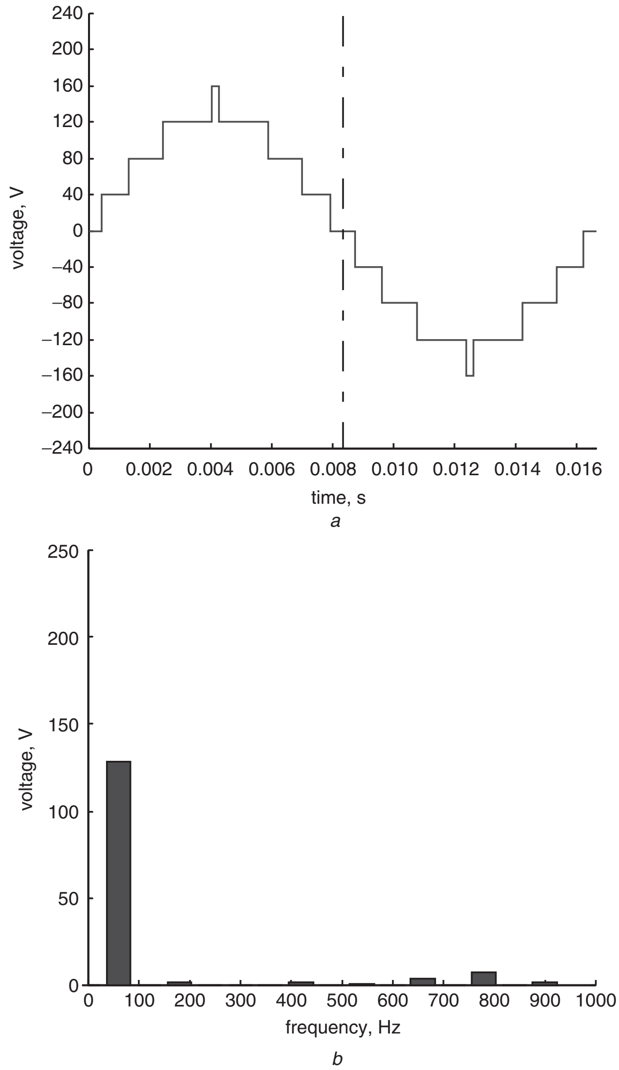


**Fig. 9** *Output step pulse and its FFT by the proposed method in the case of $M_i = 0.5$*
*a* output waveform
*b* FFT

to $A'$, $B'$, $C'$, $D'$ and $E'$ made by dummy conducting angles in Fig. 4, real conducting angles are derived as follows

Area $A'$ can be obtained from:

$$A' = (A' + B' + C' + D' + E') - (B' + C' + D' + E') = \int_0^{\frac{\pi}{2}} 5V_{dc} \sin \omega t d(\omega t) - \left\{ \int_{\theta_1'}^{\frac{\pi}{2}} 5V_{dc} \sin \omega t d(\omega t) - (\frac{\pi}{2} - \theta_1') \cdot V_{dc} \right\} \quad (7)$$

Area $A$ can be expressed as:

$$A = \left(\frac{\pi}{2} - \theta_1\right) \cdot V_{dc} \quad (8)$$

Since $A'$ is equal to $A$, the real conducting angle $\theta_1$ by the proposed method is obtained as:

$$\theta_1 = \frac{\pi}{2} - \left\{ \int_0^{\frac{\pi}{2}} 5 \sin \omega t d(\omega t). - \left( \int_{\theta_1'}^{\frac{\pi}{2}} 5 \sin \omega t d(\omega t) - \left(\frac{\pi}{2} - \theta_1'\right) \right) \right\} \quad (9)$$

In a similar manner, $\theta_2$, $\theta_3$ and $\theta_4$ can be obtained, respectively. In the case of the conducting angle $\theta_5$, there is no dummy angle $\theta_5'$. However, $\theta_5$ can be acquired in terms of area $E'$. In other words, $\theta_5$ can be found out from $E' = E$:

$$\left\{ \int_0^{\frac{\pi}{2}} 5V_{dc} \sin \omega t d(\omega t) - (A' + B' + C' + D') \right\} = (\frac{\pi}{2} - \theta_5) \cdot V_{dc} \quad (10)$$

For up to 11 levels, the generalised conducting angles are:

$$
\begin{aligned}
\theta_{k-4} =& \frac{20M_i}{\pi}\left\{\cos\left(\sin^{-1}\left(\frac{(k-4)\pi}{20M_i}\right)\right)\right.\\
& - \cos\left(\sin^{-1}\left(\frac{(k-5)\pi}{20M_i}\right)\right)\\
& + (k-4)\sin^{-1}\left(\frac{(k-4)\pi}{20M_i}\right)\\
& + (k-5)\sin^{-1}\left(\frac{(k-5)\pi}{20M_i}\right)\\
\theta_{k-3} =& \frac{20M_i}{\pi}\left\{\cos\left(\sin^{-1}\left(\frac{(k-3)\pi}{20M_i}\right)\right)\right.\\
& - \cos\left(\sin^{-1}\left(\frac{(k-4)\pi}{20M_i}\right)\right)\\
& + (k-3)\sin^{-1}\left(\frac{(k-3)\pi}{20M_i}\right)\\
& - (k-4)\sin^{-1}\left(\frac{(k-4)\pi}{20M_i}\right)\\
\theta_{k-2} =& \frac{20M_i}{\pi}\left\{\cos\left(\sin^{-1}\left(\frac{(k-2)\pi}{20M_i}\right)\right)\right.\\
& - \cos\left(\sin^{-1}\left(\frac{(k-3)\pi}{20M_i}\right)\right)\\
& + (k-3)\sin^{-1}\left(\frac{(k-2)\pi}{20M_i}\right)\\
& - (k-3)\sin^{-1}\left(\frac{(k-3)\pi}{20M_i}\right)\\
\theta_{k-1} =& \frac{20M_i}{\pi}\left\{\cos\left(\sin^{-1}\left(\frac{(k-1)\pi}{20M_i}\right)\right)\right.\\
& - \cos\left(\sin^{-1}\left(\frac{(k-2)\pi}{20M_i}\right)\right)\\
& + (k-3)\sin^{-1}\left(\frac{(k-1)\pi}{20M_i}\right)\\
& - (k-2)\sin^{-1}\left(\frac{(k-2)\pi}{20M_i}\right)\\
\theta_k =& \frac{\pi}{2}k - \frac{20M_i}{\pi}\cos\left(\sin^{-1}\left(\frac{(k-1)\pi}{20M_i}\right)\right)\\
& - (k-1)\sin^{-1}\left(\frac{(k-1)\pi}{20M_i}\right)
\end{aligned}
\tag{11}
$$

Here, if the subscript of $\theta$ is not more than 0, the conducting angle does not exist.

Table 1 shows the number of the conducting angle expressed according to $k$, depending on the modulation index. The proposed method can be easily applied to other levels in a similar manner.

To obtain conduction angles, the conventional method has to solve a lot of equations according to the modulation index. However, the proposed method determines them through simple equations such as (11). Although this equation looks somewhat complex it should be noted that the same trigonometric function is repeated. It calculates the trigonometric function ten times in

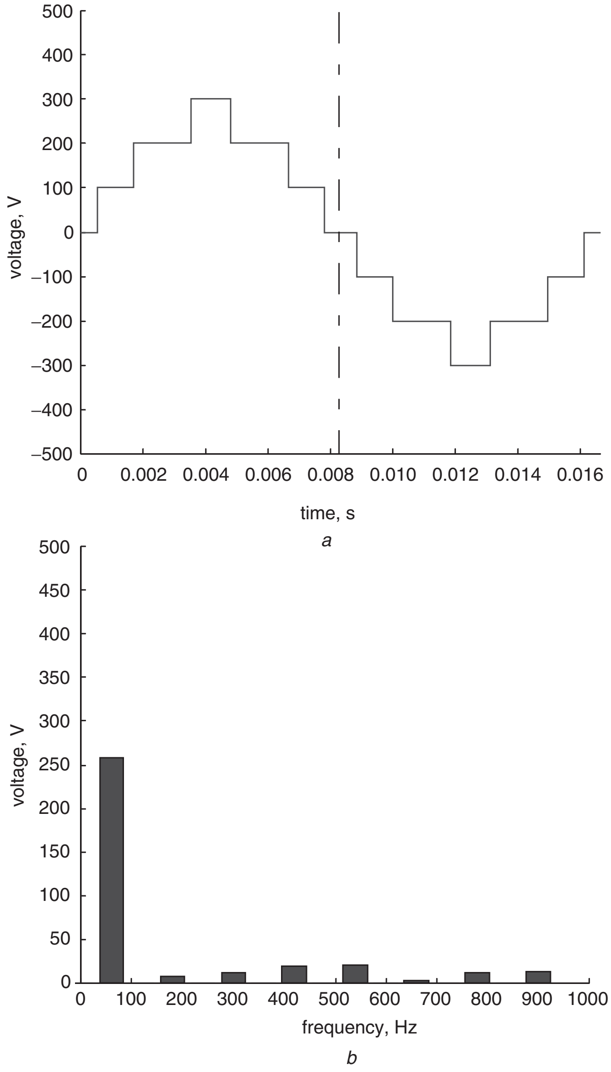


**Fig. 10** *Output step pulse and its FFT by the proposed method in the case of $M_i = 0.4$*
*a* output waveform
*b* FFT

order to obtain all the conducting angles in the case of 11-level cascaded inverter. Therefore, compared with the conventional method, the proposed method significantly decreases the amount of calculation. Moreover, it can obtain conduction angles directly through an on-line operation although the modulation index is varied.

### *4.2 Consideration*

The conducting angles obtained by the proposed are method are different from those obtained by the conventional method. As the modulation index increases, so do the conducting angles. However, the conventional method gives the same conducting angles regardless of the modulation index. The proposed method can be applied to various modulation indices. Table 2 lists the conducting angles according to the modulation indices in an 11-level cascaded inverter.

In the case that $M_i$ is more than about 0.8, the amplitude of the fundamental component in the output step-pulse wave is less than that in the reference voltage. The difference between the linear modulation region and the over-modulation region is not clear, which requires further study. In the case of the 11-level

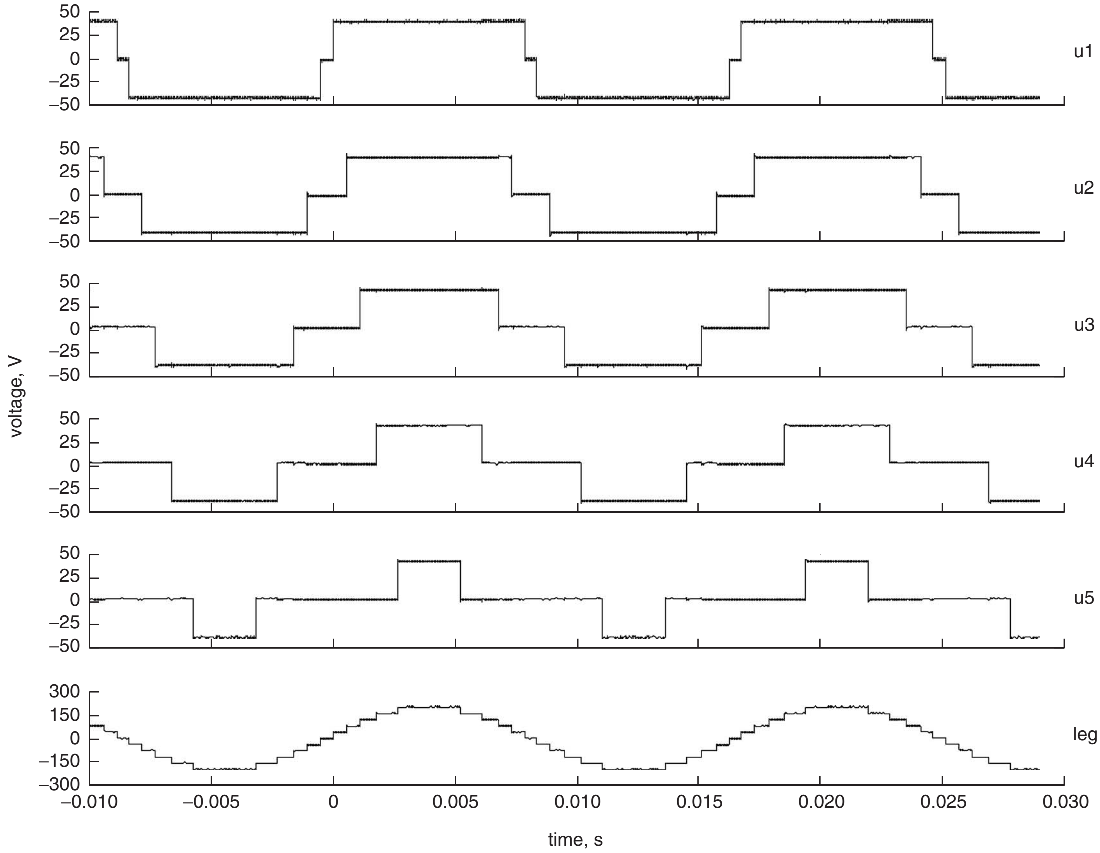


**Fig. 11** *Each output voltage of the H-bridge units and the leg voltage in the case of $M_i = 0.8$*

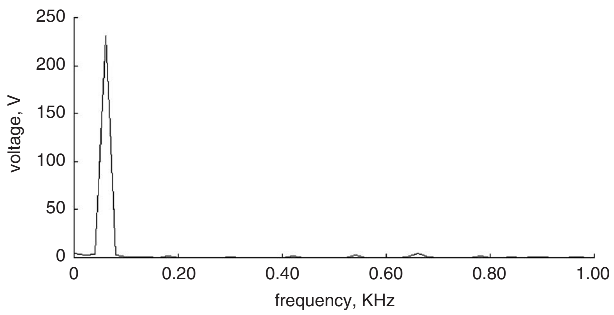


**Fig. 12** *The FFT waveform of the leg voltage in the case of $M_i = 0.8$*

cascaded inverter, the maximum area of $E'$ is $\pi/2V_{dc}$ which is the case of six-step. The DC-pulses bus utilisation factor is defined as $\pi * M_i/4$ and so it decreases with the modulation index.

## 5 simulation and experimental results

### 5.1 Simulation results

To demonstrate the validity of the proposed method, a simulation study was carried out using MATLAB/SIMULINK. Simulation conditions are shown in Table 3.

In the case of $M_i = 0.8$, Figs. 6 and 7 illustrate the output voltage waves of cascaded inverters and their FFTs by the conventional and the proposed method, respectively. The two Figures show that both simulation results are very similar. It means that the proposed method gives a very effective result, comparing with the conventional method. Each FFT displays the harmonic content as well as the fundamental wave. Here, multiples of 3rd harmonics are eliminated in the line-to-line voltage. Therefore the THD of the conventional method is near to zero and the THD of the proposed one is 1.97%.

Similarly, Figs. 8 and 9 show the output voltage waves of the cascaded inverter and their FFTs by both methods in the case of $M_i = 0.5$. Except for multiples of the 3rd harmonics, both FFT figures are very similar. Therefore the THD of the conventional method is 7.8% and the THD of the proposed one is 6.34%.

Figure 10 shows the output voltage wave and its FFT by the proposed method in the case of $M_i = 0.4$. Since the modulation index is low, only three units among five inverter ones operate. The number of conducting angles is 3 and the output levels of the cascaded inverter are 7. The lower the modulation index is, the higher the harmonic components are. Except for multiples of 3rd harmonics, the THD is 10.2%. FFT figures by the proposed method include some harmonics. However, this does not matter because the triple harmonics are eliminated in the line-to-line voltage.

### 5.2 Experiment results

One leg of the prototype 11-level cascaded inverter was implemented and tested in order to verify the proposed method. It was programmed with the TMS320C31 DSP board and the Mitsubishi CM20MD-12H was used as switching devices. The output frequency was 60Hz and the capacitance of the DC-link capacitor was 10000 μF, respectively. Each DC-link voltage was 40 V.

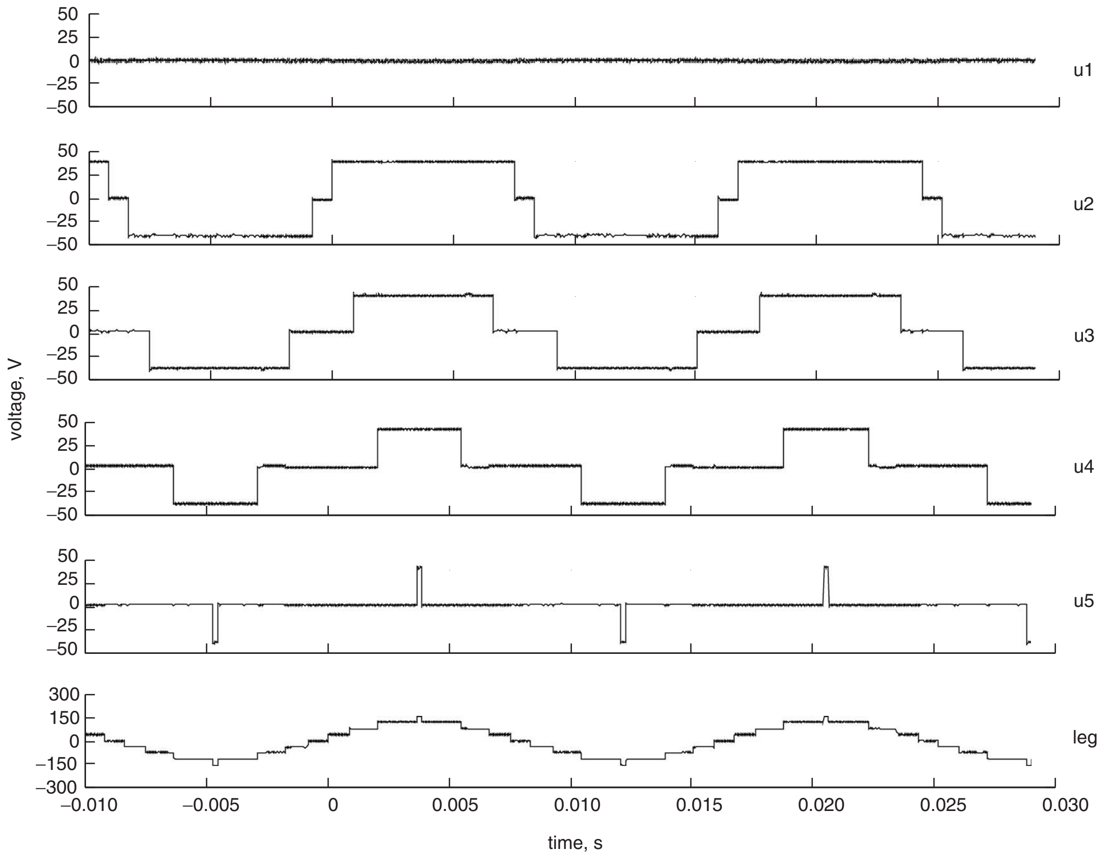


**Fig. 13** *Each output voltage of the H-bridge units and the leg voltage in the case of $M_i = 0.5$*

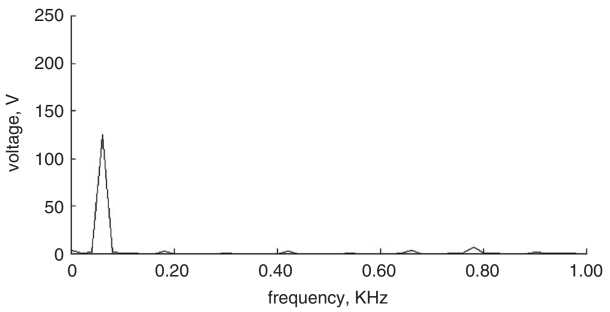


**Fig. 14** *The FFT waveform of the leg voltage in the case of $M_i = 0.5$*

Figure 11 shows each output voltage of the H-bridge units and the leg voltage in the case of $M_i = 0.8$, whereas Fig. 12 represents the FFT wave of the leg voltage in the case of $M_i = 0.8$.

Figures 13, 14, 15 and 16 show each output voltage of the H-bridge units, leg voltage and FFT waveform in the case of $M_i = 0.5$ and 0.4, respectively. These Figures demonstrate that the proposed method gives a good output voltage and obtains the conducting angles very simply.

## 6 Conclusion

This paper proposes a simple method of obtaining the conducting angles of the switching devices in a multilevel cascaded inverter with step pulses. This method makes use of the voltage-second area of the divided reference voltage waveform according to the output voltage levels of the inverter. It does not solve the set of nonlinear transcendental equations, but calculates several trigonometric functions. Therefore, it does not need look-up tables and can be calculated through an on-line operation. Moreover, the procedure to obtain the conducting angles is very simple and quick, regardless of the modulation index. The validity of the proposed method has been demonstrated according to each modulation index through simulation and experiment results. The results indicate that the harmonic components obtained by the proposed method are similar to one obtained by the conventional method.

## 7 References


1 Nabae, A., Takahashi, I., and Akagi, H.: 'A new neutral-point-clamped PWM inverter', *IEEE Trans. Ind. Appl.*, 1981, **17**, (5), pp. 518–523

2 Choi, N.S., Cho, J.G., and Cho, G.H.: 'A general circuit topology of multilevel inverter', IEEE-PESC Conf. Rec., Cambridge, MA, USA, 1991, pp. 96–103

3 Yuan, X., Stemmler, H., and Barbi, I.: 'Investigation on the clamping voltage self-balancing of the three-level capacitor clamping inverter'. IEEE-PESC Conf. Rec., South Carolina, USA, 1999, pp. 1059–1064

4 Lai, J.S., and Peng, F.Z.: 'Multilevel converters-A new breed of power converters', *IEEE Trans. Ind. Appl.*, 1996, **32**, pp. 509–517

5 Peng, F.Z., Lai, J.S., McKeever, J.W., and VanCoevering, J.: 'A multilevel voltage-source inverter with separate dc sources for static var generation', *IEEE Trans. Ind. Appl.*, 1996, **32**, pp. 1130–1138

6 Peng, F.Z., and Lai, J.S.: 'Dynamic performance and control of a static var generator using cascade multilevel inverters', *IEEE Trans. Ind. Appl.*, 1997, **33**, pp. 748–755

7 Tolbert, L.M., Peng, F.Z., and Harbetler, T.G.: 'Multilevel converters for large electric drives', *IEEE Trans. Ind. Appl.*, 1999, **35**, pp. 36–44

8 Huang, F., Lim, P.I., and Goh, E.K.: 'Design and implementation of a 3–phase voltage stablizer using 11–level voltage source inverters', IEEE–IECON Conf. Rec., New Orleans, USA, 1997, pp. 772–725

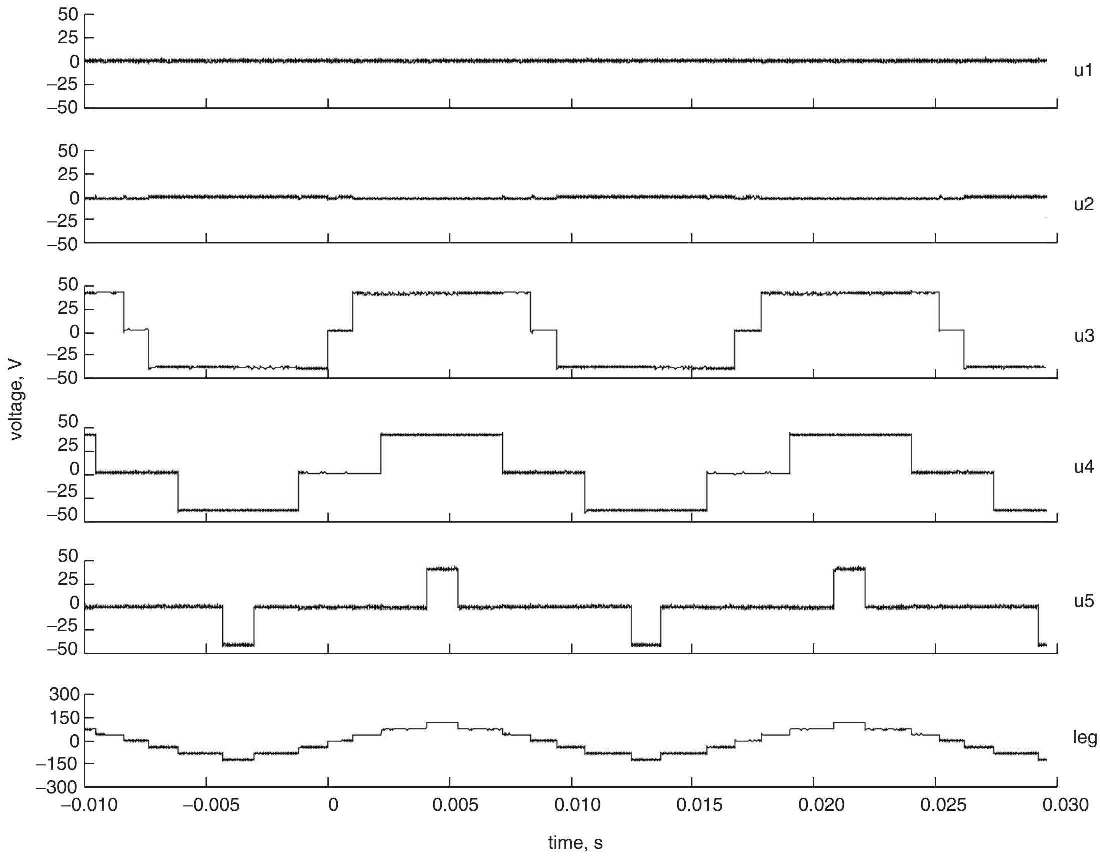


**Fig. 15** *Each output voltage of the H-bridge units and the leg voltage in the case of $M_i = 0.4$*

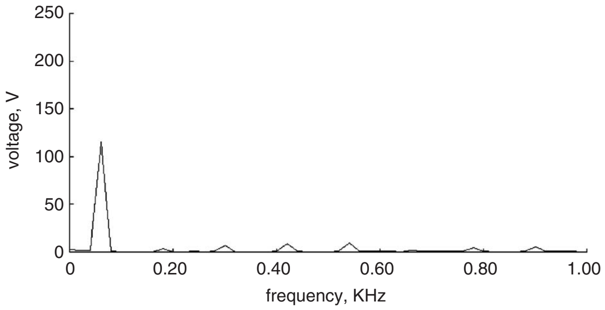


**Fig. 16** *FFT waveform of the leg voltage in the case of $M_i = 0.4$*

9 Lee, Y.H., Kim, R.Y., and Hyun, D.S.: ' A novel PWM scheme for a three-level voltage source inverter with GTO thyristors', *IEEE Trans. Ind. Appl.*, 1996, **32**, (2), pp. 2690–268
10 Marchesoni, M.: 'High performance current control techniques for applications to multilevel high power voltage source inverters', IEEE-PESC Conf. Rec., Milwaukee, USA, 1989, pp. 672–683
11 Tolbert, L.M., Peng F., Z., and Habetler, T.G.: 'Multilevel PWM methods at low modulation indices', IEEE–APEC Conf. Rec., Dallas, USA, 1999, pp. 1032–1038
12 Tolbert, L.M., Chiasson, J.N., Mckenzie, K.J., and Du, Z.: 'Control of cascaded multilevel converters with unequal voltage source of HEVs', IEEE IEMDC Conf., Madison, USA, 2003, pp. 663–669
13 Patel, H.S., and Hoft, R.G.: 'Generalized techniques of harmonic elimination and voltage control in thyristor inverters: part 1-harmonic elimination', *IEEE Trans. Ind. Appl.*, 1973, **9**, pp. 310–317