Multilevel Selective Harmonic Elimination PWM Technique in Series-Connected Voltage Inverters

Li Li, *Member, IEEE*, Dariusz Czarkowski, *Member, IEEE*, Yaguang Liu, *Member, IEEE*, and Pragasen Pillay, *Senior Member, IEEE*

***Abstract*—Selective harmonic elimination pulsewidth modulation (SHEPWM) method is systematically applied for the first time to multilevel series-connected voltage-source PWM inverters. The method is implemented based on optimization techniques. The optimization starting point is obtained using a phase-shift harmonic suppression approach. Another less computationally demanding harmonic suppression technique, called a mirror surplus harmonic method, is proposed for double-cell (five-level) inverters. Theoretical results of both methods are verified by experiments and simulations for a double-cell inverter. Simulation results for a five-cell (11-level) inverter are also presented for the multilevel SHEPWM method.**

***Index Terms*—Multilevel inverters, pulsewidth modulation inverters, selective harmonic elimination pulsewidth modulation.**

I. Introduction

MEDIUM/LARGE motor drives, uninterruptible power supply (UPS) systems, and high-power inverters in flexible alternate current transmission systems (FACTS) need switching elements which can bear high voltages and currents. To overcome the limitations of semiconductor switches, several new techniques and topologies have been developed [1]–[4], such as multiple switching elements in one leg of an inverter [5], [6], series-connected inverters [7]–[15], parallel-connected inverters [16], [17], multilevel reactive power compensators [18]–[21], multiple rectifiers for unity power factor correction [22], optimization of motor performance indexes (such as harmonic current, torque ripple, common mode voltage, and bearing currents) [23], [24], and neutral-point-clamped (NPC) inverters [25]–[27]. This paper focuses on series-connected voltage-source PWM inverters. In this area, present control techniques are based on the following methods: 1) sinusoidal PWM (SPWM) [8], [9]; 2) space-vector PWM (SVPWM) [13]; 3) nonsinusoidal carrier PWM [7]; 4) mixed PWM [12], [14]; 5) special structure of cell connections [11]; and 6) selective harmonic elimination PWM (SHEPWM) [1], [15]. The SHEPWM-based methods can theoretically provide the highest quality output among all the PWM methods. The SHEPWM method presented in [1] offers the same number of control variables as the number of inverter levels. Results given in [15] are only for a five-level inverter allowing up to seven switching angles without taking into account that inverter cells should equally share the output power.

In this paper, a SHEPWM model of a multilevel series-connected voltage-source inverter is developed which can be used for an arbitrary number of levels and switching angles. Simulation and experimental results for a five-level (or double-cell) 22-angle single-phase inverter and a five-level 20-angle three-phase inverter are presented. Simulation results for an 11-level (five-cell) 45-angle three-phase inverter are also given. A reduced-order SHEPWM method by mirror surplus harmonic shaping for five-level inverters is proposed and experimentally verified. The paper is organized in the following manner. The mirror surplus harmonic method is presented in Section II, followed by a description and results of the multilevel SHEPWM in Section III. Conclusions are given in Section IV.

II. Harmonic Suppression with Mirror Surplus Harmonics Technique

In this section, a new concept of mirror surplus harmonics is introduced for selected harmonic elimination in double-cell series-connected PWM inverters. This concept allows for reducing the amount of computations in comparison with the multilevel SHEPWM. It will be shown that the obtained output harmonic spectrum is close to that of multilevel SHEPWM.

A. Double-Cell Series-Connected Inverter Harmonic Model

In 1973, the selected harmonic elimination method for PWM inverters was introduced [28] for single-cell (two- and three-level) inverters. This method is sometimes called a programmed PWM technique. Fig. 1 illustrates the general quarter-wave symmetric triple-level programmed PWM switching pattern. The square wave is chopped m times per half cycle. Owing to the symmetries in the PWM waveform, only odd harmonics exist. The Fourier coefficients of odd harmonics

Paper IPCSD 99–60, presented at the 1998 Industry Applications Society Annual Meeting, St. Louis, MO, October 12–16, and approved for publication in the IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS by the Industrial Power Converter Committee of the IEEE Industry Applications Society. Manuscript submitted for review October 15,1998 and released for publication July 26, 1999.

L. Li was with the Department of Electrical Engineering, Polytechnic University, Brooklyn, NY 11201 USA. He is now with Lucent Technologies, Mesquite, TX 75149 USA (e-mail: lli15@lucent.com).

D. Czarkowski is with the Department of Electrical Engineering, Polytechnic University, Brooklyn, NY 11201 USA (e-mail: dcz@pl.poly.edu).

Y. Liu is with API Controls, Inc., Amherst, NY 14228 USA (e-mail: liuya@ieee.org).

P. Pillay is with the Department of Electrical and Computer Engineering, Clarkson University, Potsdam, NY 13699 USA (e-mail: pillayp@sun.soe.clarkson.edu).

Publisher Item Identifier S 0093-9994(00)00042-6.

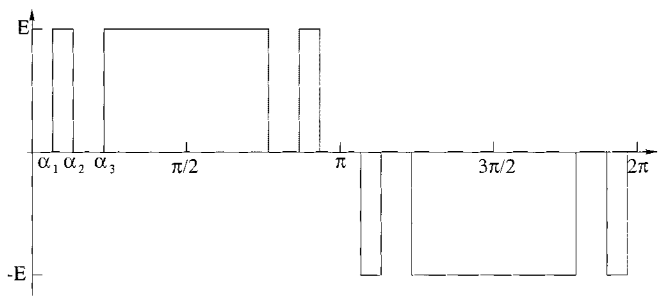


Fig. 1. Waveform of triple-level SHEPWM.

in triple-level programmed PWM inverters with odd switching angles are given by

$$b_n = \frac{4E}{n\pi}[\cos n\alpha_1 - \cos n\alpha_2 + \cdots + (-1)^{j-1} \cos n\alpha_j + \cdots + \cos n\alpha_m] \quad (1)$$

where n is the harmonic order.

Any m harmonics can be eliminated by solving the m equations obtained from setting (1) equal to zero [29]. Usually, the Newton iteration method is used to solve such systems of nonlinear equations [30]. The correct solution must satisfy the condition

$$0 < \alpha_1 < \alpha_1 < \cdots < \alpha_m < \frac{\pi}{2}. \quad (2)$$

Let us consider one leg of a double-cell series-connected PWM inverter shown in Fig. 2. Each cell of such a single-phase inverter switches m times per quarter cycle and produces a three-level {–1, 0, 1} PWM waveform. This results in a five-level –2, –1, 0, 1, 2 inverter output. Theoretically, $2m-2$ odd harmonics can be eliminated from the inverter's spectrum while keeping the fundamental components of both cells equal to each other. Even harmonics are not present due to the PWM waveform symmetry. The switching angles must be obtained from the following system of $2m$ nonlinear transcendental equations:

$$\begin{aligned} \sum_{i=1}^{m}(-1)^{i+1} \cos \alpha_i &= \frac{\pi}{4} M \\ \sum_{i=1}^{m}(-1)^{i+1} \cos \beta_i &= \frac{\pi}{4} M \\ \sum_{i=1}^{m}(-1)^{i+1} \cos 3\alpha_i + \sum_{i=1}^{m}(-1)^{i+1} \cos 3\beta_i &= 0 \\ &\cdots \\ \sum_{i=1}^{m}(-1)^{i+1} \cos(4m-3)\alpha_i + \sum_{i=1}^{m}(-1)^{i+1} \cos(4m-3)\beta_i &= 0 \end{aligned} \quad (3)$$

where α_i's are switching angles of the first cell, β_i's are switching angles of the other cell, and M is the modulation index. This paper explicitly requires an even fundamental power sharing among cells. Convergence of numerical procedures used to solve (3) is sensitive to the starting values of switching angles and requires considerable computation.

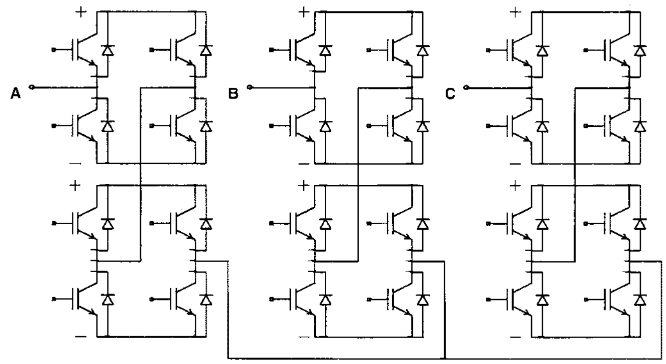


Fig. 2. Schematic diagram of a three-phase double-cell series-connected PWM inverter (each cell requires a separate dc bus).

B. Reduced-Order Model of a Double-Cell Series-Connected Inverter

Elimination of low-order harmonics from only one cell, which will be called a general SHEPWM method, can be obtained by solving a system of m equations [28]

$$\begin{aligned} \sum_{i=1}^{m}(-1)^{i+1} \cos \alpha_i &= \frac{\pi}{4} M \\ \sum_{i=1}^{m}(-1)^{i+1} \cos 3\alpha_i &= 0 \\ &\cdots \\ \sum_{i=1}^{m}(-1)^{i+1} \cos(2m-1)\alpha_i &= 0. \end{aligned} \quad (4)$$

The first significant surplus harmonic from this cell has an amplitude A_{2m+1}. If it is desired to eliminate A_{2m+1} from the output spectrum of the single-phase inverter, the other cell must produce the $2m+1$ harmonic of an amplitude $-A_{2m+1}$. To preserve the elimination of the $2m-1$ low-order odd harmonics and to set the amplitude of the $2m+1$ harmonic to $-A_{2m+1}$, the number of switching angles in the second cell must be increased by one to $m+1$. The switching angles of the second cell fulfill the following system of $m+1$ equations:

$$\begin{aligned} \sum_{i=1}^{m+1}(-1)^{i+1} \cos \beta_i &= \frac{\pi}{4} M \\ \sum_{i=1}^{m+1}(-1)^{i+1} \cos 3\beta_i &= 0 \\ &\cdots \\ \sum_{i=1}^{m+1}(-1)^{i+1} \cos(2m-1)\beta_i &= 0 \\ \sum_{i=1}^{m+1}(-1)^{i+1} \cos(2m+1)\beta_i &= -\frac{(2m+1)\pi}{4} A_{2m+1}. \end{aligned} \quad (5)$$

An unexpected benefit of such a $2m+1$ harmonic cancellation is that the whole first cluster of significant harmonics from the second cell becomes nearly a mirror image of the first cluster of

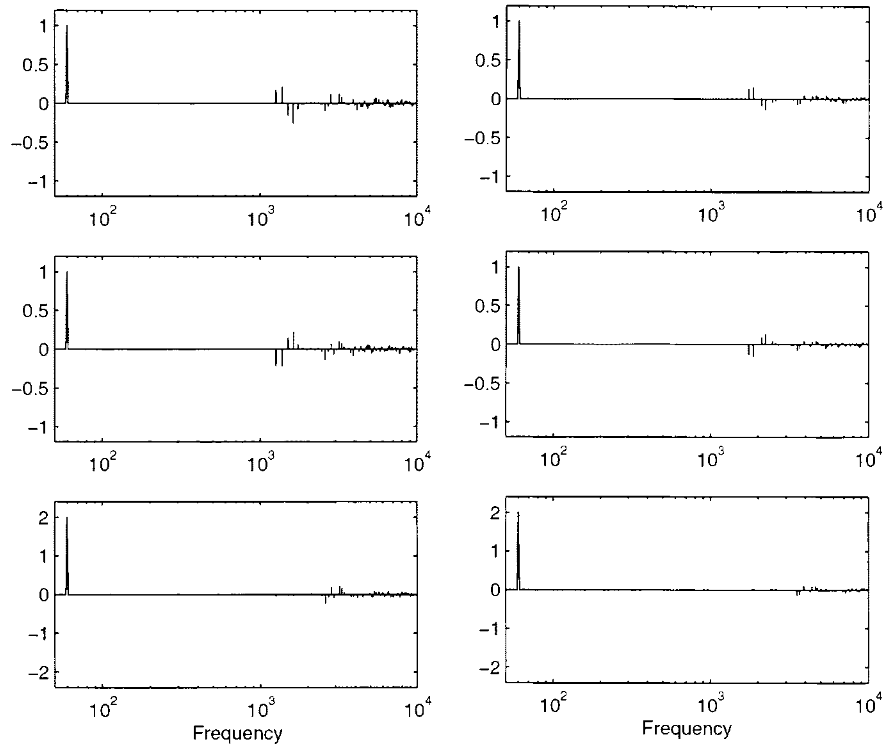


Fig. 3. Calculated frequency spectra for mirror surplus harmonic method. (a) Single phase system (from top to bottom): optimal 11 switching angles, general ten switching angles, and inverter output versus harmonic order. $M = 1.0$. (b) Three phase system (from top to bottom): optimal 11 switching angles, general nine switching angles, and inverter output versus harmonic order. $M = 1.0$.

significant harmonics from the first cell. Thus, the solution of (4) and (5) approximates very closely the solution of(3). Similar models can be developed for three-phase systems. The difference between single- and three-phase calculations is that for a three-phase system triplen harmonics need not to be included in the set of harmonics selected for elimination. Cancellation of harmonics using (4) and (5) will be called a mirror surplus harmonic PWM technique. This cancellation has been checked for several values of m and for a wide range of the modulation index M. An example for single- and three-phase inverters is given below. Since systems of equations as in (4) do not have analytical solutions, it is difficult to find a theoretical explanation for the proposed method. It will be a subject of a future research. Nonetheless, the proposed approach is a practical way of finding an approximate solution to (3) and, hence, harmonic suppression in double-cell series-connected inverters. For three and higher numbers of cells, the direct SHEPWM described in Section III should be used.

C. Simulation Example

Frequency spectra of waveforms obtained with the mirror surplus harmonic method for single-phase and three-phase systems are shown in Fig. 3(a) and (b), respectively. It can be clearly seen that first clusters of significant harmonics of two cells in Fig. 3(a) and (b) (first and second spectrum from the top) almost cancel each other in the output spectrum. In fact, the output spectra in Fig. 3(a) and (b) are very close to what can be expected if the solution of (3) is used for generation of switching patterns.

D. Experimental Results

The calculated switching patterns have been implemented using a TMS320C30 digital signal processor (DSP) unit driving IPM PM20CSJ060 power modules. The resulting inverter was loaded by an induction motor. Figs. 4(a) and 5(a) show measured voltage harmonic amplitudes of both cells and the inverter output for modulation index $M = 1.0$ in this scaled-down experimental system. Time-domain waveforms of inverter outputs are presented in Figs. 4(b) and 5(b). The experimental results are in full agreement with theoretical predictions.

III. Multilevel SHEPWM Technique

The multilevel SHEPWM technique has a theoretical potential to achieve the highest output power quality at low switching frequencies in comparison to other methods. However, because of its mathematical complexity, no significant results have been reported thus far. One of the main challenges is to obtain a good starting point when the solution of a nonlinear system of equations of the (3) type is attempted. This paper presents a concept of obtaining the starting point by means of SHEPWM with a phase shift. Then, a nonconstraint optimization [35] is used to calculate the final solution of the multilevel SHEPWM problem.

A. Specific Harmonic Elimination with Phase Shift

Phase shift is an effective and simple method to decrease harmonic content in multilevel converters. The most common application of phase shifting in PWM inverters is in carrier-based modulation schemes [8], [9]. In this paper, phase shift together

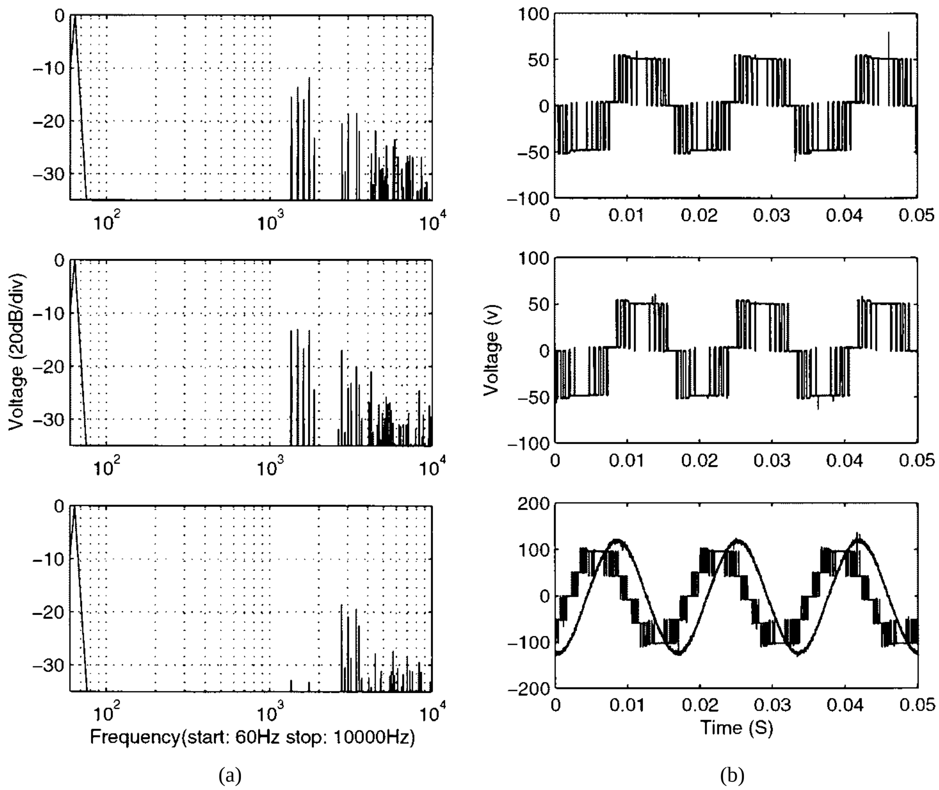


Fig. 4. Experimental single-phase spectra and waveforms with mirror surplus harmonic method. (a) Frequency spectra (from top to bottom): upper cell, lower cell, and inverter output. (b) Output voltage waveforms and load current (from top to bottom): upper cell voltage (V), lower cell voltage (V), inverter output voltage (V), and current (105 V/A).

with three-level SHEPWM is used to obtain a starting point for multilevel SHEPWM calculations.

Let us consider a three-cell single-phase PWM inverter as an example. Assume that the cells have identical voltage spectra with low-order harmonics eliminated by the classical SHEPWM. To preserve high amplitude of the fundamental, the phase-shift angle β among the three cells should be small. Using (1), the harmonic content of the reference cell $b1_n$, the lagging cell $b2_n$, and the leading cell $b3_n$ can be described as

$$\begin{aligned} b1_n &= \frac{4E}{n\pi}\left[\cos n\alpha_1 - \cos n\alpha_2 + \cdots + (-1)^{j-1}\cos n\alpha_j \right.\\ &\qquad \left. + \cdots + \cos n\alpha_m\right] \\ b2_n &= \frac{4E}{n\pi}\left[\cos n(\alpha_1-\beta) - \cos n(\alpha_2-\beta) + \cdots + (-1)^{j-1}\right.\\ &\qquad \left. \cdot \cos n(\alpha_j-\beta) + \cdots + \cos n(\alpha_m-\beta)\right] \\ b3_n &= \frac{4E}{n\pi}\left[\cos n(\alpha_1+\beta) - \cos n(\alpha_2+\beta) + \cdots + (-1)^{j-1}\right.\\ &\qquad \left. \cdot \cos n(\alpha_j+\beta) + \cdots + \cos n(\alpha_m+\beta)\right]. \end{aligned} \tag{6}$$

Adding the cell voltages in (6), the multilevel inverter output harmonics are given by

$$V_n = (1 + 2\cos n\beta)b1_n \tag{7}$$

where V_n is the nth harmonic of the inverter output voltage.

The phase shift angle β may be selected by the following heuristic approach. If the number of switching angles in a quarter period is m, the first significant harmonic crest for each cell is just above the $2m$th harmonic. One of the crest harmonics (usually, the largest one) can be eliminated by the phase shift, and others will be suppressed. If the $2m+3$ harmonic is selected for elimination, the phase-shift angle β can be obtained as

$$\beta = \frac{2\pi}{3(2m+3)}. \tag{8}$$

Fig. 6 shows the phase diagram of the fundamental and $2m+3$ harmonic. Fig. 7 presents an example of the surplus harmonic suppression with phase shift in a three-cell inverter with $m = 9$. The resulting β equals 5.71°.

B. Starting Point of Multilevel SHEPWM

The phase-shift technique for surplus harmonic suppression presented in the previous subsection can be easily generalized for a K-cell ($K = 2, 3, \cdots$) system. The obtained switching angles may be used as starting points to calculate a solution for a nonlinear system of equations of the (3) type. An application of this idea to double-cell and five-cell inverters is presented next.

C. Multilevel Selective Harmonic Elimination

Consider a two-cell series-connected PWM inverter. With m switching angles per quarter wave for each cell, there are $2m$

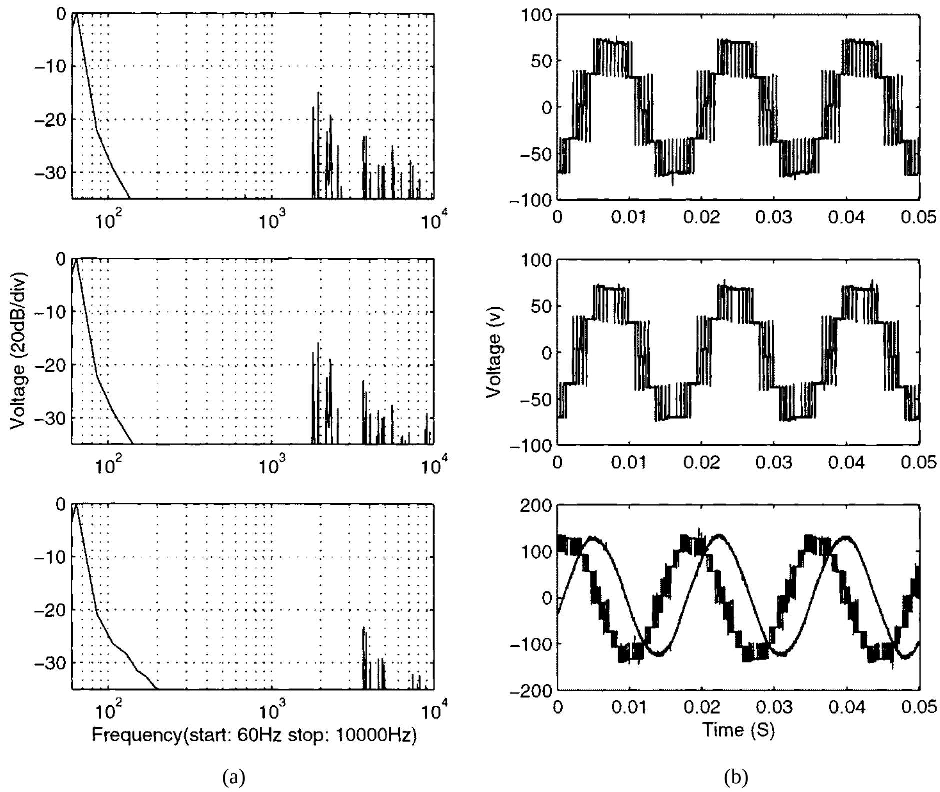


Fig. 5. Experimental three-phase spectra and waveforms with mirror surplus harmonic method. (a) Frequency spectra (from top to bottom): upper cell line-to-line voltage, lower cell line-to-line voltage, and inverter line-to-line output voltage. (b) Output voltage waveforms and load current (from top to bottom): upper cell line-to-line voltage (V), lower cell line-to-line voltage (V), inverter line-to-line output voltage (V), and phase current (125 V/A).

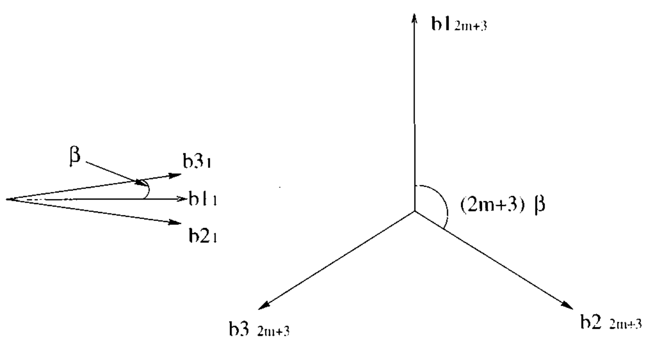


Fig. 6. Phase-shift diagram for a three-cell inverter.

variables and, consequently, $2m - 1$ harmonics can be eliminated. The amplitude of the nth voltage harmonic at the inverter output is given by

$$V_n = \frac{4E}{n\pi}\left[\cos n\alpha_1 - \cos n\alpha_2 + \cdots + (-1)^{j-1} \cos n\alpha_j + \cdots + \cos n\alpha_m\right] + \frac{4E}{n\pi}\left[\cos n\beta_1 - \cos n\beta_2 + \cdots + (-1)^{j-1} \cos n\beta_j + \cdots + \cos n\beta_m\right]. \tag{9}$$

Setting the fundamental to a desired value dictated by the modulation index M and equating selected harmonics to zero results in a system of nonlinear equations which is very difficult to solve numerically. To alleviate computational problems, a nonconstrained optimization approach [35] has been proposed. The target function of this new optimization scheme can be written as

$$F = K_1(V_1 - M)^2 + K_2 V_3^2 + \cdots + K_{2m-1} V_{2m-1}^2 \tag{10}$$

where $K_1, K_2, \cdots, K_{2m-1}$ are penalty factors. The optimization starting point is obtained by the phase shift method described above. Fig. 8 presents an example of a starting point for a three-phase two-cell inverter.

D. Simulation and Experimental Results

Simulation results of multilevel SHEPWM in a double-cell inverter are given in Fig. 9(a) and (b) for single-phase and three-phase systems, respectively. It can be observed in the first and second spectra from the top that low-order harmonics are present. However, these harmonics are out of phase and of equal amplitudes. Consequently, they do not appear at the converter output. Experimental results for the double-cell single-phase and three-phase systems are presented in Figs. 10 and 11. The experimental setup was the same as that used for the mirror surplus harmonics method (see Section II).

The multilevel SHEPWM was also verified for a five-cell series-connected PWM inverter with nine switching angles per quarter wave per cell. The simulation results at a modulation index $M = 1.0$ are presented in Figs. 12 and 13. It can be seen in Fig. 12 that the low-order harmonics are suppressed by more than 45 dB up to the 137th harmonic (8.2 kHz). Fig. 13 shows that the cells share the fundamental component of the output power equally.

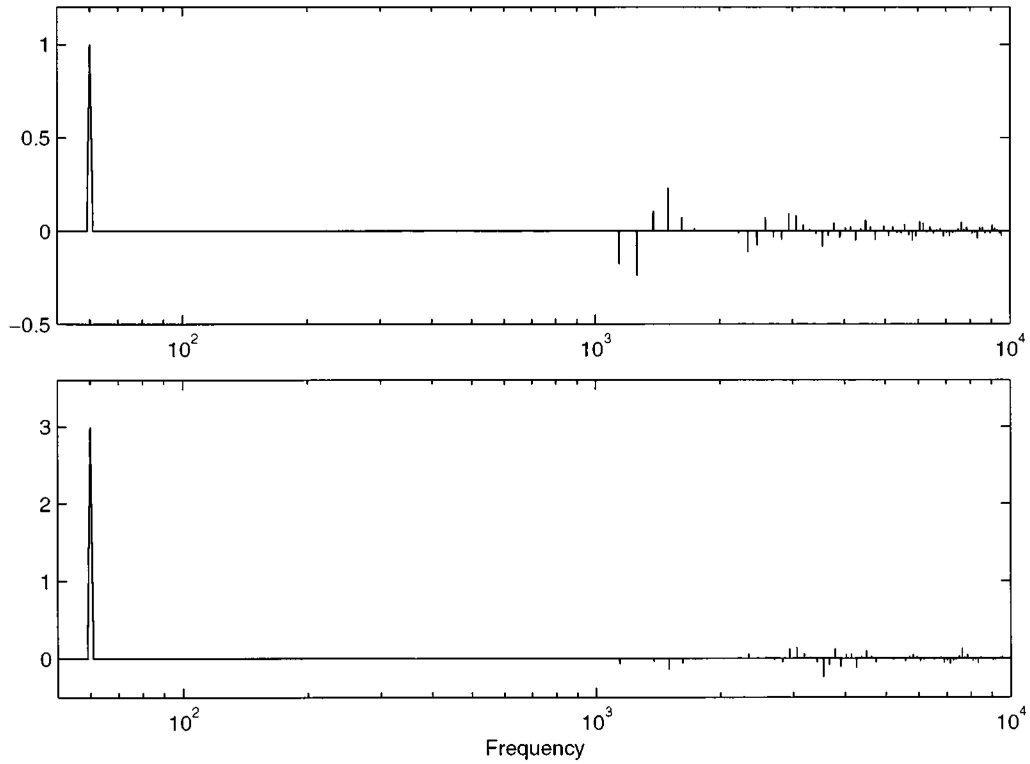


Fig. 7. Frequency spectra of phase-shift harmonic suppression in a three-cell inverter: single-cell voltage (top) and inverter voltage (bottom).

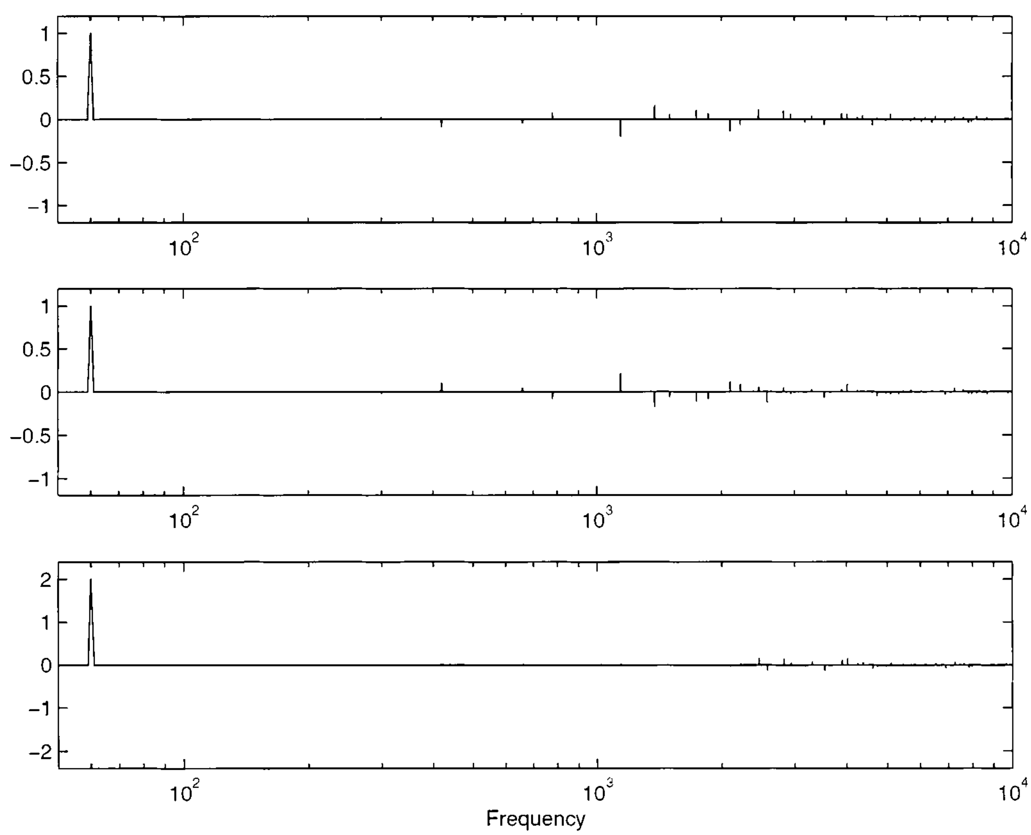


Fig. 8. Frequency spectra of an optimization starting point for a three-phase two-cell inverter obtained by phase-shift harmonic suppression (from top to bottom): upper cell, bottom cell, and inverter output.

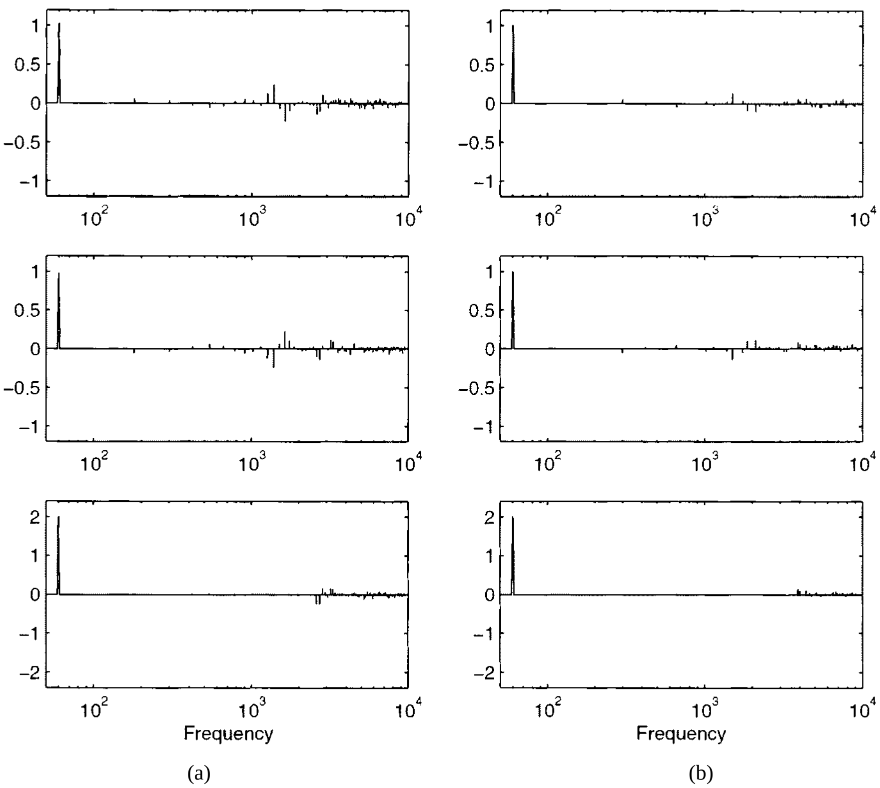


Fig. 9. Calculated frequency spectra for multilevel selective harmonic elimination PWM method. (a) Single-phase system (from top to bottom): two optimal 11 switching angles, and inverter output versus harmonic order. $M = 1.0$. (b) Three phase system (from top to bottom): two optimal 10 switching angles, and one-phase output versus harmonic order. $M = 1.0$.

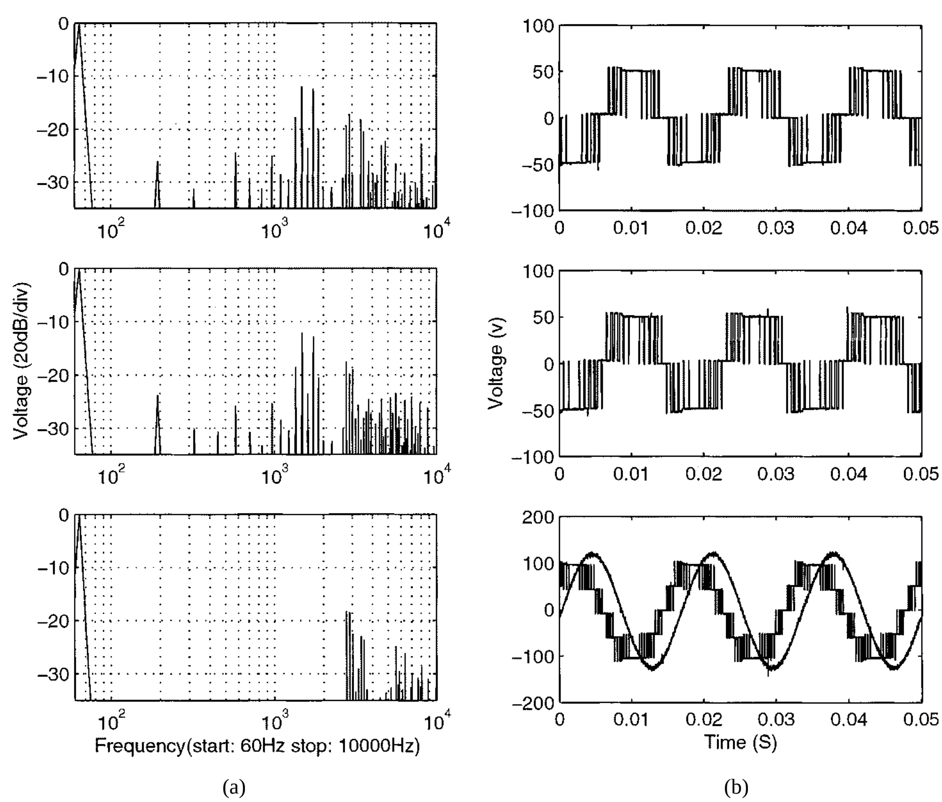


Fig. 10. Experimental single-phase spectra and waveforms with multilevel selective harmonic elimination PWM method. (a) Frequency spectra (from top to bottom): upper cell, lower cell, and inverter output. (b) Output voltage waveforms and load current (from top to bottom): upper cell voltage (V), lower cell voltage (V), inverter output voltage (V), and current (105 V/A).

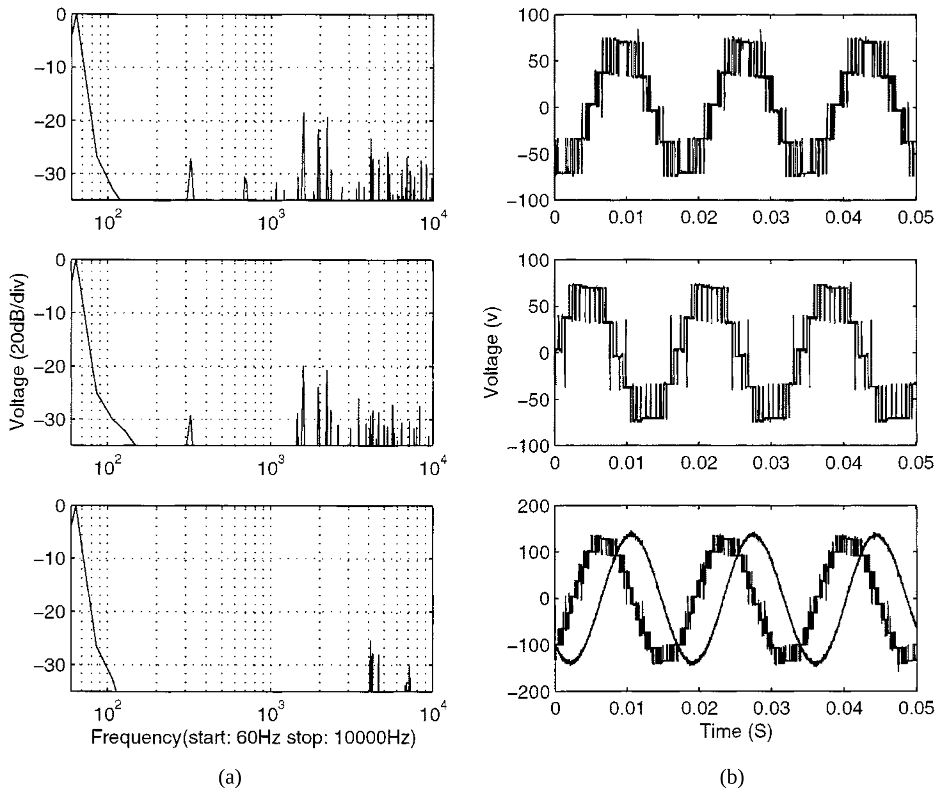


Fig. 11. Experimental three-phase spectra and waveforms with multilevel selective harmonic elimination PWM method. (a) Frequency spectra (from top to bottom): upper cell line-to-line voltage, lower cell line-to-line voltage, and inverter line-to-line output voltage. (b) Output voltage waveforms and load current (from top to bottom): upper cell line-to-line voltage (V), lower cell line-to-line voltage (V), inverter line-to-line output voltage (V), and phase current (125 V/A).

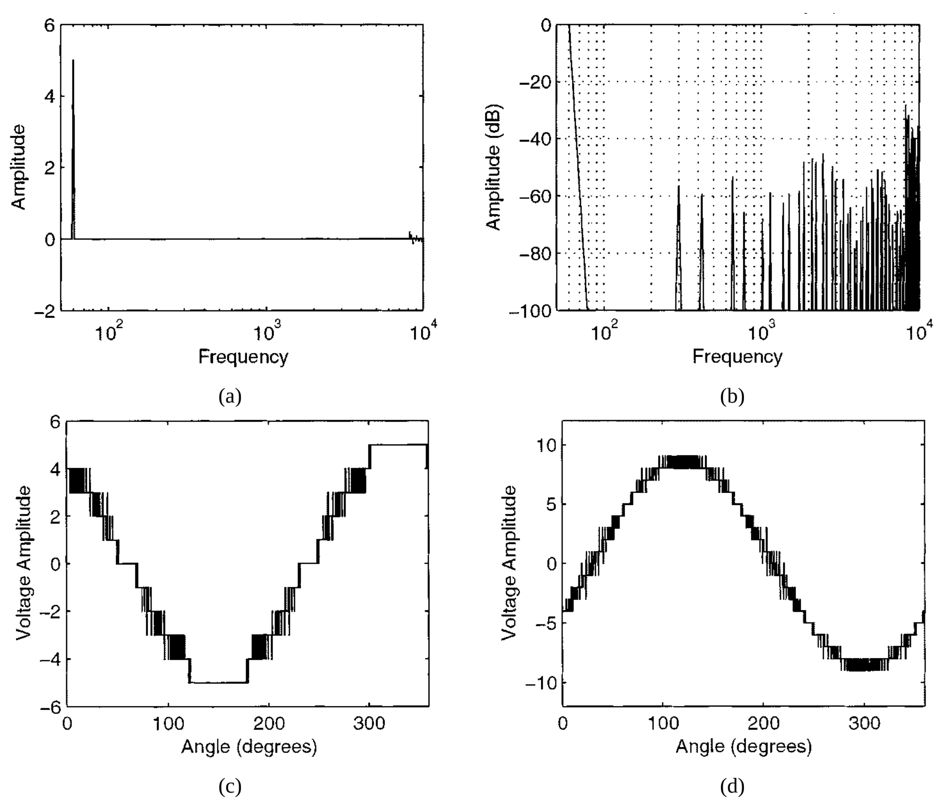
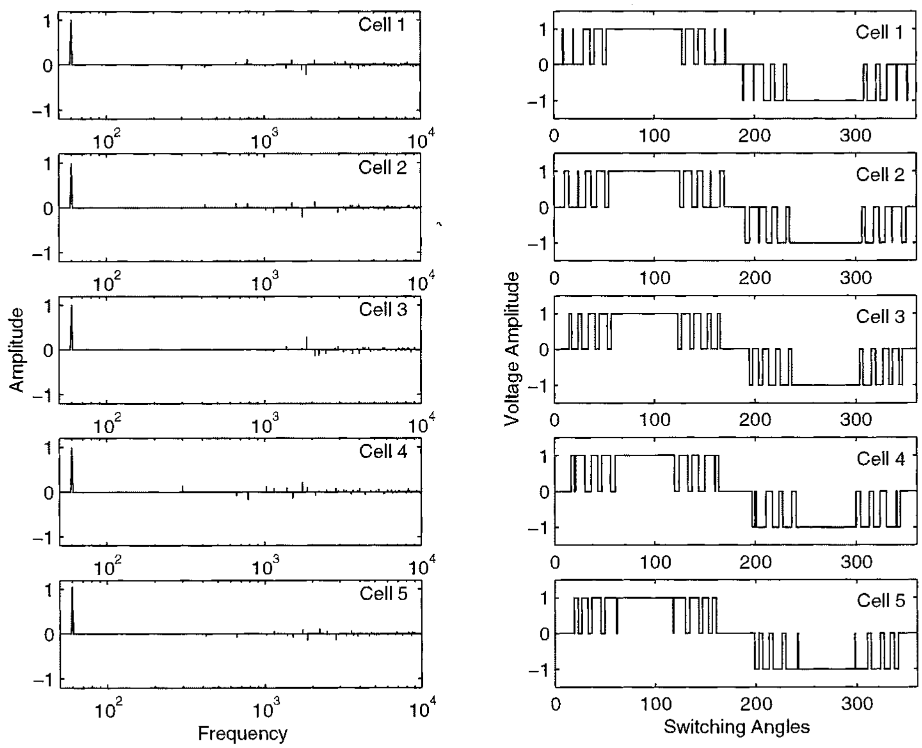
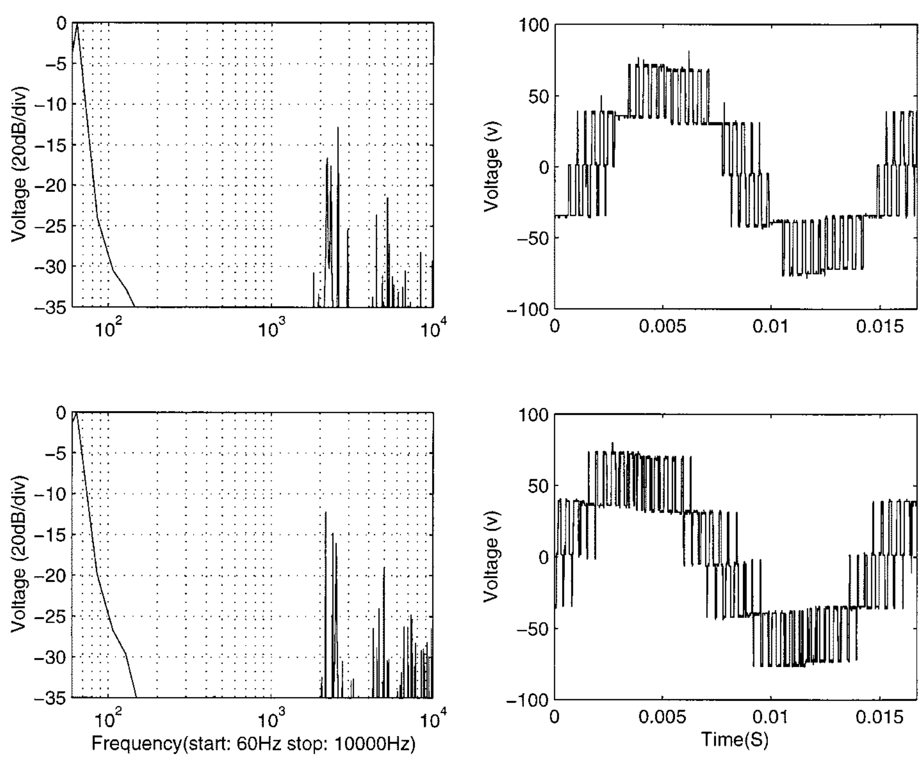


Fig. 12. Simulation results of five cell series-connected inverter. (a) Frequency-analysis PWM. (b) Bode graph of frequency spectra. (c) Phase voltage of five-cell inverters. (d) Line voltage of five-cell inverters.



(a) (b)

Fig. 13. Voltage spectra and waveforms for each cell in five-cell series-connected inverter. (a) Frequency-analysis PWM. (b) Output voltage of one cell.



(a) (b)

Fig. 14. Experimental three-phase spectra and waveforms with selective harmonic elimination PWM method and with regular-sampled PWM method. (a) Frequency spectra of inverter line-to-line output voltages: SHEPWM (top) and regular-sampled PWM (bottom). (b) Waveforms of inverter line-to-line output voltages: SHEPWM (top) and regular-sampled PWM (bottom).

E. Comparison with Sinusoidal Carrier Techniques

The advantage of the SHEPWM method over conventional sinusoidal modulation methods is that it allows the achievement of similar quality of the output waveforms at a reduced switching frequency. This feature is very important for high-power applications in which power devices cannot be switched at high frequencies due to unacceptable switching losses. Fig. 14 compares experimental frequency spectra and time-domain waveforms of line-to-line voltages of a three-phase PWM inverter controlled, in one case, with the selective harmonic elimination PWM method and with the regular-sampled PWM method in the second case. The harmonic spectra of both waveforms are similar. The main harmonic cluster starts around 2 kHz. To achieve such comparable spectra, 38 pulses per phase were needed for the regular-sampled method versus only 22 pulses for SHEPWM. That translates into reduction of the switching frequency by 42%.

The main disadvantage of the SHEPWM method is its complicated hardware implementation. Recent results from the research community [31]–[34] show two approaches to SHEPWM real-time implementation. One is to fine tune conventional PWM techniques like regular-sampled [31] or space-vector methods [33], [34] to closely approximate SHEPWM switching patterns. Another one is to simplify the nonlinear harmonic elimination equations [32] in order to obtain real-time approximate solutions using modern digital signal processors.

IV. Conclusions

A multilevel selected harmonic elimination PWM method has been proposed. The computational difficulties of multilevel SHEPWM methods are overcome by development of an inverter model for nonconstraint optimization. The optimization starting point is obtained using a phase-shift surplus harmonic suppression technique. Simulation and experimental results are presented for a double-cell series-connected voltage source PWM inverter in single-phase and three-phase configuration. Simulation results for a three-phase five-cell inverter are also given. The multilevel SHEPWM method is capable of providing very-high-quality output waveforms.

A new reduced-order method of mirror harmonic suppression in a double-cell series-connected PWM inverter is also suggested. Instead of using a difficult-to-solve system of $2m$ nonlinear equations, the two inverter cells are considered separately. $m-1$ low-order harmonics in the first cell are eliminated with a standard SHEPWM harmonic elimination scheme. An additional switching angle is allowed in the second cell to shape its frequency spectrum in such a way that it mirrors the spectrum of the first cell. The results obtained from the solution of the two systems of equations of order m and $m+1$ closely approximate the solution of a system of $2m$ equation. Hence, the difficulty and amount of calculations are greatly reduced. Experimental tests, conducted for an inverter with $m = 10$ switching angles per quarter wave in the first cell, show harmonic suppression that is comparable with that for a multilevel SHEPWM.

Further work should focus on practical real-time implementation of the multilevel SHEPWM method and extending this approach to optimization of various performance indexes in PWM multilevel inverters, e.g., distortion factor or magnetic noise.

Acknowledgment

The authors would like to thank T. Humiston for his work in implementing the experimental setup used in this paper.

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Li Li (S'97–M'98) was born in Harbin, China. He received the B.S. and M.S. degrees in electrical engineering from Harbin Institute of Technology, Harbin, China, in 1992 and 1995, respectively, and the M.S.E.E., M.S.C.S., and Ph.D. degrees from Polytechnic University, Brooklyn, NY, in 1997, 1998, and 1999, respectively. From 1995 to 1996, he was working toward the Ph.D. degree at Tsinghua University, Beijing, China.

Following receipt of the Ph.D. degree, he joined Pico Electronics, Pelham, NY, as a Design Engineer. Since November 1999, he has been a Member of Technical Staff of Bell Laboratories, Lucent Technologies, Mesquite, TX. His research interests include design and analysis of switch-mode power supplies, ac motor drives, multilevel inverters, PWM techniques, and EMI/EMC techniques.

Dr. Li was the recipient of the Richard Rosenthal Award in 1998 for outstanding performance at Polytechnic University.

Dariusz Czarkowski (M'97) received the M.S. degrees in electronics engineering and in electrical engineering from the University of Mining and Metallurgy, Cracow, Poland, in 1988 and 1989, respectively, the M.S.Egr. degree in electrical engineering from Wright State University, Dayton, OH, in 1993, and the Ph.D. degree in electrical engineering from the University of Florida, Gainesville, in 1996.

In 1989, he joined the Moszczenica Coal Mining Company and, from 1990 to 1991, he was an Instructor at the University of Mining and Metallurgy. From 1991 to 1993, he was a Research Assistant in the Department of Electrical Engineering, Wright State University. In 1993, he joined Wright State University as an adjunct faculty member. In 1994, he became a Research Associate and Doctoral Fellow with the Power Quality and Power Electronics Laboratory, University of Florida. He joined the Polytechnic University, Brooklyn, NY, as an Assistant Professor of electrical engineering in 1996. His research interests are in the areas of modeling and control of power converters, electric drives, power systems, and modern power devices. He is the coauthor of *Resonant Power Converters* (New York: Wiley, 1995).

Dr. Czarkowski is a member of Eta Kappa Nu and Sigma Xi.

Yaguang Liu (S'97–M'00) received the B.S.E.E. and M.S.E.E. degrees from Harbin Institute of Technology, Harbin, China, in 1992 and 1995, respectively, and the Ph.D. degree in electrical engineering from Clarkson University, Potsdam, NY, in 1999.

He is currently a Design Engineer with API Controls, Inc., Amherst, NY. His research interests include power electronics and motor controls.

Pragasen Pillay (S'84–M'87–SM'92) received the Bachelor's degree from the University of Durban-Westville, Durban, South Africa, in 1981, the Master's degree from the University of Natal, Durban, South Africa, in 1983, and the Ph.D. degree from Virginia Polytechnic Institute and State University, Blacksburg, in 1987, while funded by a Fulbright scholarship.

From January 1988 to August 1990, he was with the University of Newcastle, Newcastle upon Tyne, U.K. From August 1990 to August 1995, he was with the University of New Orleans, New Orleans, LA. He is currently a Professor in the Department of Electrical and Computer Engineering, Clarkson University, Potsdam, NY, where he holds the Jean Newell Distinguished Professorship in Engineering. His research and teaching interests are in modeling, design, and control of electric motors and drives.

Dr. Pillay is a member of the IEEE Power Engineering, IEEE Industry Applications, IEEE Industrial Electronics, and IEEE Power Electronics Societies. He is a member of the Electric Machines Committee, Vice Chairman-TRANSACTIONS Paper Reviews of the Industrial Drives Committee, and Vice Chairman of the Continuing Education Committee within the IEEE Industry Applications Society. He has organized and taught short courses in electric drives at the Annual Meetings of the Industry Applications Society. He is a member of the Institution of Electrical Engineers, U.K., and a Chartered Electrical Engineer in the U.K.