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Extension of VIKOR method for multi-criteria group decision making problem with linguistic information

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ABSTRACT

A new method is proposed to solve multi-criteria group decision making problems in which both the criteria values and criteria weights take the form of linguistic information based on the traditional idea of VIKOR method. Firstly, the linguistic criteria weights given by all decision makers are transformed into trapezoidal fuzzy numbers, and then aggregated and defuzzified to crisp values. Secondly, the individual linguistic decision matrix given by each decision maker (DM) is transformed into 2-tuple linguistic decision matrix, and then aggregated into collective 2-tuple linguistic decision matrix by 2-tuple linguistic arithmetic mean operation. Thirdly, the 2-tuple linguistic values (S_i, α_i), (R_i, α_i) and (Q_i, α_i) are calculated by defining the 2-tuple linguistic positive ideal solution (TL-PIS) and 2-tuple linguistic negative ideal solution (TL-NIS). Furthermore, the compromise solution can be obtained. Finally, a numerical example is used to illustrate the application of the proposed approach, and the method is verified by comparing the evaluation result with that of 2-tuple linguistic TOPSIS (TL-TOPSIS) method.

1. Introduction

Multi-criteria decision making (MCDM) is regarded as a main part of modern decision science and operational research, which contains multiple decision criteria and multiple decision alternatives. The objective of the MCDM is to find the most desirable alternative(s) from a set of available alternatives versus the selected criteria [1–4]. The increasing complexity of the engineering and management environment makes it less possible for single decision maker to consider all relevant aspects of a problem. As a result, many decision making processes, in the real world, take place in group settings [5,6]. Therefore, multiple criteria group decision making (MCGDM) problem is a hot research topic which has received a great deal of attention from researchers recently [7–15].

In classical MCDM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise [16]. A suitable approach for dealing with such a problem is to use linguistic assessments instead of numerical ones to represent the subjective judgment of decision makers by means of linguistic variables [6]. For instance, when evaluating the "rescue capacity" or "recovering capacity" of an emergency alternative, terms like "good", "medium", "poor" are usually used, and evaluating an emergency alternative's responding time, terms like "very long", "long", "short" can be used instead of numeric values [17]. Thus, in such situations, the use of linguistic approach is necessary. Since Zadeh [18] introduced fuzzy set theory to deal with vague problems, linguistic variables have been used in

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approximate reasoning within the framework of fuzzy set theory to handle the ambiguity in evaluating data and the vagueness of linguistic expression [19]. So far, many MCGDM problems were studied under linguistic environment [15,19–22].

The available approaches for dealing with linguistic terms can be classified into three categories [23,24]: (1) The extension principle [25]; (2) The symbolic method [26]; and (3) The 2-tuple fuzzy linguistic representation model [23]. In the former two approaches, an approximation process must be developed to express the result in the initial expression domain, for the computation results usually do not exactly match any of the initial linguistic terms. This produces the consequent loss of information and hence the lack of precision [27]. Whereas, the third kind of approach overcomes the above limitations. The advantage of this approach is that linguistic term is regarded as a continuous range instead of a discrete one. Therefore, the approach based on the 2-tuple fuzzy linguistic representation model is more convenient and precise to deal with linguistic terms in solving MCGDM problems.

VIKOR (the Serbian name: VlseKriterijumska Optimizacija I Kompromisno Resenje, means multi-criteria optimization and compromise solution), one of the known MCDM method, was first developed by Opricovic and Tzeng [28,29] for solving MCDM problems. The method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria to help the decision makers (DMs) to reach a final decision. Here, the compromise solution is a feasible solution which is the closest to the ideal solution, and a compromise means an agreement established by mutual concessions. The major advantages of the VIKOR method are that it can trade off the maximum "group utility" of the "majority" and the minimum of the individual regret of the "opponent", and the calculations are simple and straightforward. So, VIKOR method has been widely used to solve MCGDM problems [30–33]. However, the traditional VIKOR methods will fail in dealing with the above MCGDM problems with linguistic criterion weight information. How to evaluate the decision alternatives from the given linguistic criteria values and criteria weights by combining traditional VIKOR method and 2-tuple linguistic method is an interesting and important research topic. This is the motivation of our study.

The purpose of this paper is to extend the concept of VIKOR to develop a new methodology for solving MCGDM problems under linguistic environment, i.e., the information about criteria weights and the criteria values take the form of linguistic variables. In order to do this, the remaining of this paper is organized as follows: In Section 2, we briefly reviews basic definitions of trapezoidal fuzzy numbers, 2-tuple fuzzy linguistic approach and VIKOR technique. An extended VIKOR method is proposed to solve MCGDM problems based on 2-tuple linguistic approach in Section 3. Section 4 presents an illustrative example of emergency alternative selection problem. A comparison study of the proposed method with 2-tuple linguistic TOPSIS (TL-TOPSIS) method is given in Section 5. The paper is concluded in Section 6.

2. Preliminaries

2.1. Trapezoidal fuzzy number

Fuzzy set theory is a very feasible method to handle the imprecise and uncertain information in a real world [34,35]. It can be used to represent linguistic value, which allows the decision makers to incorporate unquantifiable information, incomplete information, non-obtainable information and partially ignorant facts into decision model. Thus, it is more suitable for subjective judgment and qualitative assessment in the evaluation processes of decision making than other classical evaluation methods applying crisp values [36,37]. A fuzzy number is a special fuzzy set $F = \{x \in R | \mu_F(x)\}$, where x takes its values on the real line $R_1 : -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from R_1 to the close interval [0,1].

Definition 2.1. A positive trapezoidal fuzzy number (PTFN) can be denoted as $\bar{A} = (a_1, a_2, a_3, a_4)$, shown in Fig. 1, and the membership function of the fuzzy number $\bar{A}$ is defined as follows:

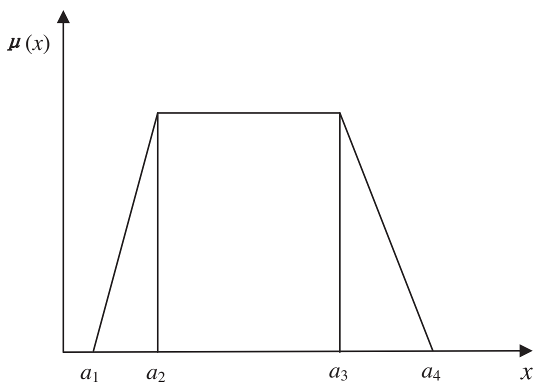

Fig. 1. Trapezoidal fuzzy number $\bar{A}$.

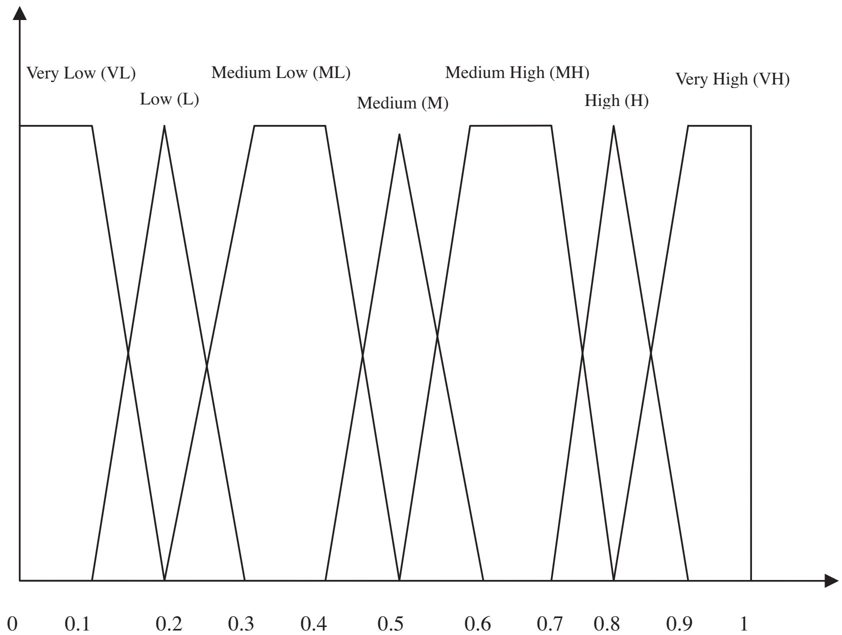

Fig. 2. Linguistic term set of seven labels with its semantics.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1, \\ \frac{x-a_1}{a_2-a_1} & a_1 \leqslant x \leqslant a_2, \\ 1 & a_2 \leqslant x \leqslant a_3, \\ \frac{x-a_4}{a_3-a_4} & a_3 \leqslant x \leqslant a_4, \\ 0 & x > a_4. \end{cases} \tag{1}$$

2.2. 2-Tuple fuzzy linguistic approach

2.2.1. 2-Tuple linguistic representation model

In the fuzzy linguistic approaches, linguistic variables are used to denote words or sentences of a natural language. In order to facilitate the computation and identify the diversity of each evaluation item, linguistic terms are often possessed of some characteristics like finite set, odd cardinality, semantic symmetric, ordinal level and compensative operation [23,38,39]. 2-Tuple linguistic representation model, presented in literature [23], is based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of 2-tuple (l_i, α), where l_i is label from predefined linguistic term set $LT = \{l_0, l_1, l_2, \ldots, l_g\}$, and α is a numerical value representing the symbolic translation. In other words, a 2-tuple linguistic variable can be denoted as (l_i, α), where l_i denotes the central value of the ith linguistic term, and α represents the distance to the central value of the ith linguistic term. For example, a set of seven terms for rating the importance of selected criteria can be given as follows:

$$LT = \{l_0 : VL, l_1 : L, l_2 : ML, l_3 : M, l_4 : MH, l_5 : H, l_6 : VH\}.$$

The meanings of linguistic terms $l_0, l_1, l_2, l_3, l_4, l_5, l_6$ are: "Very Low", "Low", "Medium Low", "Medium", "Medium High"," High", and "Very High", respectively. Each of the linguistic term is assigned one of seven trapezoidal fuzzy numbers whose membership functions are shown in Fig. 2 and Table 1.

2.2.2. Transformation between crisp value and 2-tuple linguistic variables

The 2-tuple linguistic approach was firstly proposed by Herrera and Martínez [23]. In their method the range of β is between 0 and T, which is relevant to the granularity of the linguistic term sets. To overcome this restriction, we adopt the approach proposed by Tai et al. [38,39] in this paper.

Definition 2.2. Let $LT = \{l_0, l_1, \ldots, l_g\}$ be a linguistic term set, and $l_i \in LT$ be a linguistic label. Then the function θ used to obtain the corresponding 2-tuple linguistic information of l_i is defined as follows:

$$\theta : LT \to LT \times \left[-\frac{1}{2g}, \frac{1}{2g}\right)$$

Table 1
Linguistic variable for rating the importance of selected criteria.

Linguistic variable	Trapezoidal fuzzy number
Very Low (VL)	(0.0,0.0, 0.1,0.2)
Low (L)	(0.1,0.2,0.2, 0.3)
Medium Low (ML)	(0.2,0.3,0.4, 0.5)
Medium (M)	(0.4,0.5,0.5, 0.6)
Medium High (MH)	(0.5,0.6,0.7, 0.8)
High (H)	(0.7,0.8,0.8, 0.9)
Very High (VH)	(0.8,0.9,1.0, 1.0)

$$\theta(l_i) = (l_i, 0), \quad l_i \in LT. \tag{2}$$

A crisp value $\beta \in [0, 1]$ can be transformed into the 2-tuple linguistic variable by the following definitions.

Definition 2.3 [38,39]. Let $LT = \{l_0, l_1, \ldots, l_g\}$ be a linguistic term set, $\beta \in [0, 1]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as follows:

$$\Delta : [0, 1] \rightarrow LT \times \left[-\frac{1}{2g}, \frac{1}{2g}\right)$$

$$\Delta(\beta) = (l_i, \alpha) \quad with \quad \begin{cases} l_i & i = round(\beta \times g), \\ \alpha = \beta - \frac{i}{g} & \alpha \in [-\frac{1}{2g}, \frac{1}{2g}), \end{cases} \tag{3}$$

where round (.) is the usual round operation. l_i has the closest index label to β and α is the value of the symbolic translation. The interval of value α is decided by the number of linguistic terms in LT. For example, if LT contains seven linguistic terms, then $g = 6$ and $\alpha \in [-0.083, 0.083)$.

Definition 2.4 [38,39]. If LT is a linguistic term set, $LT = \{l_0, l_1, \ldots, l_g\}$, (l_i, α) is 2-tuple linguistic information, then there exists a function Δ^{-1}, which is able to transform 2-tuple linguistic information into its equivalent numerical value $\beta \in [0, 1]$. The function Δ^{-1} is defined as follows:

$$\Delta^{-1} : LT \times \left[-\frac{1}{2g}, \frac{1}{2g}\right) \rightarrow [0, 1]$$

$$\Delta^{-1}[l_i, \alpha] = \frac{i}{g} + \alpha = \beta. \tag{4}$$

We may implement the comparison of linguistic information represented by 2-tuples. Let (l_i, α_1) and (l_j, α_2) be two 2-tuples, with each one representing a linguistic assessment:

(1) If $i < j$ then (l_i, α_1) is worse than (l_j, α_2),
(2) If $i = j$ and $\alpha_1 = \alpha_2$ then (l_i, α_1) is equal to (l_j, α_2),
(3) If $i = j$ and $\alpha_1 < \alpha_2$ then (l_i, α_1) is worse than (l_j, α_2),
(4) If $i = j$ and $\alpha_1 > \alpha_2$ then (l_i, α_1) is better than (l_j, α_2).

Definition 2.5 [38,39]. Let $LT = \{(l_1, \alpha_1), (l_1, \alpha_1), \ldots, (l_{g+1}, \alpha_{g+1})\}$ be a 2-tuple linguistic variable set, their arithmetic mean $\bar{L}$ can be calculated as follows:

$$\bar{L} = \Delta\left[\frac{1}{g+1}\sum_{i=1}^{g+1}\Delta^{-1}(l_i, \alpha_i)\right] = \Delta\left(\frac{1}{g+1}\sum_{i=1}^{g+1}\beta_i\right) = (l_m, \alpha_m), \tag{5}$$

where l_m is one linguistic term of set LT, and $-\frac{1}{2g} \leqslant \alpha_m < \frac{1}{2g}$.

2.3. VIKOR method

The basic concept of VIKOR method lies in defining the positive and negative ideal points, which was first put forth by Opricovic and Tzeng [27,28]. The VIKOR method is based on the compromise idea of MCDM. The various alternatives are denoted as $A_1, A_2, \ldots, A_m$, and all criteria are denoted as $C_1, C_2, \ldots, C_n$. For an alternative A_i, f_{ij} is the value of the jth criterion C_j. The compromise ranking algorithm is briefly reviewed as follows:

Step 1. Determine the best f_j^+ and the worst f_j^- values of all criteria. Assuming that the jth criterion represents a benefit:

$$f_j^+ = \max_{1 \leqslant i \leqslant m} f_{ij}, j = 1, 2, \ldots, n,$$

$$f_j^- = \min_{1\leqslant i\leqslant m} f_{ij}, j = 1,2,\ldots,n.$$

Step 2. Compute the values S_i and R_i, i = 1,2,3,. . .,m, by Eqs. (6) and (7).

$$S_i = \sum_{j=1}^{n}[w_j(f_j^+ - f_{ij})/(f_j^+ - f_j^-)], i = 1,2,\ldots,m, \tag{6}$$

$$R_i = \max_{1\leqslant j\leqslant n}[w_j(f_j^+ - f_{ij})/(f_j^+ - f_j^-)], i = 1,2,\ldots,m, \tag{7}$$

where

$$w_j (j = 1,2,\ldots,n)$$

are the weights of criteria, representing the decision maker's relative preference for the importance of the criteria.

Step 3. Calculate the values Q_i, i = 1,2,. . .,m, by Eq. (8).

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1 - v)(R_i - R^*)/(R^- - R^*), i = 1,2,\ldots,m, \tag{8}$$

where

$$S^* = \min_{1\leqslant i\leqslant m} S_i,$$

$$S^- = \max_{1\leqslant i\leqslant m} S_i,$$

$$R^* = \min_{1\leqslant i\leqslant m} R_i,$$

$$R^- = \max_{1\leqslant i\leqslant m} R_i.$$

and v is a weighting reference. v is introduced as the weight of the strategy of maximum group utility, whereas $(1-v)$ is the weight of the individual regret. Thus, when v is larger ($v > 0.5$), the index of Q_i will tend to majority rule.

Step 4. Rank the alternatives, sorting by the values S, R, and Q in ascending order. The results are three ranking lists.

Step 5. Propose a compromise solution, the alternative ($A^{(1)}$), which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

Con_1. Acceptable advantage:

$$Q(A^{(2)}) - Q(A^{(1)}) \geqslant DQ, \tag{9}$$

where $A^{(2)}$ is the alternative with second position in the raking list by Q,

$$DQ = 1/(n - 1)$$

and n is the number of alternatives.

Con_2. Acceptable stability in decision making:

The alternative $A^{(1)}$ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or "by consensus" ($v \approx 0.5$), or "with veto" ($v < 0.5$). Here, v is the weight of decision making strategy of maximum group utility. If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition Con_2 is not satisfied, or alternatives $A^{(1)}$, $A^{(2)}$ . . .,$A^{(M)}$ if the condition Con_1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the positions of these alternatives are "in closeness").

3. Extended VIKOR method for group decision making problem based on 2-tuple linguistic approach

In real world, determining the exact values of the criteria is difficult or impossible, it is more appropriate to consider them as linguistic variable. Therefore, we extend the VIKOR method to solve multi-criteria group decision making (MCGDM) problem with linguistic criteria values and criteria weights in this paper. The decision making matrix $D_k = (d_{ij}^{(k)})_{m\times n}$ given by the kth decision maker can be described as follows:

$$D_k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} d_{11}^{(k)} & d_{12}^{(k)} & \dots & d_{1n}^{(k)} \\ d_{21}^{(k)} & d_{21}^{(k)} & \dots & d_{2n}^{(k)} \\ \vdots & \vdots & \dots & \vdots \\ d_{m1}^{(k)} & d_{m2}^{(k)} & \dots & d_{mn}^{(k)} \end{bmatrix} \end{array}, \quad k = 1, 2, ..., K.$$

$$\lambda^{(k)} = (\lambda_1^{(k)}, \lambda_2^{(k)}, \dots, \lambda_n^{(k)}), \quad k = 1, 2, \dots, K.$$

where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria with which alternative performance is measured, K is the number of DMs, $d_{ij}^{(k)}$ is the rating of alternative A_i with respect to the criterion C_j given by the kth DM, $\lambda_j^{(k)}$ is the weight of criterion C_j given by the kth DM, where $\lambda_j^{(k)}$ and $d_{ij}^{(k)}$ take the form of linguistic variable. Decision makers can use the linguistic term set LT_1 = {Very low (VL), Low (L), Medium low (ML), medium (M), Medium high (MH), High (H) and Very high (VH)} shown in Table 1 to evaluate the importance of the criteria, and use the linguistic term set LT_2 = {Very poor (VP), Poor (P), Medium poor (MP), Fair (F), Medium good (MG), Good (G) and Very good (VG)} shown in Table 2 to evaluate the rating of decision alternative with respect to each criterion.

The procedure of the extended VIKOR method for MCGDM problem is described as follows:

Step 1. Convert linguistic decision matrix $D_k = (d_{ij}^{(k)})_{m\times n}$ into 2-tuple linguistic decision matrix $T_k = (t_{ij}^{(k)}, 0)_{m\times n}$ by Eq. (2), where $t_{ij}^{(k)} \in LT_2$.

Step 2. Calculate the collective 2-tuple linguistic decision matrix $T = (t_{ij}, \alpha_{ij})_{m\times n}$, where

$$(t_{ij}, \alpha_{ij}) = \Delta\left[\frac{1}{K}\sum_{k=1}^{K}\Delta^{-1}(t_{ij}^{(k)}, 0)\right], \qquad i = 1, 2, \dots, m; \;\; j = 1, 2, \dots, n. \tag{10}$$

Step 3. Determine the criteria weights by aggregating the criteria weights provided by DMs. Let the linguistic weight of the criterion C_j given by the kth decision maker be $\lambda_j^{(k)} \in LT_1$, and the corresponding trapezoidal fuzzy number $\tilde{w}_j^{(k)}$ can be denoted by Eq. (11).

$$\tilde{w}_j^{(k)} = (\bar{w}_{j1}^{(k)}, \bar{w}_{j2}^{(k)}, \bar{w}_{j3}^{(k)}, \bar{w}_{j4}^{(k)}) \quad j = 1, 2, \dots, n; \quad k = 1, 2, \dots, K. \tag{11}$$

The aggregated fuzzy weight of each criterion can be calculated as follows:

$$\tilde{w}_j = (\bar{w}_{j1}, \bar{w}_{j2}, \bar{w}_{j3}, \bar{w}_{j4}) \quad j = 1, 2, \dots, n, \tag{12}$$

where

$$\bar{w}_{j1} = \min_{1\leqslant k\leqslant K}\{\bar{w}_{j1}^{(k)}\},$$

$$\bar{w}_{j2} = \frac{1}{K}\sum_{k=1}^{K}\bar{w}_{j2}^{(k)},$$

$$\bar{w}_{j3} = \frac{1}{K}\sum_{k=1}^{K}\bar{w}_{j3}^{(k)},$$

$$\bar{w}_{j4} = \max_{1\leqslant k\leqslant K}\{\bar{w}_{j4}^{(k)}\}.$$

Table 2
Linguistic variables for the ratings of alternatives.

Linguistic variable	Trapezoidal fuzzy number
Very Poor (VP)	(0.0,0.0, 0.1,0.2)
Poor (P)	(0.1,0.2,0.2, 0.3)
Medium Poor (MP)	(0.2,0.3,0.4, 0.5)
Fair (F)	(0.4,0.5,0.5, 0.6)
Medium Good (MG)	(0.5,0.6,0.7, 0.8)
Good (G)	(0.7,0.8,0.8, 0.9)
Very Good (VG)	(0.8,0.9,1.0, 1.0)

To understand the importance of the criteria, the center of area (COA) method is utilized to defuzzify the trapezoidal fuzzy number into best nonfuzzy performance (BNP) value [40]. The BNP values of fuzzy number $\bar{w}_j = (\bar{w}_{j1}, \bar{w}_{j2}, \bar{w}_{j3}, \bar{w}_{j4})$, $j = 1,2,\ldots,n$, can be calculated by Eq. (13):

$$\bar{w}_j = \frac{\int xu(x)dx}{\int u(x)dx} = \frac{-\bar{w}_{j1} \times \bar{w}_{j2} + \bar{w}_{j3} \times \bar{w}_{j4} + \frac{1}{3} \times (\bar{w}_{j4} - \bar{w}_{j3})^2 - \frac{1}{3} \times (\bar{w}_{j2} - \bar{w}_{j1})^2}{-\bar{w}_{j1} - \bar{w}_{j2} + \bar{w}_{j3} + \bar{w}_{j4}}. \tag{13}$$

Then the normalized weights of criteria C_j, $j = 1,2,\ldots,n$, can be calculated by Eq. (14):

$$w_j = \frac{\bar{w}_j}{\sum_{j=1}^{n} \bar{w}_j}, j = 1,2,\ldots,n. \tag{14}$$

Obviously, $w_j (j = 1,2,\ldots,n)$ satisfy the following conditions: $w_j \geqslant 0 (j = 1,2,\ldots,n)$, and $\sum_{j=1}^{n} w_j = 1$.

Step 4. Define the 2-tuple linguistic positive ideal solution (TL-PIS) and 2-tuple linguistic negative ideal solution (TL-NIS) of the collective 2-tuple linguistic decision matrix $T = (t_{ij}, \alpha_{ij})_{m \times n}$:

$$(t^+, \alpha^+) = ((t_1^+, \alpha_1^+), (t_2^+, \alpha_2^+), \ldots, (t_n^+, \alpha_n^+)), \tag{15}$$

$$(t^-, \alpha^-) = ((t_1^-, \alpha_1^-), (t_2^-, \alpha_2^-), \ldots, (t_n^-, \alpha_n^-)), \tag{16}$$

where

$$(t_j^+, \alpha_j^+) = \max_{1 \leqslant i \leqslant m} (t_{ij}, \alpha_{ij}) \quad j = 1,2,\ldots,n,$$

$$(t_j^-, \alpha_j^-) = \min_{1 \leqslant i \leqslant m} (t_{ij}, \alpha_{ij}) \quad j = 1,2,\ldots,n,$$

Step 5. Calculate the 2-tuple linguistic values (S_i, α_i) and (R_i, α_i), $i=1,2,\ldots,m$, by Eqs. (17) and (18), respectively.

$$(S_i, \alpha_i) = \Delta\left\{ \sum_{j=1}^{n} w_j [\Delta^{-1}(t_j^+, \alpha_j^+) - \Delta^{-1}(t_{ij}, \alpha_{ij})] / [\Delta^{-1}(t_j^+, \alpha_j^+) - \Delta^{-1}(t_j^-, \alpha_j^-)] \right\}, \tag{17}$$

$$(R_i, \alpha_i) = \Delta\left\{ \max_{1 \leqslant j \leqslant n} w_j [\Delta^{-1}(t_j^+, \alpha_j^+) - \Delta^{-1}(t_{ij}, \alpha_{ij})] / [\Delta^{-1}(t_j^+, \alpha_j^+) - \Delta^{-1}(t_j^-, \alpha_j^-)] \right\}, \tag{18}$$

where w_j is the weight of criteria $C_j (j = 1,2,\ldots,n)$ in Eq. (14).

Step 6. Calculate the 2-tuple linguistic values $(Q_i, \alpha_i), i = 1,2,\ldots,m$ by Eq. (19).

$$(Q_i, \alpha_i) = \Delta[v(\beta_i - \beta^-)/(\beta^+ - \beta^-) \quad + (1 - \quad v)(\gamma_i - \gamma^-)/(\gamma^+ - \gamma^-) \ ], \tag{19}$$

where

$$\beta_i = \Delta^{-1}(S_i, \alpha_i),$$

$$\beta^+ = \Delta^{-1}[\max_{1 \leqslant i \leqslant m} (S_i, \alpha_i)],$$

$$\beta^- = \Delta^{-1}[\min_{1 \leqslant i \leqslant m} (S_i, \alpha_i)],$$

$$\gamma_i = \Delta^{-1}(R_i, \alpha_i),$$

$$\gamma^+ = \Delta^{-1}[\max_{1 \leqslant i \leqslant m} (R_i, \alpha_i)],$$

$$\gamma^- = \Delta^{-1}[\min_{1 \leqslant i \leqslant m} (R_i, \alpha_i)].$$

Step 7. Rank the alternatives, sorting by the values (S_i, α_i), (R_i, α_i), and (Q_i, α_i) in ascending order. The results are three ranking lists.

Step 8. Propose a compromise solution, the alternative ($A^{(1)}$), which is ranked the best by the measure $\min_{1 \leqslant i \leqslant m} (Q_i, \alpha_i)$ if the following two conditions are satisfied:

Con$_1$. Acceptable advantage:

$$\Delta^{-1}(Q(A^{(2)}), \alpha(A^{(2)})) - \Delta^{-1}(Q(A^{(1)}), \alpha(A^{(1)})) \geqslant DQ, \tag{20}$$

where $A^{(2)}$ is the alternative with second position in the raking list by (Q_i, α_i), $DQ = 1/(n-1)$ and n is the number of alternatives.

Con$_2$. Acceptable stability in decision making:

The alternative $A^{(1)}$ must also be the best ranked by (S_i, α_i) or/and (R_i, α_i). If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition Con_2 is not satisfied, or alternatives $A^{(1)}$, $A^{(2)} \ldots, A^{(M)}$ if the condition C_1 is not satisfied, and $A^{(M)}$ is determined by the relation:

$$\Delta^{-1}(Q(A^{(M)}), \alpha(A^{(M)})) - \Delta^{-1}(Q(A^{(1)}), \alpha(A^{(1)})) < DQ$$

for maximum M (the positions of these alternatives are "in closeness").

4. Application of the proposed method

In this section, an illustrative example is presented to illustrate the application of the proposed method for emergency alternative selection problem. To implement the rescue action of a certain urban fire happened in China, a committee of three decision makers (DM_1, DM_2 and DM_3) is formed to evaluate and select the most suitable emergency alternative. In this example, it is assumed that degrees of the importance for three decision makers are equal. Five possible emergency alternatives A_i (i = 1,2,...,5) are chosen, and six criteria C_j (j = 1,2,...,6) are selected for further evaluation, i.e., information transmission capability (C_1), command capability (C_2), rescue department speed (C_3), emergency plan and simulation exercise (C_4), collaboration capability (C_5) and forecasting capability (C_6).

The weights of the six criteria are described by using the linguistic term set: LT_1 = {Very low (VL), Low (L), Medium low (ML), Medium (M), Medium high (MH), High (H) and Very high (VH)}, which is defined in Table 1.

The performance ratings of the alternatives with respect to criteria are characterized by the linguistic term set: LT_2 = {Very poor (VP), Poor (P), Medium poor (MP), Fair (F), Medium good (MG), Good (G) and Very good (VG)}, i.e., $LT_2 = \{l_0, l_1, l_2, l_3, l_4, l_5, l_6\}$, which is defined in Table 2.

The weights of these six criteria are obtained from three decision makers according to linguistic terms in Table 1 and are presented in Table 3. For example, the linguistic weights of all criteria given by DM_1 are as follows:

$$\lambda_1^{(1)} = H, \quad \lambda_2^{(1)} = VH, \quad \lambda_3^{(1)} = VH, \quad \lambda_4^{(1)} = H, \quad \lambda_5^{(1)} = H, \quad \lambda_6^{(1)} = H.$$

and corresponding trapezoidal fuzzy numbers of each criterion given by DM_1 are as follows:

$$\tilde{w}_1^{(1)} = (0.7, 0.8, 0.8, 0.9),$$

$$\tilde{w}_2^{(1)} = (0.8, 0.9, 1.0, 1.0),$$

$$\tilde{w}_3^{(1)} = (0.8, 0.9, 1.0, 1.0),$$

$$\tilde{w}_4^{(1)} = (0.7, 0.8, 0.8, 0.9),$$

$$\tilde{w}_5^{(1)} = (0.7, 0.8, 0.8, 0.9),$$

$$\tilde{w}_6^{(1)} = (0.7, 0.8, 0.8, 0.9).$$

The ratings of alternatives with respect to criteria are given by three decision makers according to linguistic terms shown in Table 2, and the linguistic decision matrix $D_k = (d_{ij}^{(k)})_{m\times n}$ are shown in Table 4.

The computational procedure of selecting emergency alternatives is composed of the following steps.

Step 1. Convert linguistic decision matrix shown in Table 4 into 2-tuple linguistic decision matrix $T_k = (t_{ij}^{(k)}, 0)_{m\times n}$ which is shown in Table 5.

Step 2. The ratings of five emergency alternatives obtained from three DMs are aggregated by Eq. (10) and shown in Table 6. For example, the aggregated rating of alternative A_1 with respect to C_1 is computed as follows:

Table 3
Linguistic evaluation of importance of each criterion.

Criteria	DMs		
	DM_1	DM_2	DM_3
C_1	H	H	H
C_2	VH	VH	H
C_3	VH	VH	VH
C_4	H	H	MH
C_5	H	H	H
C_6	H	MH	M

Table 4
Rating of emergency alternatives with respect to each criterion.

DMs	Alternatives	Criteria					
		C_1	C_2	C_3	C_4	C_5	C_6
DM_1	A_1	G	VG	G	G	G	VG
	A_2	VG	MP	G	VG	VG	G
	A_3	VG	G	F	VG	G	G
	A_4	G	G	MG	G	G	MG
	A_5	MG	F	MG	MG	MG	F
DM_2	A_1	G	VG	G	G	G	G
	A_2	G	G	F	VG	MG	MG
	A_3	VG	G	F	VG	VG	G
	A_4	G	G	MG	G	G	F
	A_5	MG	F	MG	MG	MG	F
DM_3	A_1	VG	VG	G	G	G	VG
	A_2	G	G	MP	VG	VG	G
	A_3	G	VG	F	VG	G	VG
	A_4	G	MG	G	G	VG	MG
	A_5	MG	F	MG	G	MG	MG

$$(t_{11},\alpha_{11}) = \Delta\{\tfrac{1}{3}\times[\Delta^{-1}(l_5,0)+\Delta^{-1}(l_5,0)+\Delta^{-1}(l_6,0)]\} = \Delta[\tfrac{1}{3}\times(0.8333+0.8333+1)] = \Delta(0.8888) = (l_5,0.0556).$$

Step 3. The linguistic weights of criteria are transformed into corresponding trapezoidal fuzzy numbers shown in Table 1. The aggregated fuzzy weights of criteria are computed by Eqs. (11)–(13). For example, the aggregated fuzzy weight of criterion C_2, i.e., $\tilde{w}_2 = (0.7, 0.87, 0.93, 1.0)$, is computed as follows:

$$\bar{w}_{21} = \min\{0.7, 0.8, 0.8\} = 0.7,$$

$$\bar{w}_{22} = \frac{1}{3}\times(0.8+0.9+0.9) = 0.87,$$

$$\bar{w}_{23} = \frac{1}{3}\times(0.8+1.0+1.0) = 0.93,$$

$$\bar{w}_{24} = \max\{0.9, 1.0, 1.0\} = 1.0.$$

Similarly, we can obtain the aggregated fuzzy weights of other criteria:

$$\tilde{w}_1 = (0.70, 0.80, 0.80, 0.90),$$

$$\tilde{w}_3 = (0.80, 0.90, 1.00, 1.00),$$

$$\tilde{w}_4 = (0.50, 0.73, 0.77, 0.90),$$

$$\tilde{w}_5 = (0.70, 0.80, 0.80, 0.90),$$

$$\tilde{w}_6 = (0.40, 0.63, 0.68, 0.90).$$

The BNP value of the aggregated fuzzy number is computed by Eq. (14). For example, the BNP value of the aggregated fuzzy weight $\tilde{w}_2 = (0.7, 0.87, 0.93, 1.0)$ is computed as follows:

$$\bar{w}_2 = \frac{-0.7\times0.87+0.93\times1.0+\frac{1}{3}\times(1.0-0.93)^2-\frac{1}{3}\times(0.87-0.7)^2}{-0.7-0.87+0.93+1.0} = 0.87.$$

The BNP values of other aggregated fuzzy weights are shown in the third column of the Table 7.

Then the normalized weights of criteria can be calculated by Eq. (14) as shown in the last column of the Table 7.

Step 4.The TL-PIS and TL-NIS of the collective 2-tuple linguistic decision matrix are identified and shown in Table 8.

Step 5. The 2-tuple linguistic values (S_i, α_i), $i = 1,2,\ldots,m$, are calculated by Eq. (17) and shown in Table 9. For example, the 2-tuple linguistic value of alternative A_1 is computed as:

Table 5
2-Tuple linguistic decision matrix of three DMs.

DMs	Alternatives	Criteria					
		C_1	C_2	C_3	C_4	C_5	C_6
DM_1	A_1	(l_5,0)	(l_6,0)	(l_5,0)	(l_5,0)	(l_5,0)	(l_6,0)
	A_2	(l_6,0)	(l_2,0)	(l_5,0)	(l_6,0)	(l_6,0)	(l_5,0)
	A_3	(l_6,0)	(l_5,0)	(l_3,0)	(l_6,0)	(l_5,0)	(l_5,0)
	A_4	(l_5,0)	(l_5,0)	(l_4,0)	(l_5,0)	(l_5,0)	(l_4,0)
	A_5	(l_4,0)	(l_3,0)	(l_4,0)	(l_4,0)	(l_4,0)	(l_3,0)
DM_2	A_1	(l_5,0)	(l_6,0)	(l_5,0)	(l_5,0)	(l_5,0)	(l_5,0)
	A_2	(l_5,0)	(l_5,0)	(l_3,0)	(l_6,0)	(l_4,0)	(l_4,0)
	A_3	(l_6,0)	(l_5,0)	(l_3,0)	(l_6,0)	(l_6,0)	(l_5,0)
	A_4	(l_5,0)	(l_5,0)	(l_4,0)	(l_5,0)	(l_5,0)	(l_3,0)
	A_5	(l_4,0)	(l_3,0)	(l_4,0)	(l_4,0)	(l_4,0)	(l_3,0)
DM_3	A_1	(l_6,0)	(l_6,0)	(l_5,0)	(l_5,0)	(l_5,0)	(l_6,0)
	A_2	(l_5,0)	(l_5,0)	(l_2,0)	(l_6,0)	(l_6,0)	(l_5,0)
	A_3	(l_5,0)	(l_6,0)	(l_3,0)	(l_6,0)	(l_5,0)	(l_6,0)
	A_4	(l_5,0)	(l_4,0)	(l_5,0)	(l_5,0)	(l_6,0)	(l_4,0)
	A_5	(l_4,0)	(l_3,0)	(l_4,0)	(l_5,0)	(l_4,0)	(l_4,0)

Table 6
Aggregated 2-tuple linguistic decision matrix by three DMs.

Alternatives	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(l_5,0.0556)	(l_6, 0.0000)	(l_5,0.0000)	(l_5, 0.0000)	(l_5, 0.0000)	(l_6,–0.0556)
A_2	(l_5,0.0556)	(l_4, 0.0000)	(l_3,0.0556)	(l_6, 0.0000)	(l_5,0.0556)	(l_5,–0.0556)
A_3	(l_6,–0.0556)	(l_5,0.0556)	(l_3, 0.0000)	(l_6, 0.0000)	(l_5,0.0556)	(l_5,0.0556)
A_4	(l_5, 0.0000)	(l_5,–0.0556)	(l_4,0.0556)	(l_5, 0.0000)	(l_5,0.0556)	(l_4,–0.0556)
A_5	(l_4, 0.0000)	(l_3, 0.0000)	(l_4, 0.0000)	(l_4,0.0556)	(l_4, 0.0000)	(l_3,0.0556)

Table 7
Aggregated fuzzy weights, BNP values and normalized weights.

Criteria	Aggregated fuzzy Weights	BNP values	Normalized Weights
C_1	(0.70,0.80,0.80,0.90)	0.80	0.1681
C_2	(0.70,0.87,0.93,1.00)	0.87	0.1828
C_3	(0.80,0.90,1.00,1.00)	0.92	0.1933
C_4	(0.50,0.73,0.77,0.90)	0.72	0.1513
C_5	(0.70,0.80,0.80,0.90)	0.80	0.1681
C_6	(0.40,0.63,0.68,0.90)	0.65	0.1366

$$
\begin{aligned}
(S_1,\alpha_1) &= \Delta\{0.1681\times[\Delta^{-1}(l_6,-0.0556)-\Delta^{-1}(l_5,0.0556)]/[\Delta^{-1}(l_6,-0.0556)-\Delta^{-1}(l_4,0)]+0.1828\times[\Delta^{-1}(l_6,0)\\
&\quad-\Delta^{-1}(l_6,0)]/[\Delta^{-1}(l_6,0)-\Delta^{-1}(l_3,0)]+0.1933\times[\Delta^{-1}(l_5,0)-\Delta^{-1}(l_5,0)]/[\Delta^{-1}(l_5,0)-\Delta^{-1}(l_3,0)]+0.1513\\
&\quad\times[\Delta^{-1}(l_6,0)-\Delta^{-1}(l_5,0)]/[\Delta^{-1}(l_6,0)-\Delta^{-1}(l_4,0.0556)]+0.1681\times[\Delta^{-1}(l_5,0.0556)\\
&\quad-\Delta^{-1}(l_5,0)]/[\Delta^{-1}(l_5,0.0556)-\Delta^{-1}(l_4,0]+0.1366\times[\Delta^{-1}(l_6,-0.0556)\\
&\quad-\Delta^{-1}(l_6,-0.0556)]/[\Delta^{-1}(l_6,-0.0556)-\Delta^{-1}(l_3,0.0556]\ \}\\
&= \Delta\{0.1681\times(0.9444-0.8889)/(0.9444-0.6667)\ +0.1828\times(1-1)/(1-0.5)+0.1933\times(0.8333\\
&\quad-0.8333)/(0.8333-0.5)\ +0.1513\times(1-0.8333)/(1-0.7222)+0.1681\times(0.8889-8333)/(0.8889\\
&\quad-0.6667)+0.1366\times(0.9444-0.9444)/(0.9444-0.5556)\}\\
&= \Delta(0.1664)=(l_1,-0.0003).
\end{aligned}
$$

Similarly, the 2-tuple linguistic values (R_i,α_i), i = 1,2,…,m, are calculated by Eq. (18) and shown in Table 9.

Step 6. The 2-tuple linguistic values (Q_i,α_i) are calculated by Eq. (19) and shown in Table 9, where v = 0.5. For example, the 2-tuple linguistic value (Q_2,α_2) of alternative A_2 is computed as:

$$
\begin{aligned}
(Q_2,\alpha_2) &= \Delta\{0.5\times[\Delta^{-1}(l_2,0.0417)-\Delta^{-1}(l_1,0.0003)]/[\Delta^{-1}(l_5,0.0702)-\Delta^{-1}(l_1,0.0003)]+(1-0.5)\\
&\quad\times[\Delta^{-1}(l_1,-0.0056)-\Delta^{-1}(l_1,-0.0759)]/[\Delta^{-1}(l_1,0.0266)-\Delta^{-1}(l_1,-0.0759)]\}\\
&= \Delta(0.4844)=(l_3,-0.0156).
\end{aligned}
$$

Table 8
The TL-PIS and TL-NIS of the collective 2-tuple linguistic decision matrix.

	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
TL-PIS	$(l_6,-0.0556)$	$(l_6,0.0000)$	$(l_5,0.0000)$	$(l_6,0.0000)$	$(l_5,0.0556)$	$(l_6,-0.0556)$
TL-NIS	$(l_4, 0.0000)$	$(l_3,0.0000)$	$(l_3,0.0000)$	$(l_4,0.0556)$	$(l_4,0.0000)$	$(l_3,0.0556)$

Table 9
The values of (S_i,α_i), (R_i,α_i), and (Q_i,α_i) for all alternatives.

Alternatives	(S_i,α_i)	(R_i,α_i)	(Q_i,α_i)
A_1	$(l_1, -0.0003)$	$(l_1, -0.0759)$	$(l_0, 0.0000)$
A_2	$(l_2, 0.0417)$	$(l_1, -0.0056)$	$(l_3, -0.0156)$
A_3	$(l_2, -0.0799)$	$(l_1, 0.0266)$	$(l_3, 0.0590)$
A_4	$(l_3, -0.0792)$	$(l_1, -0.0496)$	$(l_2, -0.0325)$
A_5	$(l_5, 0.0702)$	$(l_1, 0.0161)$	$(l_6,-0.0512)$
Max	$(l_5, 0.0702)$	$(l_1, 0.0266)$	
Min	$(l_1, -0.0003)$	$(l_1, -0.0759)$	

Table 10
The ranking of the emergency alternatives by $(S_i,\alpha_i),(R_i,\alpha_i)$ and (Q_i,α_i).

	Ranking emergency alternatives				
	1	2	3	4	5
By (Q_i,α_i)	A_1	A_4	A_2	A_3	A_5
By (S_i,α_i)	A_1	A_3	A_2	A_4	A_5
By (R_i,α_i)	A_1	A_4	A_2	A_5	A_3

Step 7. The ranking of the emergency alternatives by $(S_i,\alpha_i),(R_i,\alpha_i)$ and (Q_i,α_i) are shown in Table 10.

Step 8. As we can see from Table 10, the emergency alternative A_1 is the best ranked by (Q_i,α_i). Also the condition Con_1 is satisfied:

$$\Delta^{-1}(Q(A_4),\alpha(A_4)) - \Delta^{-1}(Q(A_1),\alpha(A_1)) = 0.3008 \geqslant \frac{1}{5-1},$$

and A_1 is best ranked by (S_i,α_i) and (R_i,α_i)). So A_1 is the best choice.

5. Discussion

In order to illustrate the validity of the proposed method, we utilize 2-tuple linguistic TOPSIS (TL-TOPSIS) method proposed by Wei [41] to solve the above MCGDM problem and compare the evaluation results of the two methods. The procedure of the TL-TOPSIS method is described as follows.

Steps 1∼3. See Steps 1∼3 in Section 3.

Step 4. Utilizing the collective 2-tuple linguistic decision matrix $T=(t_{ij},\alpha_{ij})_{m\times n}$ to derive weighted collective 2-tuple linguistic decision matrix $V=(v_{ij},\delta_{ij})_{m\times n}$, where

$$(v_{ij},\delta_{ij}) = \Delta[\Delta^{-1}(t_{ij},\alpha_{ij})\times w_j] \quad i=1,2,\ldots,m, \quad j=1,2,\ldots,n. \tag{21}$$

Step 5. Define the TL-PIS $(v^+,\delta^+)=((v_1^+,\delta_1^+),(v_2^+,\delta_2^+),\ldots,(v_n^+,\delta_n^+))$ and TL-NIS $(v^-,\delta^-)=((v_1^-,\delta_1^-),(v_2^-,\delta_2^-),\ldots,(v_n^-,\delta_n^-))$. This step is similar to the Step 4 in Section 3.

Step 6. Calculating the distances of each alternative from TL-PIS and TL-NIS by Eqs. (22) and (23), respectively:

$$(\xi_i^+,\eta_i^+) = \Delta\left(\sqrt{\sum_{j=1}^{n}[\Delta^{-1}(v_{ij},\delta_{ij}) - \Delta^{-1}(v_j^+,\delta_j^+)]^2}\right), \quad i=1,2,\ldots,m, \tag{22}$$

$$(\xi_i^-,\eta_i^-) = \Delta\left(\sqrt{\sum_{j=1}^{n}[\Delta^{-1}(v_{ij},\delta_{ij}) - \Delta^{-1}(v_j^-,\delta_j^-)]^2}\right), \quad i=1,2,\ldots,m. \tag{23}$$

Table 11
The weighted collective 2-tuple linguistic decision matrix.

Alternatives	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(l_1,-0.0172)$	$(l_1,0.0161)$	$(l_1,-0.0056)$	$(l_1,-0.0406)$	$(l_1,-0.0266)$	$(l_1,-0.0377)$
A_2	$(l_1,-0.0172)$	$(l_1,-0.0448)$	$(l_1,-0.0593)$	$(l_1,-0.0154)$	$(l_1,-0.0172)$	$(l_1,-0.0604)$
A_3	$(l_1,-0.0079)$	$(l_1,-0.0042)$	$(l_1,-0.0700)$	$(l_1,-0.0154)$	$(l_1,-0.0172)$	$(l_1,-0.0452)$
A_4	$(l_1,-0.0266)$	$(l_1,-0.0245)$	$(l_1,-0.0271)$	$(l_1,-0.0406)$	$(l_1,-0.0172)$	$(l_1,-0.0832)$
A_5	$(l_1,-0.0546)$	$(l_1,-0.0753)$	$(l_1,-0.0378)$	$(l_1,-0.0574)$	$(l_1,-0.0546)$	$(l_0,0.0759)$

Table 12
The TL-PIS and TL-NIS of the weighted collective 2-tuple linguistic decision matrix.

	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
TL-PIS	$(l_1,-0.0079)$	$(l_1,0.0161)$	$(l_1,-0.0056)$	$(l_1,-0.0154)$	$(l_1,-0.0172)$	$(l_1,-0.0377)$
TL-NIS	$(l_1,-0.0546)$	$(l_1,-0.0753)$	$(l_1,-0.0700)$	$(l_1,-0.0574)$	$(l_1,-0.0546)$	$(l_0,0.0759)$

Step 7. Calculating the relative closeness degree of each alternative from TL-PIS by Eq. (24).

$$(\xi_i,\eta_i)=\Delta\left(\frac{\Delta^{-1}(\xi_i^-,\eta_i^-)}{\Delta^{-1}(\xi_i^-,\eta_i^-)+\Delta^{-1}(\xi_i^+,\eta_i^+)}\right),\quad i=1,2,\ldots,m. \tag{24}$$

Step 8. According to the relative closeness degree (ξ_i,η_i), the ranking order of all alternatives can be determined. If any alternative has the highest (ξ_i,η_i) value, then, it is the most desirable alternative.

In the following, we give some main calculation results. Firstly, by Steps 1–3, we get the 2-tuple linguistic decision matrix as shown in Table 5, the collective 2-tuple linguistic decision matrix as shown in Table 6, and the criteria weight vector $w=(0.1681,0.1828,0.1933,0.1513,0.1681,0.1366)$. Secondly, by Steps 4-5, we obtain the weighted collective 2-tuple linguistic decision matrix $V=(v_{ij},\delta_{ij})_{m\times n}$ as shown in Table 11, and the TL-PIS and TL-NIS of the weighted collective 2-tuple linguistic decision matrix as shown in Table 12. Thirdly, by Step 6, we calculate the distances of each alternative from TL-PIS and TL-NIS, respectively. The results are shown as follows:

$$(\xi_1^+,\eta_1^+)=(l_0,0.0285),(\xi_2^+,\eta_2^+)=(l_1,-0.0818),(\xi_3^+,\eta_3^+)=(l_0,0.0680),$$

$$(\xi_4^+,\eta_4^+)=(l_0,0.0719),(\xi_5^+,\eta_5^+)=(l_1,-0.0342);$$

$$(\xi_1^-,\eta_1^-)=(l_1,-0.0333),(\xi_2^-,\eta_2^-)=(l_0,0.0808),(\xi_3^-,\eta_3^-)=(l_1,-0.0550),$$

$$(\xi_4^-,\eta_4^-)=(l_1,-0.0833),(\xi_5^-,\eta_5^-)=(l_0,0.0322).$$

Fourthly, by Step 7, we calculate the relative closeness degree of each alternative from TL-PIS as follows: $(\xi_1,\eta_1)=(l_6,\ 0.0000)$, $(\xi_2,\eta_2)=(l_4,\ -0.0749)$, $(\xi_3,\eta_3)=(l_5,\ -0.0791)$,

$$(\xi_4,\eta_4)=(l_4,-0.0153),(\xi_5,\eta_5)=(l_1,0.0707).$$

Finally, we rank all the alternatives $A_i(i=1,2,3,4,5)$ in accordance with the relative closeness degree (ξ_i,η_i):

$$A_1\succ A_3\succ A_4\succ A_2\succ A_5,$$

and thus the most desirable alternative is also A_1.

As we can see, the most desirable alternative obtained from the proposed method and the TL-TOPSIS method is similar. This demonstrates the validity of the proposed approach. But there are some differences in the rank of other alternatives by the two approaches. The difference mainly results from the idea of the VIKOR method: achieving an acceptable compromise of the maximum "group utility" of the "majority" and the minimum of the individual regret of the "opponent". It is worth pointing out that the TL-TOPSIS method just provide sole rank of the alternatives, and can't effectively reflect the experts' preferences. However, the proposed method can help DMs to achieve an acceptable compromise by assigning suitable value of v according to their preferences: if they are concerned about both group utility and individual regret, then $v=0.5$ would be selected; if they are concerned about group utility, then $0.5<v\leqslant 1$ would be used; if they are concerned about individual regret, then $0\leqslant v<0.5$ would be utilized.

6. Conclusion

Many practical problems are often characterized by MCGDM. Because of time pressure, lack of knowledge or data, and the DMs' limited expertise about the problem domain, the criteria values and weights given by decision makers often take the form of linguistic variable. In this paper, a new method is proposed to solve multi-criteria group decision making problems in which both the criteria values and weights take the form of linguistic information based on the traditional idea of VIKOR method. The criteria weights are determined by transforming linguistic weights into trapezoidal fuzzy numbers, and defuzzifying the trapezoidal fuzzy numbers to crisps. The collective decision matrix, which is in the form of 2-tuple linguistic, is obtained by transforming individual linguistic decision matrix into 2-tuple linguistic decision matrix and aggregating them. The compromise solution is determined by comparing 2-tuple linguistic values of (Q_i, α_i), (S_i, α_i) and (R_i, α_i).

The proposed method is straightforward and can be performed on computer easily. It has characteristic of avoiding information distortion and losing which occur formerly in the linguistic information processing, and it can be utilized for making a best decision in any other areas of engineering and management problems.

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