Transformer Internal Faults Simulation

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Abstract—this paper presents a novel method of modeling internal faults in a power transformer. The method leads to a model which is compatible with commercial phasor-based software packages. Consequently; it enables calculation of fault currents in any branch of the network due to a winding fault of a power transformer. These currents can be used for evaluation of protective relays' performance and can lead to better setting of protective functions.

Index Terms—Transformers, transformer windings, modeling, short circuit currents, internal fault

I. INTRODUCTION

Here is an ongoing need to develop analyses and systems for better protection of power systems. Transformers are widely used in the power system and are among the most important and the most expensive equipments. Failure of a power transformer may lead to loss of power in parts of a system, which can be costly and inconvenient for customers. Therefore; the protection of power transformers is of high importance.

The internal short circuit currents for a transformer may be larger than its terminal short circuit currents. Accordingly, high current stage protection is used in transformer differential protection in order to protect transformers against such large internal short circuit currents immediately.

Therefore, for adequate power transformer protection, an accurate method of calculation of internal faults should be available.

There are several researches in the area of simulation of internal faults in a power transformer [1]-[4]. Nevertheless; all of these methods consider that the transformer is separated from the network. However; transformers are always used inside an actual power system. Therefore; the contribution of the network in an internal fault in a transformer should be regarded. Moreover; none of the previous methods is capable of considering the transformer internal faults in short circuit analysis, and none has been addressed in the practical fault analysis standards (such as IEC, IEEE, etc.).

Calculation of fault currents due to a winding fault of a power transformer, not only at the terminals of the power transformer but also in any branch of the network, was the motivation for doing this study.

This paper presents a method of modeling internal faults of power transformers in phasor-based fault analysis. The model is fully compatible with software packages which are using phasor-based fault analysis. Moreover; it considers the contribution of the network surrounding the transformer, and can calculate the fault currents in any branch of the network.

The winding fault can be single phase to ground or two phase to ground or three phase to ground. Although internal two phase to ground fault and three phase to ground fault rarely happen in a power transformer, the model enables calculation of fault currents in these situations too.

II. TRANSFORMER MODEL

A. Basic Transformer Model

The basic transformer model used is the one presented in [5]. In this model, the excitation and short circuit tests in positive and zero sequences are used to compute two matrices [R] and [L] modeling for a healthy transformer. In the case of a three phase transformer, with two windings for each phase, these matrices are of order 6 as shown in (1)-(2).

\[
\begin{bmatrix}
R_1 & 0 & 0 & 0 & 0 & 0 \\
0 & R_2 & 0 & 0 & 0 & 0 \\
0 & 0 & R_3 & 0 & 0 & 0 \\
0 & 0 & 0 & R_4 & 0 & 0 \\
0 & 0 & 0 & 0 & R_5 & 0 \\
0 & 0 & 0 & 0 & 0 & R_6 \\
\end{bmatrix}
\] (1)

\[
\begin{bmatrix}
L_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\
M_{21} & L_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\
M_{31} & M_{32} & L_{33} & M_{34} & M_{35} & M_{36} \\
M_{41} & M_{42} & M_{43} & L_{44} & M_{45} & M_{46} \\
M_{51} & M_{52} & M_{53} & M_{54} & L_{55} & M_{56} \\
M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & L_{66} \\
\end{bmatrix}
\] (2)

Where \(R_i\) and \(L_i\) are the resistance and self-inductance of coil i, and \(M_{ij}\) is the mutual inductance between coils i and j, see Figure 1. This model is based on the physical concept of representing windings as mutually coupled coils. The method of calculating the two matrices' elements is summarized below. However; all theoretical details relating to this model are explained in [5].

If the winding resistances are known, the [R] matrix is known. If they are not known, they could be calculated from load losses. For two-winding transformers, one could assume that per unit values of primary and secondary resistances are equal [5].

Figure 1. Three phase transformer windings.
For calculation of the \([L]\) matrix, at first, it is assumed that it is of order 2, as shown in (3).

\[
L = \begin{bmatrix}
L_{aa} & L_{ab} \\
L_{ba} & L_{bb}
\end{bmatrix}
\]  

(3)

Where \(a, b\) are the primary and secondary windings of the three-phase power transformer. Each of the elements of (3) is replaced by a \(3 \times 3\) sub matrix so that \([L]\) will be of order 6 as in (2). The \(3 \times 3\) sub matrices are as shown in (4).

\[
\begin{bmatrix}
L_s & L_m & L_m \\
L_m & L_s & L_m \\
L_m & L_m & L_s
\end{bmatrix}
\]  

(4)

Where \(L_s\) is the self inductance of a phase or leg and \(L_m\) is the mutual inductance among the three phases or legs. These self and mutual inductances are related to the positive and zero sequence values \(L_1\) and \(L_0\) by

\[
L_s = (L_n + 2L_e)/3
\]  

(5)

\[
L_m = (L_0 - L_n)/3
\]  

(6)

Positive and zero sequence inductances of the winding "a" are calculated from the exciting current of the positive and zero sequence excitation tests, if "a" is the excited winding.

\[
L_{0-\text{exc}} = 1/I_{\text{exc} - 0}
\]  

(7)

\[
L_{1-\text{exc}} = 1/I_{\text{exc} - 1}
\]  

(8)

Equations (7), (8) are valid in per unit values. For the other windings, it is reasonable to assume that the per unit inductances are practically the same as in (7), (8). With the diagonal element pairs known, the off-diagonal element pair values \((L_{s-ab}, L_{m-ab})\) are calculated from the short circuit input inductance with (9).

\[
L_{s-ab} = L_{m-ab} = \sqrt{(L_{s} - L_{m})/L_{s}L_{m}}
\]  

(9)

These three sets of equations (of three sequences) can be rewritten as the following equations:

\[
\begin{bmatrix}
V_{p1} \\
V_{p2} \\
V_{p0}
\end{bmatrix} = \begin{bmatrix}
Z_{pp-1} & Z_{pq-1} & -I_{p1} \\
0 & Z_{qq-1} & -I_{q1} \\
0 & 0 & Z_{pp-0}
\end{bmatrix} \begin{bmatrix}
I_{p1} \\
I_{q1} \\
I_{p0}
\end{bmatrix} + \begin{bmatrix}
V^0_p \\
V^0_q \\
0
\end{bmatrix}
\]  

(13)

B. Faulty Transformer Model

In order to model a transformer with a fault between a coil and earth, the faulty coil is divided into two parts [1]. See Figure 2. therefore; \([R]\) and \([L]\) matrices of a transformer with one faulty winding are of order 7.

The \(7 \times 7\) diagonal matrix \([R]\) will be determined with the help of following relations:

\[
R_1 = \frac{n_1}{n} \times R_s
\]  

(10)

\[
R_2 = \frac{n_2}{n} \times R_s
\]  

(11)

Where \(n_1, n_2\) are the number of turns of coils 1,2, \(n\) is the number of turns of whole phase, and \(R_s\) is the total resistance of the healthy phase (\(R_1\) in (1)).

For determining the \(7 \times 7\) matrix \([L]\), the elements related to faulty sub-coils 1 and 2 must be calculated. The other elements are known from the basic transformer model, which is described in the previous section. The method used for calculation of elements associated with faulty sub-coils is like [1], which is verified by experimental results. The difference between model used in this paper and the model presented in [1] is that the model in [1] is a transient model, but the model here is its steady state version.

III. MODELING THE SURROUNDING NETWORK

In order to model the power system surrounding the transformer, two-port thevenin equivalent circuit concept is used [6]. Consider the transformer shown in Figure 3. It is located between busbars p and q in a power system. In order to model the surrounding network by mathematical equations, the transformer is separated from the power system first. Then the thevenin equivalent circuit is calculated:

\[
\begin{bmatrix}
V_{p1} \\
V_{q1} \\
V_{p0}
\end{bmatrix} = \begin{bmatrix}
Z_{pp-1} & Z_{pq-1} & -I_{p1} \\
0 & Z_{qq-1} & -I_{q1} \\
0 & 0 & Z_{pp-0}
\end{bmatrix} \begin{bmatrix}
I_{p1} \\
I_{q1} \\
I_{p0}
\end{bmatrix} + \begin{bmatrix}
V^0_p \\
V^0_q \\
0
\end{bmatrix}
\]  

(12)

Where \(Z_{pp-1}, Z_{pq-1}, Z_{qp-1}, Z_{qq-1}\) are positive sequence elements of the \(Z\)BUS matrix. \(V_p^0, V_q^0\) are pre-fault voltages. The same relation exists for negative and zero sequence networks. However, pre-fault voltages of these networks will be zero.

Figure 2. Three phase transformer with a turn to earth fault.

Figure 3. A transformer between busbars p, q in a power system.
By using the Fortescue transformation, the above equations may be transformed into phase quantities:

\[
\begin{bmatrix}
  V_{p1}^0 \\
  V_{p2}^0 \\
  V_{p0}^0
\end{bmatrix} = \begin{bmatrix}
  Z_{q1} & 0 & 0 \\
  0 & Z_{q2} & 0 \\
  0 & 0 & Z_{q0}
\end{bmatrix} \begin{bmatrix}
  I_{p1} \\
  I_{p2} \\
  I_{p0}
\end{bmatrix} + \begin{bmatrix}
  -I_{p1} \\
  -I_{p2} \\
  -I_{p0}
\end{bmatrix}
\]

(14)

Where \( V_{p1}, V_{p2}, V_{p0} \) are phase voltages of the busbar "p".
\( V_{q1}, V_{q2}, V_{q0} \) are phase voltages of the busbar "q".
\( I_{p1}, I_{p2}, I_{p0} \), \( I_{q1}, I_{q2}, I_{q0} \) are phase currents of I_p and I_q which are shown in Figure 3.

IV. COMBINING TWO MODELS

Transformer and network models are described in the previous sections. The two models must be combined so that the currents in both sides of the transformer can be calculated. The transformer equations with one faulty phase are:

\[
\begin{bmatrix}
  V_{p1} \\
  V_{p2} \\
  V_{p0}
\end{bmatrix} = \begin{bmatrix}
  Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} \\
  Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} \\
  Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37}
\end{bmatrix} \begin{bmatrix}
  I_{1} \\
  I_{2} \\
  I_{3}
\end{bmatrix}
\]

(15)

Where \( V_{p1}, V_{p2}, V_{p0} \) are phase voltages of the busbar "p".
\( V_{q1}, V_{q2}, V_{q0} \) are phase voltages of the busbar "q".
\( I_{p1}, I_{p2}, I_{p0} \), \( I_{q1}, I_{q2}, I_{q0} \) are phase currents of I_p and I_q which are shown in Figure 3.

The elements of the \([A], [B] matrices are dependent upon the transformer's winding connection.

Using (17), (18), the two sets of equations can be solved together. In the case of star winding and not grounded star point, both sets of equations and \([A], [B] matrices are changed slightly since a new variable appears in this case (neutral point voltage). The relations (17), (18) will become:

\[
\begin{bmatrix}
  V_{1} \\
  V_{2} \\
  V_{3}
\end{bmatrix} = \begin{bmatrix}
  A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} \\
  A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} \\
  A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37}
\end{bmatrix} \begin{bmatrix}
  I_{1} \\
  I_{2} \\
  I_{3}
\end{bmatrix}
\]

(19)

In this case, by combining the transformer and the network equations there will be 7 equations and 8 variables. Therefore, an additional equation is needed, for the system will be deterministic as written in (19), (20):

\[
I_{pa} + I_{pb} + I_{pc} = 0
\]

(21)

\[
I_{qa} + I_{qb} + I_{qc} = 0
\]

(22)

The method used for analysis of two or three phase to ground faults is similar to the method which is already presented.

In order to calculate fault currents in other parts of the power system, two sets of currents in primary and secondary sides are transformed into the sequence components. Then, these sequence currents will be injected into the appropriate sequence ZBUS matrices. Therefore; voltages and currents in other parts of power system will be calculated. It should be noted that the corresponding ZBUS matrices of the power system is calculated without considering transformer, i.e. the transformer is separated from the power system.
V. SIMULATION RESULTS

For demonstration of performance of proposed method two simulations has been carried out.

A. Comparison between two faulty transformer models

A comparison is made between transient [1] and steady state models of faulty power transformer, and Figure 4 shows the system model used for this comparison. The transformer is 230kv/109.8kv with YgYg winding connection.

Transformer parameters are as follows:

\[ Z_{s11}=0.2054666+i0.41432E+05 \]
\[ Z_{m11}=i*0.5416527496E-01 \]
\[ Z_{s12}=Z_{s21}=i*0.1977101943E+05 \]
\[ Z_{m12}=Z_{m21}=i*0.9553782211 \]
\[ Z_{s22}=0.742333E-01+i*0.9437875034E+0.4 \]
\[ Z_{m22}=i*0.736830605 \]

Where windings 1, 2 are a, b in (3), and \( Z_{s11}=R_1+i*L_{s11} \). (\( R_1 \) and \( L_{s11} \) are defined in (10), (5) respectively). Load and terminal impedances are as follows:

\[ Z_{LO}=32.55+i*15.76, Z_{TL}=3+i*3 \]

Fault has occurred in 50% percent of phase "a" of primary side winding, and fault time is 0.2 seconds. Figs. 5-11 show the winding’s currents. Table I show the result of this comparison. The closeness of currents calculated by two models validates use of the steady state model.

![Figure 4. System model for an internal fault in winding ap.](image)

![Figure 5. Winding ap1 current for an internal single phase to ground fault.](image)

![Figure 6. Winding ap2 current for an internal single phase to ground fault.](image)

![Figure 7. Winding bp current for an internal single phase to ground fault.](image)

![Figure 8. Winding cp current for an internal single phase to ground fault.](image)

![Figure 9. Winding as current for an internal single phase to ground fault.](image)

![Figure 10. Winding bs current for an internal single phase to ground fault.](image)

![Figure 11. Winding cs current for an internal single phase to ground fault.](image)

<table>
<thead>
<tr>
<th>( i_{ap1}(A) )</th>
<th>( i_{ap2}(A) )</th>
<th>( i_{bp}(A) )</th>
<th>( i_{cp}(A) )</th>
<th>( i_{bp}(A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.020E4</td>
<td>3.017E4</td>
<td>787.009</td>
<td>776.403</td>
<td>1639.1</td>
</tr>
<tr>
<td>3.024E4</td>
<td>3.018E4</td>
<td>783.154</td>
<td>777.580</td>
<td>1637.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80.5395</td>
<td>80.4138</td>
<td>1628.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1626.2</td>
</tr>
</tbody>
</table>

TABLE I. COMPARISON BETWEEN THE RESULTS OF TRANSIENT AND STEADY STATE TRANSFORMER MODELS FOR AN INTERNAL SINGLE PHASE TO GROUND FAULT.
B. Simulation of the proposed method

A sample network is shown in Figure 13. The faulty transformer is located between sub1 and sub2 busbars, to which primary side winding and secondary side winding of transformer are connected respectively. A single phase to ground fault is located in the primary side winding as shown in Figure 12. The transformer's winding connection is DYg1.

Figure 12. A DYg1 transformer with a turn to earth fault.

The transformer and network data are listed in tables II, III.

![Figure 13. A sample network.](image.png)

**TABLE II. TRANSFORMER DATA**

<table>
<thead>
<tr>
<th>Short circuit data</th>
<th>Positive sequence impedance (p.u.)</th>
<th>0.01+i*0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zero sequence impedance (p.u.)</td>
<td>0.01+i*0.1</td>
</tr>
<tr>
<td>Excitation data</td>
<td>Positive sequence current (p.u.)</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>Zero sequence current (p.u.)</td>
<td>0.004</td>
</tr>
</tbody>
</table>

**TABLE III. NETWORK DATA**

<table>
<thead>
<tr>
<th>Positive sequence impedance (p.u.)</th>
<th>0.0261+i*0.0502</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zpp</td>
<td>0.0281+i*0.1294</td>
</tr>
<tr>
<td>Zpq</td>
<td>0.000882+i*0.00173</td>
</tr>
<tr>
<td>Negative sequence impedance (p.u.)</td>
<td>0.0281+i*0.0507</td>
</tr>
<tr>
<td>Zpp</td>
<td>0.0058+i*0.083</td>
</tr>
<tr>
<td>Zpq</td>
<td>0.000241+i*0.001</td>
</tr>
<tr>
<td>Zero sequence impedance (p.u.)</td>
<td>0.0401+i*0.0706</td>
</tr>
<tr>
<td>Zpp</td>
<td>0.0058+i*0.4978</td>
</tr>
<tr>
<td>Zpq</td>
<td>0</td>
</tr>
</tbody>
</table>

The [A], [B] matrices for this transformer are as follows:

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
-\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 & 0 \\
-\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 \\
0 & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]

TABLE IV. COMPARISON OF TERMINAL FAULT CURRENTS

<table>
<thead>
<tr>
<th>Proposed model</th>
<th>Phasor-based software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ipa</td>
<td>15.01&lt;60.71</td>
</tr>
<tr>
<td>Ipb</td>
<td>1.2439&lt;92.49</td>
</tr>
<tr>
<td>Ipc</td>
<td>1.351&lt;69.4</td>
</tr>
<tr>
<td>Ipa</td>
<td>15.02&lt;60.77</td>
</tr>
<tr>
<td>Ipb</td>
<td>1.2325&lt;90.54</td>
</tr>
<tr>
<td>Ipc</td>
<td>1.3717&lt;71.74</td>
</tr>
<tr>
<td>Ipa</td>
<td>2.207&lt;103.230</td>
</tr>
<tr>
<td>Ipb</td>
<td>2.25&lt;102.72</td>
</tr>
<tr>
<td>Ipc</td>
<td>2.1566&lt;84.441</td>
</tr>
<tr>
<td>Ipa</td>
<td>2.1718&lt;83.92</td>
</tr>
<tr>
<td>Ipb</td>
<td>0.2964&lt;7.347</td>
</tr>
<tr>
<td>Ipc</td>
<td>0.2697&lt;8.55</td>
</tr>
</tbody>
</table>

Then, the fault point is moved from terminal of phase a to the terminal of phase c. Magnitude of the fault currents are shown in Figs. 14, 15. If fault point is in the middle of the faulty phase, fault currents of the phases a, c of the primary side are expected to be equal as in Fig.14. Fault currents in phases b, c of the secondary side are also equal, which shows that the model is correctly performed.

VI. CONCLUSION

A new modeling approach has been developed which enables calculation of the transformer internal faults in short circuit analysis. The model is validated and is compatible with phasor-based software packages. Therefore; it can be widely used for the purpose of protection of power transformers and power systems.
Figure 14. Fault currents as a function of fault position (percentage of coil from the terminal of phase a) in the primary side.

Figure 15. Fault currents as a function of fault position (percentage of coil from the terminal of phase a) in the secondary side.

REFERENCES


