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Multi-choice goal programming formulation based on the conic scalarizing function

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ABSTRACT

The multi-choice goal programming allows the decision maker to set multi-choice aspiration levels for each goal to avoid underestimation of the decision. In this paper, we propose an alternative multi-choice goal programming formulation based on the conic scalarizing function with three contributions: (1) the alternative formulation allows the decision maker to set multi-choice aspiration levels for each goal to obtain an efficient solution in the global region, (2) the proposed formulation reduces auxiliary constraints and additional variables, and (3) the proposed model guarantees to obtain a properly efficient (in the sense of Benson) point. Finally, to demonstrate the usefulness of the proposed formulation, illustrative examples and test problems are included.

1. Introduction

Goal programming (GP) is an analytical approach devised to address decision-making problems where targets have been assigned to all the attributes and where the decision-maker (DM) is interested in minimizing the non-achievement of the corresponding goals. In other words, the DM seeks a Simonian satisficing solution (i.e., satisfactory and sufficient) with this strategy [1].

As has been stated by Romero [1], the core of GP lies in the works by Charnes et al. [2] and Charnes and Cooper [3]. Since the mid 70s, and chiefly due to the seminal works by Lee [4] and Ignizio [5,6], an impressive boom of GP applications and theoretical developments have arisen. For an updated presentation of the GP paradigm see Tamiz et al. [7], Lee and Olson [8], Jones and Tamiz [9], Ignizio and Romero [10], and Romero [1].

A key element of a goal programming model is the achievement function that measures the degree of minimization of the unwanted deviation variables of the goals considered in the model. This function has a typical "less is better behavior" (i.e., each argument of the function decreases monotonically). Each type of achievement function leads to a different GP variant. The three oldest and still most widely used forms of achievement functions are weighted (Archimedean), preemptive (lexicographic) and MINMAX (Chebyshev). Tamiz et al. [11] show that around 65% of GP applications reported in the literature use preemptive achievement functions, 21% weighted achievement functions and the rest other types of achievement functions, such as a MINMAX structure in which the maximum deviation is minimized. The purpose of GP is to minimize the deviations between the achievement of goals and their aspiration levels. It can be expressed as the following program:

(GP)

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} w_i |f_i(x) - a_i| \\ &\text{s.t. } \; x \in X (X \text{ is a feasible set}), \end{aligned}$$

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where w_i is the respective positive weights attached to these deviations in the achievement function; $f_i(x)$ is the function of the ith goal, and a_i is the aspiration level of the ith goal.

The above minimization process can be accomplished with various types of methods such as those of weighted GP (WGP), Lexicographic GP (LGP) and MINMAX GP. On the other hand, the most important weakness of traditional forms of GP does not guarantee efficient points. The reason for the fact that GP models can produce inefficient points is that the decision maker may set target values which are too pessimistic, i.e., objectives which are easily achieved with respect to the restrictions (constraints and conflicting objectives) imposed. This disadvantage has, in the past, caused great concern and doubt regarding the use of GP, as detailed in the studies of Zeleny and Cochrane [12] and Zeleny [13].

To overcome this drawback, Hannan [14,15] proposed a remedy to restore Pareto efficiency. His method is based on the production of a set of efficient points which dominate the standard inefficient GP optimization point. Further developments to Hannan's method were carried out by Romero [16], in order to generate efficient points, while preventing the degradation of any objective's achieved value from the standard inefficient GP point. Tamiz and Jones [17] propose an alternative technique for Pareto efficiency and inefficiency detection and implement it within a GP optimization package GPSYS [18]. The technique consists of a series of tests which are designed to categorize objectives into Pareto efficient, inefficient or unbounded states [17]. These tests investigate the possibility of improving the objectives from the initial optimal solution in order to detect efficiency or inefficiency. Also, Tamiz et al. [19] proposed the Pareto efficiency detection and restoration analysis for obtaining final solution. Also the WGP as well as LGP and MINMAX GP require the Pareto efficiency detection and restoration analysis for obtaining final solution [19]. Main reason of the inefficient solution in traditional GP approaches is the underestimation of initial aspiration levels.

Chang [20] has recently proposed a novel approach namely multi-choice goal programming (MCGP), which allows DMs to set multi-choice aspiration levels (MCAL) for each goal (i.e., one goal mapping multiple aspiration levels) to avoid underestimation of decision making. The conceptual expression of MCGP is as follows:

(MCGP)

$$\begin{aligned}&\text{Min } \sum_{i=1}^{n} w_i |f_i(x) - a_{i1} \quad \text{or} \quad a_{i2} \quad \text{or} \ \dots \ \text{or} \quad a_{im}| \\ &\text{s.t. } \ x \in X (X \text{ is a feasible set}),\end{aligned}$$

where a_{ij} ($i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$) is the jth aspiration level of the ith goal, $a_{ij-1} \leqslant a_{ij} \leqslant a_{ij-1}$; other variables are defined as in GP.

According to MCGP, DMs not only must consider the only single aspiration level in the local region, but also develop multiple aspiration levels under given constraints to obtain the global optimal solution in the global region.

The achievement function of the MCGP can be expressed as follows [20]:

$$\begin{aligned}&\text{Min } \sum_{i=1}^{n} w_i (d_i^+ + d_i^-) \\ &\text{s.t. } \ f_i(x) - d_i^+ + d_i^- = \sum_{j=1}^{m} a_{ij} S_{ij}(B), \quad i = 1,2,\ldots,n, \\ &\qquad d_i^+, d_i^- \geqslant 0, \quad i = 1,2,\ldots,n, \\ &\qquad S_{ij}(B) \in R_i(X), \quad i = 1,2,\ldots,n, \\ &\qquad x \in X (X \text{ is a feasible set}),\end{aligned}$$

where $d_i^+ = \max(0, f_i(x) - \sum_{j=1}^{m} a_{ij} S_{ij}(B))$ and $d_i^- = \max(0, \sum_{j=1}^{m} a_{ij} S_{ij}(B) - f_i(x))$ are, respectively, over- and under-achievements of ith goal; $S_{ij}(B)$ represents a function of binary serial numbers. Other variables are defined as in the GP or the MCGP.

Chang [21] proposed an alternative method to formulate the MCGP in which the new approach does not involve multiplicative terms of binary variables for solving such problem. These two alternative MCGP-achievement functions can be given as follows:

The first case: "the more the better" is formulated as:

(Revised MCGP)

$$\begin{aligned}&\text{Min } \sum_{i=1}^{n} \left[w_i (d_i^+ + d_i^-) + \alpha_i (e_i^+ + e_i^-) \right] \\ &\text{s.t. } \ f_i(x) - d_i^+ + d_i^- = y_i \quad i = 1,2,\ldots,n, &(1) \\ &\qquad y_i - e_i^+ + e_i^- = a_{i,max}, \quad i = 1,2,\ldots,n, &(2) \\ &\qquad a_{i,min} \leqslant y_i \leqslant a_{i,max}, \\ &\qquad d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, \quad i = 1,2,\ldots,n, \\ &\qquad x \in X (X \text{ is a feasible set}),\end{aligned}$$

where the ith aspiration level y_i is the continuous variable restricted between the upper ($a_{i,max}$) bound and lower ($a_{i,min}$) bound ($a_{i,min} \leqslant y_i \leqslant a_{i,max}$); d_i^+ and d_i^- are positive and negative deviations attached to the ith goal $|f_i(x) - y_i|$ in Eq. (1); e_i^+

and e_i^- are positive and negative deviations attached to the ith goal $|y_i - a_{i,max}|$ in Eq. (2); α_i is the weight attached to the sum of the deviations of $|y_i - a_{i,max}|$; other variables are defined as in MCGP.

The second case: "the less the better" is formulated as:

$$\begin{aligned}
&\text{Min } \sum_{i=1}^{n} \left[w_i(d_i^+ + d_i^-) + \alpha_i(e_i^+ + e_i^-)\right] \\
&\text{s.t. } f_i(x) - d_i^+ + d_i^- = y_i, \quad i = 1,2,\ldots,n, && (3)\\
&\quad y_i - e_i^+ + e_i^- = a_{i,min}, \quad i = 1,2,\ldots,n, && (4)\\
&\quad a_{i,min} \leqslant y_i \leqslant a_{i,max}, \\
&\quad d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, \quad i = 1,2,\ldots,n, \\
&\quad x \in X (X \text{ is a feasible set}),
\end{aligned}$$

where d_i^+ and d_i^- are positive and negative deviations attached to the ith goal $|f_i(x) - y_i|$ in Eq. (3); e_i^+ and e_i^- are positive and negative deviations attached to the ith goal $|y_i - a_{i,min}|$ in Eq. (4); α_i is the weight attached to the sum of the deviations of $|y_i - a_{i,min}|$; other variables are defined as in MCGP.

The rapid development of MCGP has led to an enormous diversity in models and applications. In practice, the MCGP has been applied to the real-world multi-criteria decision-making problems, such as supplier selection [22,23], an evaluation of framework for product planning [24], the plotting a quality management system [25].

The conic scalarization is one of the reference point approaches for general multi-objective problems. Wierzbicki [26] produced seminal research on reference point (aspiration level) methods, including an investigation of the characteristics of various achievement functions for allowing the search for attractive efficient solutions to be controlled by reference points. These achievement functions were designed to have a significant advantage over goal programming by producing only efficient, or Pareto-optimal, points. In addition to their desirable structural features, reference point methods have also appeared useful from a methodological or operational perspective. In general, reference point approaches for multi-objective problems (considering discrete variables or not) rely on the definition of an achievement scalarizing function - as suggested by Wierzbicki [26] -by means of aspiration levels (reference point) for the objective functions. Two forms of reference points exist: aspiration points (desirable levels of achievement) and reservation points (levels of achievement that should be attained, if at all possible).

Reference point methodology provides the foundation for many methods in multiple objective programming [27]. A variety of scalarization methods used reference point for finding efficient solutions of multiple objective programs (MOPs) have been developed over last decades. Most of the mathematical programming models of the real life problems have non-convex structures such as discrete variables. Since the set of efficient points for problems with discrete variables is not convex, weighted sums of the objective functions do not provide a way of reaching every efficient point. Besides supported there exist unsupported efficient points - points that are dominated by convex combinations of other efficient points. Conic scalarization-based techniques have the advantage over weighted-sums programs of being able to reach, not only supported, but also unsupported efficient points. The conic scalarizing function is also called "conic scalarization" that a general characterization for the Benson proper efficient point set was firstly proposed by Gasimov [28]. Gasimov [28] introduced a class of increasing convex functions which serve for combining different objectives to a single one without any restrictions on objectives and constraints of the problem under consideration. The other advantage of this approach is that it preserves the convexity, if the objective functions of the initial problem are linear or convex. DM's preferences can be accurately reflected to the mathematical programming model as weights of the objectives and reference point (aspiration levels) by using the conic scalarizing function. The conic scalarizing function has been successfully applied to the non-convex multi-objective faculty course assignment problem [29], and the 1.5 dimensional multi-objective assortment problems [30]. The existing approaches based on the conic scalarizing function allow using a single reference point (aspiration level). However, in some cases the DMs would like to make a decision on the problem, with the goal that can be achieved from some specific aspiration levels (i.e., one goal mapping many aspiration levels) indicated by Chang [20]. To the best knowledge of the author, this problem cannot be solved by the current conic scalarization approaches.

In this study, we combine the multi-choice reference points (aspiration levels) with the conic scalarizing function to obtain more satisfactory solutions. The paper is outlined as follows: Mathematical backgrounds of the multi-choice conic goal programming formulation are given in Section 2. Some definitions and results related to the proposed formulation are presented in Section 3. Section 4 presents illustrative examples and computational results. Finally Section 5 presents the important conclusions of this study and suggestions for further research.

2. Preliminaries

In this section, we introduce some definitions and results of multi-objective programming and the conic scalarizing function. A multi-objective programming problem can be written as

(MOP)

$$\begin{aligned}
&\text{Min } f(x) \\
&\text{s.t. } x \in X,
\end{aligned} \tag{5}$$

where $X \subset R^p$ is the feasible set in decision space R^p and $f: R^p \to R^n$ is a vector valued objective function mapping a feasible solution x to a point = $(f_1(x), \ldots, f_n(x))$ in objective space R^n. We denote by $Y := f(X)$ the feasible set in objective space.

Let $R^n_{\geqq} = \{y \in R^n : y_i \leqslant 0, i = 1, \ldots, n\}$ and $R^n_{>} = \{y \in R^n : y_i > 0, i = 1, \ldots, n\}$. For any $y^1, y^2 \in R^n$ we define

$$y^1 \leqq y^2 \quad \text{if } y^2 - y^1 \in R^n_{\geqq},$$
$$y^1 \leqslant y^2 \quad \text{if } y^1 \leqq y^2 \quad \text{and} \quad y^1 \neq y^2,$$
$$y^1 < y^2 \quad \text{if } y^2 - y^1 \in \text{int } R^n_{\geqq} = R^n_{>},$$

Definition 1. Let Y be a non-empty subset of R^n.

1. An element $y \in Y$ is called a efficient point if $(\{y\} - R^n_{\geqq}) \cap Y = \{y\}$, i.e. there is no $y^* \in Y$ such that $y^* \leqslant y$.
2. An element $y \in Y$ is called a properly efficient (in the sense of Benson) if y is a efficient point of Y and the zero element of R^n is a efficient point of $clcone\left(Y + R^n_{\geqq} - y\right)$, where $cl(Y)$ denotes the closure of a set Y and cone $(Y) = \{\alpha y : \alpha \geqslant 0, y \in Y\}$.

The set of all efficient points of Y is denoted Y_N, the set of all properly efficient points Y_{pN}. A feasible solution $x \in X$ is called (properly) efficient solution if $y = f(x)$ is a (properly) efficient point of Y. The set of (properly) efficient solutions of a multi-objective programming problem is denoted X_E (X_{pE}).

Now we briefly present the main conic scalarization results introduced by Gasimov [28].

$$\text{Let } W = \{(\beta, w) \in R \times R^n_{\geqq} : 0 \leqslant \beta < \min\{w_1, \ldots, w_n\}\}.$$

Theorem 1 (see Gasimov [28]). *Suppose that for some $(\beta, w) \in W$ a feasible solution $\hat{x} \in X$ is an optimal solution to the scalar minimization problem*

$$\begin{aligned} &\text{Min } \beta \sum_{i=1}^{n} |f_i(x)| + \sum_{i=1}^{n} w_i f_i(x) \\ &\text{s.t. } \ x \in X (X \textit{ is a feasible set}), \end{aligned} \tag{6}$$

then $\hat{x}$ is a Benson proper efficient solution to (5).

Theorem 2 (see Gasimov [28]). *Let $\hat{x} \in X$ is a Benson proper efficient solution to (5). Then there exist a vector $(\beta, w) \in W$ such that $\hat{x}$ is an optimal solution to the scalar minimization problem*

$$\begin{aligned} &\text{Min } \beta \sum_{i=1}^{n} |f_i(x) - f_i(\hat{x})| + \sum_{i=1}^{n} w_i (f_i(x) - f_i(\hat{x})) \\ &\text{s.t. } \ x \in X (X \textit{ is a feasible set}), \end{aligned} \tag{7}$$

In non-convex multi-objective programs the distinction between supported and unsupported efficient solutions is important. An efficient solution $\hat{x} \in X_E$ is called supported, if there is $w \in R^n_{>}$ such that $\hat{x}$ is an optimal solution to

(WSP)

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} w_i f_i(x) \\ &\text{s.t. } \ x \in X (X \text{ is a feasible set}), \end{aligned} \tag{8}$$

It is well known [25] that if X is convex and all $f_i(x)$, i = 1, …, n are convex functions, then all Benson proper efficient solutions are supported, see e.g. [31]. However, for non-convex problems there exist unsupported efficient solutions.

It is evident that if $\hat{x} \in X$ is an efficient solution to problem (5) then it is also an efficient solution to the shifted multi-objective program

(SMOP)

$$\begin{aligned} &\text{Min } (f_1(x) - a_1, \ldots, f_n(x) - a_n) \\ &\text{s.t. } \ x \in X, \end{aligned} \tag{9}$$

where $a \in R^n$ is an arbitrary vector. Such a shifting can be used in situations when objectives do not change sign on the whole efficient solution set X_E in order to make the absolute value used in the scalarized problem (6) sensible. In this case we can formulate the following scalarized problem, which is similar to that in (7) and can be used even if we do not know any efficient solution.

(CSP)

$$\begin{aligned} &\text{Min } \beta\sum_{i=1}^{n}|f_i(x)-a_i|+\sum_{i=1}^{n}w_i(f_i(x)-a_i) \\ &\text{s.t. } \ x\in X(X \text{ is a feasible set}). \end{aligned} \tag{10}$$

We can therefore completely characterize Benson proper efficient solutions through Gasimov's scalarization.

Corollary 1. *A feasible solution $\hat{x}\in X$ is a Benson proper efficient if and only if there are $a\in R^n$ and $(\beta,w)\in W$ such that is an optimal solution to the scalar minimization problem*

(CSP)

$$\begin{aligned} &\text{Min } \beta\sum_{i=1}^{n}|f_i(x)-a_i|+\sum_{i=1}^{n}w_i(f_i(x)-a_i) \\ &\text{s.t. } \ x\in X(X \text{ is a feasible set}). \end{aligned}$$

3. Multi-choice conic goal programming formulation

In this section, we propose goal programming and multi-choice goal programming formulations of (CSP).

Proposition 1. *The conic scalarization problem (10) with $a\in R^n$ and $(\beta,w)\in W$ is equivalent the following conic goal programming formulation:*

(CGP)

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n}\left[(\beta+w_i)d_i^+ + (\beta-w_i)d_i^-\right], \\ &\text{s.t. } \ f_i(x)-d_i^+ + d_i^- = a_i, \quad i=1,2,\ldots,n, \\ &\qquad d_i^+, d_i^- \geqslant 0, \quad i=1,2,\ldots,n, \\ &\qquad x\in X(X \text{ is a feasible set}), \end{aligned} \tag{11}$$

where $d_i^+=\max(0,f_i(x)-a_i)$ and $d_i^-=\max(0,a_i-f_i(x))$ are, respectively, over- and under-achievements of ith goal; where a_i is aspiration or target level for ith goal; $f_i(x)$ is defined as in CSP.

Proof. The goal programming formulation (11) can be easily accomplished by introducing the variables

$$d_i^+ = \frac{1}{2}\{|f_i(x)-a_i|+(f_i(x)-a_i)\} \tag{12}$$

and

$$d_i^- = \frac{1}{2}\{|f_i(x)-a_i|+(f_i(x)-a_i)\} \tag{13}$$

for each $i = 1,\ldots,n$. We note that

$$d_i^+ + d_i^- = |f_i(x)-a_i|, \tag{14}$$
$$d_i^+ - d_i^- = f_i(x)-a_i \tag{15}$$
$$d_i^+.d_i^- = 0 \tag{16}$$

and $d_i^+\geqslant 0$ and $d_i^-\geqslant 0$ for each $i = 1,\ldots,n$. Therefore the CSP can be transformed into CGP model as follows:

$$\begin{aligned} &\min \ \ \beta\sum_{i=1}^{n}\left(d_i^+ + d_i^-\right)+\sum_{i=1}^{n}w_i\left(d_i^+ - d_i^-\right)=\sum_{i=1}^{n}\left[(\beta+w_i)d_i^+ + (\beta-w_i)d_i^-\right], \\ &\text{s.t.} \ \ f_i(x)-d_i^+ + d_i^- = a_i, \quad i=1,2,\ldots,n, \\ &\qquad d_i^+, d_i^- \geqslant 0, \quad i=1,2,\ldots,n, \\ &\qquad x\in X(X \text{ is a feasible set}), \end{aligned}$$

where all variables are defined as in CGP. □

Thus the CSP is reduced to the new goal programming form without absolute value. Deviational variables d_i^+ and d_i^- in the CGP formulation can be interpreted as the over-achievement and the under-achievement, respectively, of the *i*th goal level. Over-achievement and under-achievement can clearly not occur simultaneously. That is, if $d_i^+>0, d_i^-$ must be zero, and vice versa. Depending on the decision situation, some DMs may prefer over-achievement whereas others may prefer under-achievement. In order to accommodate this preference, weights (β+w_i) and (β-w_i) may be assigned to d_i^+ and d_i^- , respectively.

The weight $(\beta + w_i)$ is strictly positive and other weight $(\beta - w_i)$ is strictly negative because of $(\beta, w) \in W$. It is clearly that the inequality $|\beta + w_i| > |\beta - w_i|$ satisfies for all i. If over-achievement is considered more desirable than under-achievement then β should be selected close to w_i as soon as possible by considering $(\alpha, w) \in W$.

We know that the most important weakness of traditional forms of GP such as the WGP and the LGP does not guarantee efficient points. The CGP produce only Benson proper efficient solutions for convex and non-convex MOP problems. We also propose the multi-choice conic scalarization function.

The conceptual expression of Multi-Choice CSP is as follows:

(MCSP)

$$\begin{aligned}&\text{Min}\ \beta\sum_{i=1}^{n}|f_i(x)-a_{i1}\ \text{ or }\ a_{i2}\ \text{ or }\ \dots\ \text{ or }\ a_{im}|+\sum_{i=1}^{n}w_i(f_i(x)-a_{i1}\ \text{ or }\ a_{i2}\ \text{ or}\dots\text{or }\ a_{im})\\&\text{s.t. } x\in X(X\ \text{is a feasible set}).\end{aligned} \tag{17}$$

where a_{ij} ($i = 1,2,\dots,n$ and $j = 1, 2,\dots,m$) is the jth aspiration level of the ith goal, $a_{ij-1} \leqslant a_{ij} \leqslant a_{ij-1}$; other variables are defined as in CSP.

For the existing studies in the conic scalarization literature, DMs must select a single reference point (aspiration level). According to MCSP, DMs not only must consider the only single aspiration level in the local region, but also develop multiple aspiration levels under given constraints to obtain the global optimal solution in the global region.

We propose the multi-choice CGP to allow the usage of multiple aspiration levels. Multi-choice CGP can be given as follows:

The first case: "the less the better" is formulated as:

$$\begin{aligned}&\text{Min}\ \sum_{i=1}^{n}\left[(\beta+w_i)d_i^+ + (\beta-w_i)d_i^-\right],\\&\text{s.t. } f_i(x)-d_i^+ + d_i^- = y_i,\quad i=1,2,\dots,n,\\&\qquad a_{i,min}\leqslant y_i\leqslant a_{i,max},\quad i=1,2,\dots,n,\\&\qquad d_i^+,d_i^-\geqslant 0,\quad i=1,2,\dots,n,\\&\qquad x\in X(X\ \text{is a feasible set}),\end{aligned} \tag{18}$$

where the ith aspiration level y_i is the continuous variable restricted between the upper ($a_{i,max}$) bound and lower ($a_{i,min}$) bound ($a_{i,min} \leqslant y_i \leqslant a_{i,max}$); and are positive and negative deviations attached to the ith goal $|f_i(x)$-$y_i|$ in Eq. (18); other variables are defined as in the CGP.

In the second case: "the more the better", objective functions which are maximized in the model can be easily transformed to "the less the better" form by multiplying -1.

Proposition 2. *A feasible solution $\hat{x} \in X$ is a Benson proper efficient if and only if there are $y_i \in [a_{i,min}, a_{i,max}]$ for $i = 1,2,\dots,n$ and $(\beta, w) \in W$ such that $\hat{x}$ is an optimal solution to the multi-choice CGP:*

$$\begin{aligned}&\text{Min}\ \sum_{i=1}^{n}\left[(\beta+w_i)d_i^+ + (\beta-w_i)d_i^-\right],\\&\text{s.t. } f_i(x)-d_i^+ + d_i^- = y_i,\quad i=1,2,\dots,n,\\&\qquad a_{i,min}\leqslant y_i\leqslant a_{i,max},\quad i=1,2,\dots,n,\\&\qquad d_i^+,d_i^-\geqslant 0,\quad i=1,2,\dots,n,\\&\qquad x\in X(X\ \text{is a feasible set}).\end{aligned} \tag{19}$$

Proof. It is evident that if $\hat{x} \in X$ is an efficient solution to problem (5) then it is also an efficient solution to the shifted multi-objective program (9) where $a \in R^n$ is an arbitrary vector. Theorem 1 indicates that for some $(\beta, w) \in W$ a feasible solution $\hat{x} \in X$ is an optimal solution to the scalar minimization problem (6) then $\hat{x}$ is a Benson proper efficient solution to (5). This result is valid for any reference point $a \in R^n$ chosen at the interval $[a_{i,min}, a_{i,max}]$ for $i = 1,2,\dots,n$. □

To deal with a MCGP problem with n goals and each goal has m aspiration levels, the number of additional variables, auxiliary constraints and extra binary variables used in Chang [20], Chang [21] and the proposed model are compared in Table 1.

Table 1 indicates that the proposed model reduces n auxiliary constraints and $2n$ additional variables from Chang's model [21]. The proposed model represents a linear form without adding any extra binary variables to formulate the multiple aspirations level. This gives the proposed model more computational efficiency.

4. Illustrative examples

We now examine the nature of GP and MCGP approaches based on the achievement functions WGP, CGP, the revised MCGP and the multi-choice CGP in the context of some convex or non-convex MOP problems. Additive utility function

Table 1
Size of constraints and variables.

Model	MCGP [20]	The Revised MCGP [21]	The proposed model
Auxiliary constraints	$2n$	$2n$ (plus $2n$ bounded constraints)	n (plus $2n$ bounded constraints)
Additional variables	$2n$	$4n$	$2n$
Extra binary variables	$n\left\lceil \frac{\ln m}{\ln 2} \right\rceil$	0	0

was used to simulate DM preferences with different aspiration levels or/and weights of goals. Thus the solution quality of the goal programming approaches can be evaluated and compared by using DM's additive utility function and aspiration levels. The optimal solutions of mathematical models have been found by using LINGO 11.0 Solver [32] on an Intel (R) Core 2 Duo CPU 2.26 GHz-based computer in a few seconds of computer time for illustrative examples. We compare the performances of the approaches for two cases as follows:

Case I: The CGP is compared with WGP for a single aspiration level,
Case II: The multi-choice CGP is compared with the revised MCGP for multiple aspiration levels.

4.1. Case I: the performance of CGP versus WGP for a single aspiration level

The Example 1 is given to compare the performances of CGP with WGP for a single aspiration level.

Example 1. Let us consider a multi-objective integer linear programming problem as follows [19]:

$$\begin{aligned} \max \quad & f_1(x) = x_1, \\ \max \quad & f_2(x) = x_2, \\ \max \quad & f_3(x) = 2x_1 + 3x_2, \\ \text{subject to} \quad & x_1 \leqslant 10.5, \\ & 0.6x_1 + x_2 \leqslant 20.5, \\ & x_1, x_2 \geqslant 0 \text{ and integer.} \end{aligned}$$

In this example, we assume that DM's target (aspiration) levels according to the objective functions are 6.5, 7.5 and 7.5, respectively. Let the weights of the goals be 2, 1 and 1, respectively. The mathematical model of the WGP of Example 1 can be given as follows:

$$\begin{aligned} \min \quad & z = 2d_1^- + d_2^- + d_3^-, \\ \text{subject to} \quad & x_1 - d_1^+ + d_1^- = 6.5 \\ & x_2 - d_2^+ + d_2^- = 7.5 \\ & 2x_1 + 3x_2 - d_3^+ + d_3^- = 7.5 \\ & x_1 \leqslant 10.5, \\ & 0.6x_1 + x_2 \leqslant 20.5, \\ & d_i^+, d_i^- \geqslant 0, \quad i = 1, 2, 3, \\ & x_1, x_2 \geqslant 0 \text{ and integer.} \end{aligned}$$

Fig. 1 is given to illustrate above example diagrammatically by Tamiz et al. [19]. The shaded area OABC represents the feasible region for the model with three objectives and two constraints. Point F (6.5, 7.5) is the initial GP optimum solution. DBEF is the feasible dominating area of this solution. It contains substantial number of integer points, few of which are marked in Fig. 1. By applying branch and bound algorithm, the initial integer GP optimum solution, point G (7, 8), is obtained. But this point is inefficient for all objectives in the feasible region. It is clear that the point G (7, 8) is dominated by the point H (10, 14) for all objectives.

Again, we assume that Decision Maker's target (aspiration) levels according to the objective functions are 6.5, 7.5 and 7.5, respectively. Let the weights of the goals be 2, 1 and 1, respectively. The mathematical model of the CGP for the parameter values $\beta = 0.99$ of this example can be given as follows:

$$\begin{aligned} \min \quad & z = 2.99d_1^- + 1.99(d_2^- + d_3^-) - 1.01d_1^+ - 0.01(d_2^+ + d_3^+), \\ \text{subject to} \quad & x_1 - d_1^+ + d_1^- = 6.5, \\ & x_2 - d_2^+ + d_2^- = 7.5, \\ & 2x_1 + 3x_2 - d_3^+ + d_3^- = 7.5, \\ & x_1 \leqslant 10.5, \\ & 0.6x_1 + x_2 \geqslant 20.5 \\ & d_i^+, d_i^- \geqslant 0, \quad i = 1, 2, 3, \\ & x_1, x_2 \geqslant 0 \text{ and integer.} \end{aligned}$$

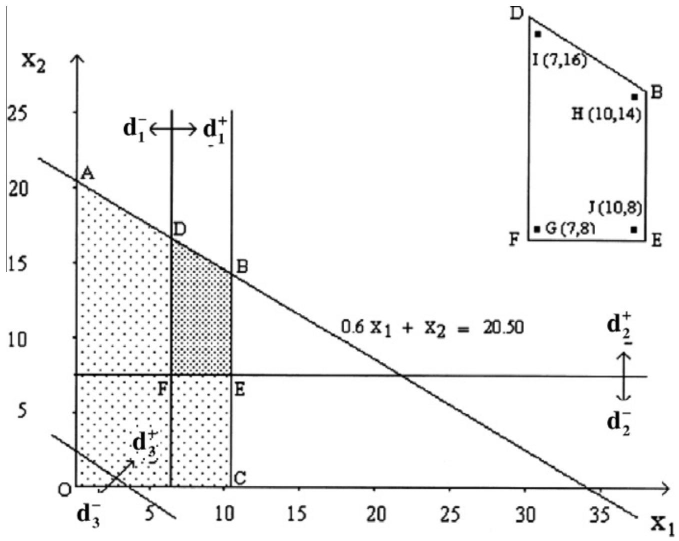

Fig. 1. Illustration of the feasible region and feasible dominating area of the WGP.

The efficient solution of the CGP is the point H (10,14). The values of negative deviations are zero $\left(d_i^- = 0,\ \text{for } i = 1,2,3\right)$ and the values of positive deviations are $d_1^+ = 3.5, d_2^+ = 6.5$ and $d_3^+ = 54.5$.

Note that the CGP guarantees an efficient point and does not requires any past processing procedure such as the Pareto efficiency detection and restoration analysis [19] for obtaining final solution. Additionally, it is clear that finding the optimal solution to the CGP is not difficult than the weighted GP, because both of the models are an integer linear programming model. In this example, an additive utility function was used to simulate DM preferences with weights of 2, 1 and 1. The additive utility value of the CGP's optimal solution is $U(10,14) = 2\ f_1(10,14) + f_2(10,14) + f_3(10,14) = 2 \times 10 + 1 \times 14 + 1 \times 62 = 96$. The weighted GP optimal solution's additive utility value is $U(7,8) = 2 \times 7 + 1 \times 8 + 1 \times 38 = 60$. The solution of the CGP is more satisfactory than the weighted GP for DMs.

4.2. Case II: the multi-choice CGP is compared with the revised MCGP for multiple aspiration levels

Example 2. Let us consider a multi-choice goal programming problem [22] as follows:

$$f_1(x) = 62.7x_1 + 79.38x_2 + 24.5x_3 + 55.13x_4 + 48.02x_5 \geqslant 225400 \quad \text{and} \quad \leqslant 237650$$
$$\text{(For quality goal, the less the better)},$$
$$f_2(x) = 24x_1 + 6x_3 + 15.36x_4 + 54x_5 \geqslant 2352 \quad \text{and} \quad \leqslant 2388$$
$$\text{(For price goal, the less the better)},$$
$$f_3(x) = 3.95x_1 + 4.96x_2 + 7.67x_3 + 3.26x_4 + 5.8x_5 \geqslant 1.428 \quad \text{and} \quad \leqslant 1.433$$
$$\text{(For delivery goal, the less the better)},$$
$$f_4(x) = 27.7x_1 + 37.18x_2 + 44.44x_3 + 51.02x_4 + 59.17x_5 \geqslant 24.5 \quad \text{and} \quad \leqslant 24.88$$
$$\text{(For service goal, the less the better)},$$
$$f_5(x) = 22.22x_1 + 23.24x_2 + 24.91x_3 + 28.13x_4 + 26.77x_5 \geqslant 17.18 \quad \text{and} \quad \leqslant 17.91$$
$$\text{(For warranty goal, the less the better)},$$
$$f_6(x) = 5x_1 + 9x_2 + 8x_3 + 10x_4 + 12x_5 \geqslant 5 \quad \text{and} \quad \leqslant 12$$
$$\text{(For experience goal, the more the better)},$$
$$f_7(x) = 7x_1 + 10x_2 + 14x_3 + 11x_4 + 6x_5 \geqslant 6 \quad \text{and} \quad \leqslant 14$$
$$\text{(For financial goal, the less the better)},$$
$$\text{subject to } x_1 + x_2 + x_3 + x_4 + x_5 = 1 \text{ (select a supplier)}, \quad x_i \geqslant 0, \quad i = 1,2,\ldots,5.$$

The revised MCGP model formulation of Example 2 is given in [22] as follows:

$$\begin{aligned}
\min\quad & z = 0.446(d_1^- + e_1^-) + 0.285(d_2^- + e_2^-) + 0.110(d_3^- + e_3^-) + 0.159(d_4^- + e_4^-) \\
& \quad + 0.129(d_5^- e_5^-) + d_6^+ + e_6^+ + d_7^+ + e_7^+, \\
\text{subject to}\quad & 62.7x_1 + 79.38x_2 + 24.5x_3 + 55.13x_4 + 48.02x_5 - d_1^+ + d_1^- = y_1, \\
& y_1 - e_1^+ + e_1^- = 225400, \\
& 225400 \leqslant y_1 \leqslant 237650, \\
& 24x_1 + 6x_3 + 15.36x_4 + 54x_5 - d_2^+ + d_2^- = y_2, \\
& y_2 - e_2^+ + e_2^- = 2352, \\
& 2352 \leqslant y_2 \leqslant 2388, \\
& 3.95x_1 + 4.96x_2 + 7.67x_3 + 3.26x_4 + 5.8x_5 - d_3^+ + d_3^- = y_3, \\
& y_3 - e_3^+ + e_3^- = 1.428, \\
& 1.428 \leqslant y_3 \leqslant 1.433, \\
& 27.7x_1 + 37.18x_2 + 44.44x_3 + 51.02x_4 + 59.17x_5 - d_4^+ + d_4^- = y_4, \\
& y_4 - e_4^+ + e_4^- = 24.5, \\
& 24.5 \leqslant y_4 \leqslant 24.88, \\
& 22.22x_1 + 23.24x_2 + 24.91x_3 + 28.13x_4 + 26.77x_5 - d_5^+ + d_5^- = y_5, \\
& y_5 - e_5^+ + e_5^- = 17.18, \\
& 17.18 \leqslant y_5 \leqslant 17.91, \\
& 5x_1 + 9x_2 + 8x_3 + 10x_4 + 12x_5 - d_6^+ + d_6^- = y_6, \\
& y_6 - e_6^+ + e_6^- = 12, \\
& 5 \leqslant y_6 \leqslant 12, \\
& 7x_1 + 10x_2 + 14x_3 + 11x_4 + 6x_5 - d_7^+ + d_7^- = y_7, \\
& y_7 - e_7^+ + e_7^- = 14, \\
& 6 \leqslant y_7 \leqslant 14, \\
& x_1 + x_2 + x_3 + x_4 + x_5 = 1, \\
& x_i \geqslant 0, \quad i = 1, 2, \ldots, 5, \\
& d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, \quad i = 1, 2, \ldots, 7.
\end{aligned}$$

The multi-choice CGP formulation for the parameter value $\beta = 0.109$ of Example 3 can be given as follows:

$$\begin{aligned}
\min\quad & z = (0.446 + 0.109)d_1^+ + (0.109 - 0.446)d_1^- + (0.285 + 0.109)d_2^+ + (0.109 - 0.285)d_2^- \\
& \quad +(0.110 + 0.109)d_3^+ + (0.109 - 0.110)d_3^- + (0.159 + 0.109)d_4^+ \\
& \quad +(0.109 - 0.159)d_4^- + (0.129 + 0.109)d_5^+ + (0.109 - 0.129)d_5^- \\
& \quad +(1 + 0.109)d_6^+ + (0.109 - 1)d_6^- + (1 + 0.109)d_7^+ + (0.109 - 1)d_7^-, \\
\text{subject to}\quad & 62.7x_1 + 79.38x_2 + 24.5x_3 + 55.13x_4 + 48.02x_5 - d_1^+ + d_1^- = y_1, \\
& 225400 \leqslant y_1 \leqslant 237650, \\
& 24x_1 + 6x_3 + 15.36x_4 + 54x_5 - d_2^+ + d_2^- = y_2, \\
& 2352 \leqslant y_2 \leqslant 2388, \\
& 3.95x_1 + 4.96x_2 + 7.67x_3 + 3.26x_4 + 5.8x_5 - d_3^+ + d_3^- = y_3, \\
& 1.428 \leqslant y_3 \leqslant 1.433, \\
& 27.7x_1 + 37.18x_2 + 44.44x_3 + 51.02x_4 + 59.17x_5 - d_4^+ + d_4^- = y_4, \\
& 24.5 \leqslant y_4 \leqslant 24.88, \\
& 22.22x_1 + 23.24x_2 + 24.91x_3 + 28.13x_4 + 26.77x_5 - d_5^+ + d_5^- = y_5, \\
& 17.18 \leqslant y_5 \leqslant 17.91, \\
& -5x_1 - 9x_2 - 8x_3 - 10x_4 - 12x_5 - d_6^+ + d_6^- = y_6, \\
& -12 \leqslant y_6 \leqslant -5, \\
& -7x_1 - 10x_2 + 14x_3 + 11x_4 + 6x_5 - d_7^+ + d_7^- = y_7, \\
& -14 \leqslant y_7 \leqslant -6, \\
& x_1 + x_2 + x_3 + x_4 + x_5 = 1, \\
& x_i \geqslant 0, \quad i = 1, 2, \ldots, 5, \\
& d_i^+, d_i^- \geqslant 0, \quad i = 1, 2, \ldots, 7.
\end{aligned}$$

The optimal solutions and the values of objective functions for the revised MCGP model are as follows:

$$x^* : x_1 = x_2 = x_3 = x_4 = 0, \quad x_5 = 1,$$

$$y_1 = 225400, \quad y_2 = 2352, \quad y_3 = 1.428, \quad y_4 = 24.5, \quad y_5 = 17.18, \quad y_6 = 12, \quad y_7 = 6,$$

$$f_1(x^*) = 48.02, \quad f_2(x^*) = 54, \quad f_3(x^*) = 5.8, \quad f_4(x^*) = 59.17, \quad f_5(x^*) = 26.77, \quad f_6(x^*) = 12, \quad f_7(x^*) = 6.$$

The optimal solution and the related values of objective functions for the multi-choice CGP model are as follows:

$$x^\circ : x_1 = x_2 = x_4 = x_5 = 0, \quad x_3 = 1,$$

$$y_1 = 237650, \quad y_2 = 2388, \quad y_3 = 1.433, \quad y_4 = 24.88, \quad y_5 = 17.91, \quad y_6 = -5, \quad y_7 = -6,$$

$$f_1(x^\circ) = 24.5, \quad f_2(x^\circ) = 6, \quad f_3(x^\circ) = 7.67, \quad f_4(x^\circ) = 44.44, \quad f_5(x^\circ) = 24.91, \quad f_6(x^\circ) = 8, \quad f_7(x^\circ) = 14.$$

The solution quality of the revised MCGP and the multi-choice CGP can be evaluated and compared by using DM's additive utility function.

The additive utility value of the revised MCGP's optimal solution is $U(x^*) = -0.446\ f_1(x^*) - 0.285\ f_2(x^*) - 0.110\ f_3(x^*) - 0.159\ f_4(x^*) - 0.129\ f_5(x^*) + f_6(x^*) + f_7(x^*) = -32.306$. The multi-choice CGP optimal solution's additive utility value is $U(x^\circ) = -0.446\ f_1(x^\circ) - 0.285\ f_2(x^\circ) - 0.110\ f_3(x^\circ) - 0.159\ f_4(x^\circ) - 0.129\ f_5(x^\circ) + f_6(x^\circ) + f_7(x^\circ) = -1.760$. The solution of the multi-choice CGP is more satisfactory than the revised MCGP for DMs.

Example 3. Let us consider the Example 1 with multi-choice aspiration levels as follows:

$$f_1(x) = x_1 \geqslant 5 \quad \text{and} \quad \leqslant 10 \text{ (for first goal, the more the better)}$$

$$f_2(x) = x_2 \geqslant 5 \quad \text{and} \quad \leqslant 10 \text{ (for second goal, the more the better)}$$

$$f_3(x) = 2x_1 + 3x_2 \geqslant 5 \quad \text{and} \quad \leqslant 10 \text{ (for the first goal, the more the better)}$$

subject to

$$x_1 \leqslant 10.5,$$

$$0.6x_1 + x_2 \leqslant 20.5,$$

$$x_1, x_2 \geqslant 0 \text{ and integer.}$$

In this example, we assume that DM's weights of the goals be 2, 1 and 1, respectively.

The revised MCGP model formulation of the Example 1 with multiple aspiration levels as follows:

$$\min\ z = 2(d_1^+ d_1^- + e_1^+ + e_1^-,) + d_2^+ + d_2^- + e_2^+ + e_2^- + d_3^+ + d_3^- + e_3^+ + e_3^-,$$

$$x_1 - d_1^+ + d_1^- = y_1,$$

$$y_1 - e_1^+ + e_1^- = 10,$$

$$5 \leqslant y_1 \leqslant 10,$$

$$x_2 - d_2^+ + d_2^- = y_2,$$

$$y_2 - e_2^+ + e_2^- = 10,$$

$$5 \leqslant y_2 \leqslant 10,$$

$$2x_1 + 3x_2 - d_3^+ + d_3^- = y_3,$$

$$y_3 - e_3^+ + e_3^- = 10,$$

$$5 \leqslant y_3 \leqslant 10,$$

$$x_1 \leqslant 10.5,$$

$$0.6x_1 + x_2 \leqslant 20.5,$$

$$d_i^+, d_i^-, e_i^+, e_i^- \geqslant 0, i = 1, 2, 3,$$

$$x_1, x_2 \geqslant 0 \text{ and integer.}$$

This problem is solved using LINGO 11.0 to obtain the optimal solutions as $(x_1, x_2, f_1(x), f_2(x), f_1(x)) = (10, 0, 10, 0, 20)$.

Let the weights of the goals be 2, 1 and 1, respectively and $\beta = 0.99$. The multi-choice CGP model formulation of the Example 1 with multiple aspiration levels as follows:

$$\begin{aligned}
&\min\ z = 2.99d_1^+ + 1.99\left(d_2^+ + d_3^+\right) - 1.01d_1^- - 0.01\left(d_2^- + d_3^-\right),\\
&-x_1 - d_1^+ + d_1^- = y_1,\\
&-10 \leqslant y_1 \leqslant -5,\\
&-x_2 - d_2^+ + d_2^- = y_2,\\
&-10 \leqslant y_2 \leqslant -5,\\
&-2x_1 - 3x_2 - d_3^+ + d_3^- = y_3,\\
&-10 \leqslant y_3 \leqslant -5,\\
&x_1 \leqslant 10.5,\\
&0.6x_1 + x_2 \leqslant 20.5,\\
&d_i^+, d_i^- \leqslant 0, \quad i = 1,2,3,\\
&x_1, x_2 \geqslant 0 \text{ and integer.}
\end{aligned}$$

The multi-choice CGP model for Example 1 is solved using LINGO 11.0 to obtain the optimal solutions as $(x_1, x_2, f_1(x), f_2(x), f_1(x)) = (10,14,10,14,62)$.

The additive utility value of the multi-choice CGP's optimal solution is $U(10,14) = 2f_1(10,14) + f_2(10,14) + f_3(10,14) = 96$. The additive utility value of the revised MCGP's optimal solution's is $U(10,0) = 2f_1(10,0) + f_2(10,0) + f_3(10,0) = 40$. The solution of the multi-choice CGP is more satisfactory than the revised MCGP for DMs.

4.3. Computational experience

The advantage of the proposed approach can be also observed through some test examples. These examples are derived from the test problems given by Miettinen et al. [33]. Further information about the problems can be found in the study of Miettinen et al. (2006). The selected test problems are the first 8 problems given in the study [33]. It is clear that the original problems are multi-objective programming problems. A summary of the 8 test problems used is given in Table 2. In Table 2, after the number of problem, the number of variables is denoted by p and the number of objective functions by n. The next columns indicate the numbers of linear constraints lc and nonlinear constraints nc, respectively. The problems classified to be of a linear (lin), quadratic (quad), nonlinear (nonl), or nonsmooth (nons) type. The problem is regarded as linear if all the functions involved are linear and quadratic if at least one of the objective functions is quadratic. In the same way, problems are classified as nonlinear or nonsmooth. The next column specifies whether the problem is convex (conv) or not (nonc). The last column gives the ideal solutions of the problems that are obtained by using the global solver of LINGO 11. The ideal solutions can be obtained by minimizing the each individual objective function subject to the given constrains.

In order to compare the computational performances of the revised MCGP and the multi-choice CGP, we carried out 24 experiments for 8 problems. Three different ranges of interval values have been constructed for each objective function in these problems. These ranges of interval values given in Appendix A are determined by considering the related ideal solutions. Each of test problems is formulated as the revised MCGP and the multi-choice CGP due to the range of interval values. We assume that the goals are equally important for all test examples. So, the weights of goals can be calculated by using the formula $w_i = 1/n$, $i = 1,2,\ldots,n$. The values of β parameter are selected as 10^{-4} in the multi-choice CGP formulations. The value of parameter α_i is taken equal to the value of weight w_i in the revised MCGP formulations for $i = 1,2,\ldots,n$. The performances of the revised MCGP and the multi-choice CGP, measured by additive utility function value, is compared in Table 3. The additive utility function value of any feasible solution x can be calculated as $U(f(x)) = -(\sum_{i=1}^{n} f_i(x))/n$, where n is the number of the objective functions. It is clear that DM wants to maximize the own additive utility function. As it can be seen in Table 3, the solutions of the multi-choice CGP are more satisfactory than the revised MCGP solutions for all the ranges of interval values.

Additionally, the sensitivity analyses are conducted to determine the effect of the changes of the value of β parameter and the value of α_i $(i = 1,2,\ldots,7)$ parameters for the problem 6 in the interval I, II or III, respectively. The horizontal axis in Figs. 2–4 indicates the values of parameters $\alpha_i = \beta$, for $i = 1,2,\ldots,7$. The vertical axis in these figures indicates the values of additive

Table 2
Summary of the test problems.

Problem	p	n	lc	nc	Type	Convexity	Ideal solution
1	3	3	0	1	nonl	nonc	(49,672, 48,996, 48,996)
2	2	5	0	0	nons	nonc	$(-6.34, -3.445, -7.5, 0, 0)$
3	2	3	0	0	nonl	nonc	$(9.1 \times 10^{-5}, 5 \times 10^{-5}, -100.6785)$
4	2	3	1	0	quad	conv	(0,0,0)
5	3	5	1	1	nonl	nonc	$(72.106, 1,301, 0.084, 101346.7, -2,136)$
6	2	7	0	0	nons	nonc	$(-10,919, 0.375 \times 10^{-7}, 2.934, -3, 7.2, 0, 0)$
7	3	3	0	1	quad	conv	(2525, 48,996, 48,996)
8	3	6	0	1	nonl	nonc	$(-6.339, -6.792, -6.597, -7.5, 0, 0.958)$

Table 3
The additive utility values of the optimal solutions of the revised MCGP and the multi-choice CGP.

Problem	Method	Interval I $\left[a^1_{i,min}, a^1_{i,max}\right]$	Interval II $\left[a^2_{i,min}, a^2_{i,max}\right]$	Interval III $\left[a^3_{i,min}, a^3_{i,max}\right]$
1	The revised MCGP	−50353.809	−51142.308	−52213.92
	The multi-choice CGP	−50353.809	−50353.809	−50353.809
2	The revised MCGP	3.053	2.996	2.786
	The multi-choice CGP	3.053	3.053	3.053
3	The revised MCGP	-1.37493×10^{-5}	−0.275	−2.750
	The multi-choice CGP	-1.37493×10^{-5}	-1.37493×10^{-5}	-1.37493×10^{-5}
4	The revised MCGP	−2.200	−2.328	−3.163
	The multi-choice CGP	−2.199	−2.200	−2.200
5	The revised MCGP	−20474.341	−24113.674	−30060.29
	The multi-choice CGP	−20474.341	−20474.341	−20474.341
6	The revised MCGP	1079.384	540.489	−50.976
	The multi-choice CGP	1079.384	1079.384	1079.384
7	The revised MCGP	−34829.160	−34829.160	−34860.88
	The multi-choice CGP	−34829.160	−34829.163	−34829.163
8	The revised MCGP	3.379	3.242	2.744
	The multi-choice CGP	3.379	3.379	3.379

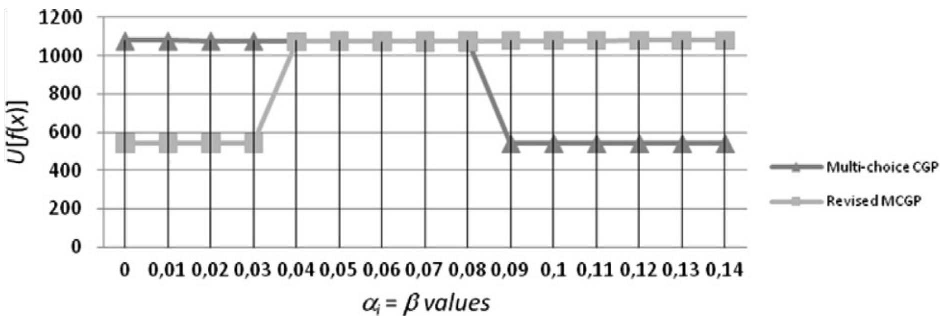

Fig. 2. Additive utility function values for the interval I corresponding to the different $\alpha_i = \beta$ values for $i = 1,2,\ldots,7$.

utility function of the optimal solutions obtained by using the multi-choice CGP and the revised MCGP related to the values of parameters $\alpha_i = \beta$, for $i = 1,2,\ldots,7$.

Figs. 2–4 shows that the additive utility function values of the optimal solutions are very sensitive to the changes in the values of α_i and β parameters. The additive utility function values of the multi-choice CGP has downward trend while the values of β parameters increases in Figs. 2–4. On the other hand, the additive utility function values of the revised MCGP has upward trend while the values of α_i parameters increases in Figs. 2, 3 for $i = 1,2,\ldots,7$. The additive utility function values of the multi-choice CGP are better than the revised MCGP for all the values of α_i and β parameters in Fig. 4.

5. Conclusions

In this paper, we propose the multi-choice goal programming formulation of the conic scalarization function which guarantees an efficient solution and reduces auxiliary constraints and additional variables. This makes it easier to obtain more satisfactory solutions in practice. The theoretical superiority of the proposed model is also supported by a computational experiment conducted on the test examples. The more satisfactory solution is obtained for the literature test problem given in Example 2 by using the multi-choice CGP. The promising results are achieved in the test problems.

The solution methodology of the revised MCGP is different from the multi-choice CGP. While the revised MCGP tries to improve the values of the objective functions from the lower bounds to upper bounds in the related interval values, the multi-choice CGP tries to improve the values of objective functions from lower bounds to efficient frontier. Because the revised MCGP does not guarantee an efficient solution, the multi-choice CGP formulations provides more satisfactory solutions in “less the better” or “more the better” case. On the other hand, if DM wants to obtain a solution provides that values of objective functions should be in the related interval values, the revised MCGP can be preferred by DM to obtain the more promising solutions. The multi-choice CGP formulation is also more suitable than the revised MCGP formulation for non-smooth or non-convex problems.

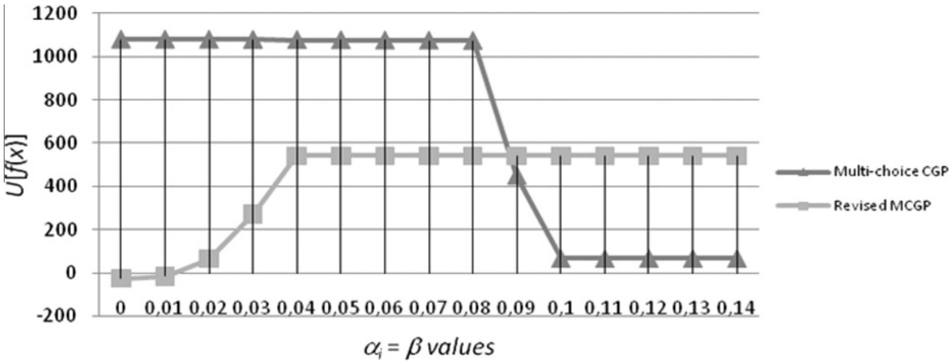

Fig. 3. Additive utility function values for the interval II corresponding to the different $\alpha_i = \beta$ values for $i = 1,2,\ldots,7$.

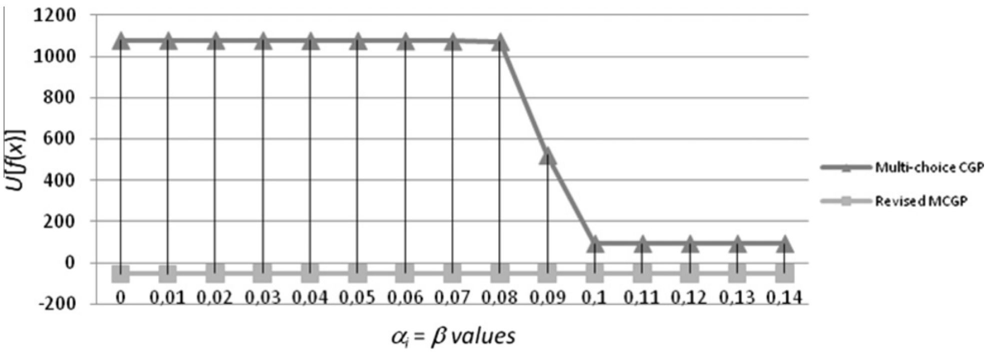

Fig. 4. Additive utility function values for the interval III corresponding to the different $\alpha_i = \beta$ values for $i = 1,2,\ldots,7$.

Additionally, an important question about the usage of the revised MCGP or the multi-choice CGP in practice is that how can be determined the values of the parameters α_i for $i = 1,2,\ldots,n$, or β in practice. The values of the parameters w_i, for $i = 1,2,\ldots,n$, can be obtained by using a multi-criteria decision making methodology such as the simple additive weighting (SAW), Analytic Hierarchy Process (AHP) or Analytic Network Process (ANP). After that the value of the parameter β will be interactively determined by considering DMs preferences. DM may want to change the initial rage of interval values and the initial values of parameters due to the effect of learning after an efficient solution is obtained. An interactive multi-choice CGP procedure can be proposed to allow a flexible decision-making process. Additionally, the mullti-choice CGP formulation can be extended to the other traditional form of GP such as the LGP or the MINMAX GP.

Appendix A

Ranges of interval values related to the test problems.

Problem	Objective functions	$a^1_{i,min}$	$a^1_{i,max}$	$a^2_{i,min}$	$a^2_{i,max}$	$a^3_{i,min}$	$a^3_{i,max}$
1	$f_1(x)$	49,000	52,000	52,000	55,000	55,000	58,000
	$f_2(x)$	49,000	52,000	52,000	55,000	55,000	58,000
	$f_3(x)$	49,000	52,000	52,000	55,000	55,000	58,000
2	$f_1(x)$	−6	−4	−5	−3	−4	−2
	$f_2(x)$	−4	−2	−3	−1	−2	0
	$f_3(x)$	−7.5	0	−6.5	0	−5.5	0
	$f_4(x)$	0	1	0.2	1	0.4	1
	$f_5(x)$	0	1	0.2	1	0.4	1
3	$f_1(x)$	9.1×10^{-5}	1	1	10	10	20
	$f_2(x)$	5×10^{-5}	1	1	10	10	20
	$f_3(x)$	−100	−80	−80	−60	−60	−40

Appendix A (*continued*)

Problem	Objective functions	$a^1_{i,min}$	$a^1_{i,max}$	$a^2_{i,min}$	$a^2_{i,max}$	$a^3_{i,min}$	$a^3_{i,max}$
4	$f_1(x)$	0	2	2	4	4	6
	$f_2(x)$	0	2	2	4	4	6
	$f_3(x)$	0	2	2	4	4	6
5	$f_1(x)$	70	100	100	150	150	200
	$f_2(x)$	0	50	50	100	100	150
	$f_3(x)$	0	0.2	0.2	0.4	0.4	0.6
	$f_4(x)$	100,000	120,000	120,000	150,000	150,000	200,000
	$f_5(x)$	−2	0	0	2	2	4
6	$f_1(x)$	−11,000	−5000	−5000	0	0	5000
	$f_2(x)$	0	5	5	10	10	15
	$f_3(x)$	3	50	50	100	100	150
	$f_4(x)$	−3	20	20	40	40	60
	$f_5(x)$	7.2	50	50	100	100	150
	$f_6(x)$	0	10	10	20	20	30
	$f_7(x)$	0	50	50	100	100	150
7	$f_1(x)$	2500	2600	2600	2700	2700	2800
	$f_2(x)$	49,000	50,000	50,000	51,000	51,000	52,000
	$f_3(x)$	49,000	50,000	50,000	51,000	51,000	52,000
8	$f_1(x)$	−6	−5	−5	−4	−4	−3
	$f_2(x)$	−7	−6	−6	−5	−5	−4
	$f_3(x)$	−7	−6	−6	−5	−5	−4
	$f_4(x)$	−7.5	−6.5	−6.5	−5.5	−5.5	−4.5
	$f_5(x)$	0	2	2	4	4	6
	$f_6(x)$	1	2	2	3	3	4

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