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Comparative analysis of SAW and TOPSIS based on interval-valued fuzzy sets: Discussions on score functions and weight constraints

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ABSTRACT

Interval-valued fuzzy sets involve more uncertainties than ordinary fuzzy sets and can be used to capture imprecise or uncertain decision information in fields that require multiple-criteria decision analysis (MCDA). This paper takes the simple additive weighting (SAW) method and the technique for order preference by similarity to an ideal solution (TOPSIS) as the main structure to deal with interval-valued fuzzy evaluation information. Using an interval-valued fuzzy framework, this paper presents SAW-based and TOPSIS-based MCDA methods and conducts a comparative study through computational experiments. Comprehensive discussions have been made on the influence of score functions and weight constraints, where the score function represents an aggregated effect of positive and negative evaluations in performance ratings and the weight constraint consists of the unbiased condition, positivity bias, and negativity bias. The correlations and contradiction rates obtained in the experiments suggest that evident similarities exist between the interval-valued fuzzy SAW and TOPSIS rankings.

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1. Introduction

The concept of interval-valued fuzzy sets (IVFSs) is defined by an interval-valued membership function (Sambuc, 1975; Zadeh, 1975), and an element's degree of membership in a set is characterized as a closed subinterval of [0, 1]. Because it may be difficult for decision-makers to exactly quantify their opinions of subjective judgments as a number within the interval [0, 1], it is better to represent the degree of membership by an interval rather than a single number. For this reason, IVFSs can be used to capture imprecise or uncertain decision information and many useful methods have been developed to enrich IVFS theory. Wang and Li (1998) defined interval-valued fuzzy numbers and interval-distributed numbers and provided a starting point for real-world applications. Deschrijver (2007) introduced some arithmetic operators for IVFSs. Vlachos and Sergiadis (2007) established a unified framework that includes the concepts of subsets, entropy, and cardinality for IVFSs. Wu and Mendel (2007) provided definitions of the centroid, cardinality, fuzziness, variance, and skewness of interval type-2 fuzzy sets. Zeng and Guo (2008) proposed a new axiomatic definition of the IVFS inclusion measure and examined relationships among the normalized distance, similarity measure, inclusion measure, and entropy of IVFSs. Sun, Gong, and Chen (2008) defined an interval-valued relation and built an interval-valued fuzzy information system. Bustince, Barrenechea, Pagola, and Fernandez (2009) presented a method for constructing IVFSs (or interval type-2 fuzzy sets) from a matrix (or image) and analyzed the application of IVFSs to edge detection in grayscale images. Bigand and Colot (2010) proposed a new fuzzy image filter, controlled by IVFSs, to remove noise from images. Yakhchali and Ghodsypour (2010) addressed the determination of possible values of the earliest and latest starting times of an activity in an interval-valued network with minimal time lag. Lu, Huang, and He (2010) developed an interval-valued fuzzy linear-programming method based on infinite α-cuts, and they applied this method to water resource management.

IVFSs involve more uncertainties than ordinary fuzzy sets. They allow for additional degrees of freedom to represent the uncertainty and fuzziness of the real world (Chen & Lee, 2010). Because IVFS theory is valuable in modeling imprecision and due to its ability to easily reflect the ambiguous nature of subjective judgments, IVFSs are suitable for capturing imprecise or uncertain information in fields that require multiple-criteria decision analysis (MCDA). Wei, Wang, and Lin (2011) introduced a correlation and correlation coefficients for interval-valued intuitionistic fuzzy sets. They then established an optimization model based on the negative ideal solution and max-min operator to solve multiple-attribute decision-making problems. Ye (2009) proposed a novel accuracy function for interval-valued intuitionistic fuzzy sets and applied weighted arithmetic average operator in MCDA. Yang, Lin, Yang, Li, and Yu (2009) combined IVFSs and soft sets to obtain an interval-valued fuzzy soft set. They defined the complement and the "and" and "or" operations, proved DeMorgan's associative and

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distribution laws, and applied these to a decision-making problem. Ashtiani, Haghighirad, Makui, and Montazer (2009) presented an interval-valued fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) for solving MCDA problems. Wei and Chen (2009) applied their proposed similarity measure between interval-valued fuzzy numbers to develop a new fuzzy risk analysis algorithm. Chen and Chen (2009) presented a fuzzy risk analysis method based on a similarity measure between interval-valued fuzzy numbers and interval-valued fuzzy number arithmetic operators. Chen and Lee (2010) presented an interval type-2 fuzzy TOPSIS method to handle fuzzy multiple-attribute group decision-making problems based on interval type-2 fuzzy sets. To aggregate interval-valued intuitionistic fuzzy information, Xu (2010) proposed correlated averaging and geometric operators for interval-valued intuitionistic fuzzy processes. In the context of interval-valued intuitionistic fuzzy sets, Li (2010a) constructed a pair of nonlinear fractional programming models to calculate the relative closeness coefficient intervals of alternatives to the ideal solutions. In a similar manner, Li (2010b) developed TOPSIS-based nonlinear-programming methodology.

As a whole, the above-mentioned studies have focused on the extended simple additive weighting (SAW) or TOPSIS methods underlying interval-valued fuzzy information. The SAW method (Harsanyi, 1955) is a commonly known and very widely used method for providing a comparative evaluation procedure in MCDA. SAW uses all criterion values of an alternative and employs the regular arithmetical operations of multiplication and addition. Further, it is also necessary to determine a reasonable basis on which to form the weights reflecting the importance of each criterion. Einhorn and McCoach (1977) investigated the properties of SAW, including conditionally monotonic with utility and risk neutrality of the decision behavior. On the other hand, TOPSIS, developed by Hwang and Yoon (1981), is a well-known MCDA method. The basic concept of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. TOPSIS assumes that each criterion takes either monotonically increasing or monotonically decreasing utility. Both SAW and TOPSIS require the same input data and they can lead to a unique choice by comparing overall evaluations in SAW or closeness coefficients in TOPSIS.

In the decision context of IVFSs, substantial research took the SAW method or TOPSIS technique as the main structure to deal with multi-criteria evaluation information and to construct a priority ranking for a best alternative. The advantage of SAW is simple and easy to use and understand, while TOPSIS considers positive and negative ideal solutions as anchor points to reflect the contrast of the currently achievable criterion performances. Using an interval-valued fuzzy framework, the purpose of this study is to separately establish two MCDA methods using SAW and TOPSIS and then conduct a comparative study through computational experiments. Additional discussions have been made on the influence of score functions and weight constraints. First, a series of score functions for interval-valued evaluations is proposed from various perspectives to identify the mixed results of the outcome expectations. Based on the score functions, the degree of suitability to which each alternative satisfies the decision-maker's requirements, or instead, the relative degree of closeness of each alternative with respect to the positive ideal solution is defined. Because the information available on the relative importance of the multiple criteria for decision-making is often incomplete, this study proposes several optimization models with suitability functions or closeness coefficients for ill-known membership grades. To cope with different weight constraints of criterion importance, an integrated programming model is developed, utilizing both deviation variables and weighted suitability functions (or closeness coefficients). Furthermore, objective information in the decision matrix and subjective information of the criterion importance are combined to construct procedural steps using the SAW and TOPSIS methods for acquiring optimal decisions. Finally, a large set of random MCDA problems are generated, and computational studies are undertaken to compare preference orders determined by interval-valued fuzzy SAW and TOPSIS methods with several score functions and different conditions for the criterion weights.

2. Decision environment and weight assessment

Definition 1. Let Int([0, 1]) stand for the set of all closed subintervals of [0, 1]. Let X be an ordinary finite non-empty set. An IVFS A in X is an expression given by:

$$A = \{\langle x, M_A(x)\rangle | x \in X\}, \tag{1}$$

where the function M_A: $X \to \text{Int}([0,1])$ defines the degree of membership of an element x in A, such that $x \to M_A(x) = [M_A^-(x), M_A^+(x)]$.

Definition 2. For each IVFS A in X, the value of

$$W_A(x) = M_A^+(x) - M_A^-(x) \tag{2}$$

represents the width of the interval $M_A(x)$. $W_A(x)$ can be considered as the degree of uncertainty (or indeterminacy) or the degree of hesitancy associated with the membership of element $x \in X$ in IVFS A. Let $IVFS(X)$ denote the class of IVFSs in the universe X.

2.1. Decision matrix based on IVFSs

In the work presented here, evaluations of each alternative in an MCDA problem with respect to each criterion of the fuzzy concept "excellence" are given using IVFSs. Suppose that there exists a non-dominated set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$. Each alternative is assessed on n criteria, which are denoted by $X = \{x_1, x_2, \ldots, x_n\}$.Let M_{ij}: $X \to \text{Int}([0,1])$ such that $x_j \to M_{ij} = [M_{ij}^-, M_{ij}^+]$, where M_{ij}^- and M_{ij}^+ are the lower extreme and upper extreme, respectively, of the membership degrees of the alternative $A_i \in A$ with respect to the criterion $x_j \in X$ for the fuzzy concept "excellence." In addition, let $X_{ij} = \{\langle x_j, [M_{ij}^-, M_{ij}^+]\rangle\}$. The degree of uncertainty in alternative A_i in the set X_{ij} is defined by $W_{ij} = M_{ij}^+ - M_{ij}^-$. The interval-valued decision matrix D is defined in the following form:

$$D = \begin{bmatrix} & x_1 & x_2 & \cdots & x_n \\ A_1 & [M_{11}^-, M_{11}^+] & [M_{12}^-, M_{12}^+] & \cdots & [M_{1n}^-, M_{1n}^+] \\ A_2 & [M_{21}^-, M_{21}^+] & [M_{22}^-, M_{22}^+] & \cdots & [M_{2n}^-, M_{2n}^+] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & [M_{m1}^-, M_{m1}^+] & [M_{m2}^-, M_{m2}^+] & \cdots & [M_{mn}^-, M_{mn}^+] \end{bmatrix}, \tag{3}$$

where the characteristics of the alternative A_i can be represented by the IVFS as follows:

$$\begin{aligned} A_i &= \{\langle x_1, [M_{i1}^-, M_{i1}^+]\rangle, \langle x_2, [M_{i2}^-, M_{i2}^+]\rangle, \ldots, \langle x_n, [M_{in}^-, M_{in}^+]\rangle\} \\ &= \{\langle x_j, [M_{ij}^-, M_{ij}^+]\rangle | x_j \in X\}. \end{aligned} \tag{4}$$

In a similar manner, the decision-maker's weight lies in the closed interval $[w_j^l,\ w_j^u]$, where $0 \leqslant w_j^l \leqslant w_j^u \leqslant 1$ for each criterion $x_j \in X$. Because there is no objection in the literature to considering normalized weights, the criterion weights should be normalized to sum to one in general. Therefore, $\sum_{j=1}^n w_j^l \leqslant 1$ and $\sum_{j=1}^n w_j^u \geqslant 1$ are required to determine the weights $w_j \in [0, 1]$ $(j = 1, 2, \ldots, n)$ that satisfy $w_j^l \leqslant w_j \leqslant w_j^u$ and $\sum_{j=1}^n w_j = 1$.

2.2. Goal programming model for assessing weights

Assume that a decision-maker provides his/her preferences on X as a set of n interval values $[w_1^l, w_1^u], [w_2^l, w_2^u], \ldots, [w_n^l, w_n^u]$, where $0 \leqslant w_j^l \leqslant w_j^u \leqslant 1$ for each criterion $x_j \in X$. The criterion weight, w_j, belongs to the interval $[w_j^l,\ w_j^u]$, i.e., $w_j^l \leqslant w_j \leqslant w_j^u$ for $j = 1, 2, \ldots, n$. The necessary conditions for acquiring feasible weights are $\sum_{j=1}^n w_j^l \leqslant 1$ and $\sum_{j=1}^n w_j^u \geqslant 1$ for $j = 1, 2, \ldots, n$. Nevertheless, if a decision-maker tends to give positive connotations to the importance of most criteria, this will lead to positivity bias and overestimation of the criterion weights. On the other hand, negativity bias will result in underestimated ratings of criterion importance. Because the positivity or negativity bias leads to distributional errors, an assessment method was developed to determine the criterion weights.

It is possible that $\sum_{j=1}^n w_j^l > 1$ (positivity bias) or $\sum_{j=1}^n w_j^u < 1$ (negativity bias) in the decision-maker's subjective judgments. However, this condition is not permitted by the constraint $\sum_{j=1}^n w_j = 1$. If this is the case, it follows that there are no feasible solutions for the criterion weights. To overcome this difficulty, the condition of $w_j^l \leqslant w_j \leqslant w_j^u$ can be relaxed by introducing the deviation variables d_j^- and d_j^+ for $j = 1, 2, \ldots, n$ as follows:

$$w_j^l - d_j^- \leqslant w_j \leqslant w_j^u + d_j^+, \quad \text{for} \quad j = 1, 2, \ldots, n, \tag{5}$$

where d_j^- and d_j^+ are both non-negative real numbers. If both d_j^- and d_j^+ are equal to zero, then (5) reduces to $w_j^l \leqslant w_j \leqslant w_j^u$ .The smaller the deviation variables d_j^- and d_j^+, the closer the criterion weights w_j to the interval value $[w_j^l, w_j^u]$. Furthermore, the fact that all of the deviation variables turn out to be close to zero indicates that there is no gross violation of the necessary conditions. Thus, the following goal programming model was established:

$$\min \left\{ \sum_{j=1}^n (d_j^- + d_j^+) \right\}$$
$$\text{s.t.} \begin{cases} w_j + d_j^- \geqslant w_j^l & (j = 1, 2, \ldots, n), \\ w_j - d_j^+ \leqslant w_j^u & (j = 1, 2, \ldots, n), \\ d_j^-, d_j^+, w_j \geqslant 0 & (j = 1, 2, \ldots, n), \\ \sum_{j=1}^n w_j = 1 \end{cases} \tag{6}$$

The optimal deviation values $\bar{d}_j^-$ and $\bar{d}_j^+$ for each criterion can be determined by solving the programming problem (6). If the sum of all optimal deviation values is equal to zero, then the criterion weights are consistent with the interval values; otherwise, the weights are inconsistent with the interval values. The information on the criterion weights is included in the range of $w_j^l \leqslant w_j \leqslant w_j^u$ $(j = 1, 2, \ldots, n)$ for the consistent case and in the range of $w_j^l - \bar{d}_j^- \leqslant w_j \leqslant w_j^u + \bar{d}_j^+$ $(j = 1, 2, \ldots, n)$ for the inconsistent case.

3. Score function of interval-valued evaluations

The main feature of IVFSs is that its membership function assigns an interval to each element in a universal set. This interval approximates the correct but not precisely known grades of membership by employing lower and upper approximations. The lower approximations of the membership grades contained in the definition of the IVFSs are exact without any assumption on indeterminacy; thus, they can represent positive outcome expectations of performance ratings. On the other hand, the exact value of non-membership grades equals one minus the upper approximation, and this value represents negative outcome expectations of performance ratings. Therefore, there are two evaluation dimensions in the representation of an interval-valued decision environment: one for a positive outcome expectation and another for a negative outcome expectation. The bi-dimensional framework is appropriate for separately evaluating the advantages and disadvantages, or pros and cons, of the non-dominated alternatives. It can decompose positive and negative comprehensive evaluations and further aggregate these evaluations into a net result. However, the presence of both positive and negative evaluations might cause ambiguity and difficulty in decision-making. One solution is to map the positive and negative evaluations onto a bi-dimensional scale to acquire an overall result. This comprehensive final evaluation can be viewed as the score function.

This study used the score function to represent an aggregated effect of positive and negative evaluations in performance ratings based on IVFS data. The evaluation value of alternative A_i with respect to criterion x_j was determined by the score function $S(X_{ij})$, which was earlier conceptualized in the cumulative prospect theory (CPT) introduced by Tversky and Kahneman (1992) and was named as net predisposition. CPT is an example of a decision-making model that computes a net predisposition in a simple manner (Grabisch, Greco, & Pirlot, 2008). More specifically, the net predisposition is computed as a difference of positive and negative outcomes. That is, the net predisposition is defined as the degree of membership minus the degree of non-membership. In the bi-dimensional framework of IVFSs, a CPT-type function was used to identify the mixed result of positive and negative outcome expectations for $X_{ij} \in IVFS(X)$, which was denoted by $S_{\mathrm{I}}(X_{ij})$.

Furthermore, other score functions for calculating the net predisposition were considered by taking into account the degree of uncertainty (i.e., the hesitancy degree) of IVFSs. By analogy to the definitions proposed by De, Biswas, and Roy (2001) and Kharal (2009), three score functions, denoted by $S_{\mathrm{II}}(X_{ij})$, $S_{\mathrm{III}}(X_{ij})$, and $S_{\mathrm{IV}}(X_{ij})$, were also used to compute the net predisposition of X_{ij}. The function $S_{\mathrm{II}}(X_{ij})$ is defined as the degree of membership minus the product of the non-membership and hesitancy degrees. $S_{\mathrm{III}}(X_{ij})$ is similar, but subtracts the arithmetic mean of the non-membership and hesitancy degrees. In contrast, $S_{\mathrm{IV}}(X_{ij})$ is defined as the arithmetic mean of the membership and non-membership degrees minus the hesitancy degree.

The CPT model and Kharal's (2009) approach are very simple ways of computing the score function, but they do not exhibit steeper slopes for the negative outcome. There may be an asymmetry in the effect of positive and negative information on overall evaluations. As noted, Cacioppo, Gardner, and Berntson (1997) and Grabisch et al. (2008) indicated that negative information has more weight than positive information. Nevertheless, an individual's personal characteristics and own perception of self also influence the resulting ratings of positive and negative information. For example, persons who are promotion-focused are interested in their growth and development, have more hopes and aspirations, and favor the presence of positive outcomes (Chernev, 2004). Following the discussion above, the relative weights or worth of positive and negative parts were considered in computing the score function. Let $\gamma \in [0, 1]$ be a coefficient reflecting the decision-maker's valuation of the importance of positive outcomes relative to that of negative outcomes. By analogy to the concepts of the expected decision matrix (Xu, 2007) and the expected preference relation (Xu, 2006), a parameterized score function of X_{ij} was defined to represent a mixed result of positive and negative outcome expectations, denoted by $S_{\mathrm{V}}(X_{ij})$.

The preceding score functions in the bi-dimensional framework of interval-valued evaluations are presented in Definition 3.

Definition 3. Let $X_{ij} \in IVFS(X)$ for each $A_i \in A$ and $x_j \in X$. The score function of X_{ij} is defined as follows:

$$\text{(i) } S_{\mathrm{I}}(X_{ij}) = M_{ij}^- - (1 - M_{ij}^+) = M_{ij}^- + M_{ij}^+ - 1, \tag{7}$$

(ii) $S_{\mathrm{II}}(X_{ij}) = M_{ij}^- - (1 - M_{ij}^+) \cdot W_{ij} = M_{ij}^- - W_{ij} + M_{ij}^+ \cdot W_{ij}$, (8)

(iii) $S_{\mathrm{III}}(X_{ij}) = M_{ij}^- - \left(\dfrac{1 - M_{ij}^+ + W_{ij}}{2}\right) = \dfrac{1}{2}(3 \cdot M_{ij}^- - 1)$, (9)

(iv) $S_{\mathrm{IV}}(X_{ij}) = \left(\dfrac{M_{ij}^- + 1 - M_{ij}^+}{2}\right) - W_{ij} = \dfrac{1}{2}(1 - 3 \cdot W_{ij})$, (10)

(v) $S_{\mathrm{V}}(X_{ij}) = \gamma \cdot M_{ij}^- + (1-\gamma) \cdot M_{ij}^+, \quad \gamma \in [0,1]$, (11)

where $S_{\mathrm{I}}(X_{ij}),\ S_{\mathrm{II}}(X_{ij}),\ S_{\mathrm{V}}(X_{ij}) \in [-1, 1]$, $S_{\mathrm{III}}(X_{ij}) \in [-0.5, 1]$, and $S_{\mathrm{IV}}(X_{ij}) \in [-1, 0.5]$.

4. MCDA methods based on IVFSs

In a bi-dimensional framework defined over IVFSs, the degree of suitability to which a given alternative satisfies the decision-maker's requirements can be determined by aggregating the score functions of interval-valued evaluations. Instead, we can apply score functions to calculate the separation measures of each alternative from the positive and negative ideal solutions to determine the closeness coefficient. By simultaneously considering the objectives of maximal suitability (or maximal closeness coefficient) and minimal deviation, the optimal weights for criteria were determined by solving an integrated programming model. Then the optimal values of the suitability functions (or closeness coefficients) were utilized to rank all of the non-dominated alternatives. Finally, we present the detailed steps for solving the multi-criteria decision-making problem with SAW and TOPSIS methods.

4.1. Interval-valued fuzzy SAW method

Based on the SAW method, a weighted sum for each alternative can be obtained simply by multiplying the score function for each criterion by the importance weight assigned to the criterion and then summing these products over all criteria. If one denotes the weighted sum of the score functions as the suitability function, it can be used to determine the degree to which an alternative satisfies the decision-maker's requirements. Let $Z(A_i)$ denote the suitability function of alternative A_i and be defined as follows:

Definition 4. Let $X_{ij} \in IVFS(X)$ for each $A_i \in A$ and $x_j \in X$. Let the criterion weight $w_j \in [0,1]$ for each $x_j \in X$. The suitability function of alternative A_i is defined as follows:

(i) $Z_{\mathrm{I}}(A_i) = \sum_{j=1}^{n} w_j \cdot S_{\mathrm{I}}(X_{ij}) = \sum_{j=1}^{n} w_j \cdot (M_{ij}^- + M_{ij}^+ - 1)$, (12)

(ii) $Z_{\mathrm{II}}(A_i) = \sum_{j=1}^{n} w_j \cdot S_{\mathrm{II}}(X_{ij}) = \sum_{j=1}^{n} w_j \cdot (M_{ij}^- - W_{ij} + M_{ij}^+ \cdot W_{ij})$, (13)

(iii) $Z_{\mathrm{III}}(A_i) = \sum_{j=1}^{n} w_j \cdot S_{\mathrm{III}}(X_{ij}) = \frac{1}{2}\sum_{j=1}^{n} w_j \cdot (3 \cdot M_{ij}^- - 1)$, (14)

(iv) $Z_{\mathrm{IV}}(A_i) = \sum_{j=1}^{n} w_j \cdot S_{\mathrm{IV}}(X_{ij}) = \frac{1}{2}\sum_{j=1}^{n} w_j \cdot (1 - 3 \cdot W_{ij})$, (15)

(v) $$Z_{\mathrm{V}}(A_i) = \sum_{j=1}^{n} w_j \cdot S_{\mathrm{V}}(X_{ij}) = \sum_{j=1}^{n} (\gamma \cdot w_j \cdot M_{ij}^- + (1-\gamma) \cdot w_j \cdot M_{ij}^+), \quad \gamma \in [0,1]. \tag{16}$$

After comparing the suitability functions of all alternatives, the alternative with the highest value is prescribed to the decision-maker. However, the information on the multiple criteria corresponding to decision importance may be incomplete or unknown in real applications. Accordingly, an integrated programming model was developed for multi-criteria decision-making in the environment of IVFSs, with the criteria explicitly taken into account.

The optimal value of the suitability function for determining the degree to which the alternative A_i satisfies the decision-maker's requirements can be measured by an optimization model with weighted score functions. Because there are m alternatives in the set A, a total of m linear programming models must be solved to provide m degrees of optimal suitability. Although the optimal weight vector for each alternative can be computed, these optimal weights may be different in general, and thus, the corresponding optimal values of the suitability functions for all m alternatives cannot be compared. In view of the fact that the decision-maker cannot easily or evidently judge the preference relations among all of the non-dominated alternatives, it is reasonable to assume that all non-dominated alternatives are of equal importance. Hence, by assigning these alternatives equal weights of $1/m$, the m linear programming models can be aggregated into one programming model. In the following, a linear programming model was constructed for each $l \in \{\mathrm{I, II, III, IV, V}\}$:

$$\max\left\{\frac{1}{m}\sum_{i=1}^{m} Z_l(A_i) = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j \cdot S_l(X_{ij})\right\}$$
$$\text{s.t.}\begin{cases} w_j^l \leqslant w_j \leqslant w_j^u \ (j = 1, 2, \ldots, n), \\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{17}$$

In general, the optimal solutions of (6) and (17) are different. Thus, a unique criterion weight vector cannot be derived to compute the optimal suitability function of each alternative. To determine a consistent weight vector based on the models in (6) and (17), the following integrated multi-objective programming model was constructed for each $l \in \{\mathrm{I, II, III, IV, V}\}$:

$$\max\left\{\frac{1}{m}\sum_{i=1}^{m} Z_l(A_i) = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n} w_j \cdot S_l(X_{ij})\right\}$$
$$\min\left\{\sum_{j=1}^{n}(d_j^- + d_j^+)\right\}$$
$$\text{s.t.}\begin{cases} w_j + d_j^- \geqslant w_j^l & (j = 1, 2, \ldots, n), \\ w_j - d_j^+ \leqslant w_j^u & (j = 1, 2, \ldots, n), \\ d_j^-, d_j^+, w_j \geqslant 0 & (j = 1, 2, \ldots, n), \\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{18}$$

Let the weighted suitability function be $Z_l(A_i)/m = d_i^l$ for $i = 1, 2, \ldots, m$, where $l \in \{\mathrm{I, II, III, IV, V}\}$. In addition, $\min\{\sum_{j=1}^{n}(d_j^- + d_j^+)\} = \max\{-\sum_{j=1}^{n}(d_j^- + d_j^+)\}$. Following the linear equal-weighted summation method, the model in (18) can be transformed into the following single objective optimization model for each $l \in \{\mathrm{I, II, III, IV, V}\}$:

$$\max\left\{\sum_{i=1}^{m} d_i^l - \sum_{j=1}^{n}(d_j^- + d_j^+)\right\}$$
$$\text{s.t.}\begin{cases} \frac{1}{m} Z_l(A_i) = d_i^l & (i = 1, 2, \ldots, m), \\ w_j + d_j^- \geqslant w_j^l & (j = 1, 2, \ldots, n), \\ w_j - d_j^+ \leqslant w_j^u & (j = 1, 2, \ldots, n), \\ d_j^-, d_j^+, w_j \geqslant 0 & (j = 1, 2, \ldots, n), \\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{19}$$

The solution of (19) yields the optimal values of the suitability functions $\overline{Z}_l(A_i) (= m \cdot \overline{d}_i^l)$ $(i = 1, 2, \ldots, m)$, the optimal weight vector $\overline{w} = (\overline{w}_1, \overline{w}_2, \ldots, \overline{w}_n)$, and the optimal deviation values $\overline{d}_j^-$ and $\overline{d}_j^+$

($j = 1, 2, \ldots, n$). The ranking order of all alternatives is preceded by the degrees of suitability for individual alternatives in order to draw the conclusion of the best alternative. A higher value of $\overline{Z}_l(A_i)$ indicates a better alternative A_i. Thus, the best alternative $\overline{A}^l \in A$ can be generated such that:

$$\overline{A}^l = \{A_i \in A | \max_i \overline{Z}_l(A_i)\}. \tag{20}$$

Moreover, the m alternatives can be ranked in decreasing order of their $\overline{Z}_l(A_i)$ values for all $A_i \in A$.

4.2. Interval-valued fuzzy TOPSIS method

TOPSIS, a compromising model developed by Hwang and Yoon (1981), is a widely used MCDA method. In addition to the SAW method, this study applied TOPSIS to develop another MCDA method based on IVFSs. Based on score functions, we convert the interval-valued decision matrix D into the following form for each $l \in \{\text{I, II, III, IV, V}\}$:

$$D_l = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ A_1 & S_l(X_{11}) & S_l(X_{12}) & \cdots & S_l(X_{1n}) \\ A_2 & S_l(X_{21}) & S_l(X_{22}) & \cdots & S_l(X_{2n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & S_l(X_{m1}) & S_l(X_{m2}) & \cdots & S_l(X_{mn}) \end{matrix}. \tag{21}$$

Then, the weighted decision matrix with score functions is calculated as follows:

$$D'_l = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ A_1 & w_1 \cdot S_l(X_{11}) & w_2 \cdot S_l(X_{12}) & \cdots & w_n \cdot S_l(X_{1n}) \\ A_2 & w_1 \cdot S_l(X_{21}) & w_2 \cdot S_l(X_{22}) & \cdots & w_n \cdot S_l(X_{2n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & w_1 \cdot S_l(X_{m1}) & w_2 \cdot S_l(X_{m2}) & \cdots & w_n \cdot S_l(X_{mn}) \end{matrix}. \tag{22}$$

Regarding the anchor points, the specification of ideal solutions in this paper is predetermined. The positive ideal solution, denoted as A_l^*, and the negative ideal solution, denoted as A_l^*, are defined as follows:

$$A_l^* = \begin{cases} \{\langle x_j, 1\rangle | x_j \in X\} & \text{for } l \in \{\text{I, II, III, V}\}; \\ \{\langle x_j, 0.5\rangle | x_j \in X\} & \text{for } l = \text{IV}, \end{cases} \tag{23}$$

$$A_l^* = \begin{cases} \{\langle x_j, -1\rangle | x_j \in X\} & \text{for } l \in \{\text{I, II, IV, V}\}; \\ \{\langle x_j, -0.5\rangle | x_j \in X\} & \text{for } l = \text{III}. \end{cases} \tag{24}$$

The separation measures, $E_l^*(A_i)$ and $E_l^*(A_i)$, of each alternative from the positive ideal and negative ideal solutions, respectively, are derived from:

$$E_l^*(A_i) = \begin{cases} \sqrt{\sum_{j=1}^n (1 - w_j \cdot S_l(X_{ij}))^2} & \text{for } l \in \{\text{I, II, III, V}\}; \\ \sqrt{\sum_{j=1}^n (0.5 - w_j \cdot S_l(X_{ij}))^2} & \text{for } l = \text{IV} \end{cases} \tag{25}$$

$$E_l^*(A_i) = \begin{cases} \sqrt{\sum_{j=1}^n (-1 - w_j \cdot S_l(X_{ij}))^2} & \text{for } l \in \{\text{I, II, IV, V}\}; \\ \sqrt{\sum_{j=1}^n (-0.5 - w_j \cdot S_l(X_{ij}))^2} & \text{for } l = \text{III}, \end{cases} \tag{26}$$

where $i = 1, 2, \ldots, m$.

The relative closeness of an alternative A_i with respect to the positive ideal solution A_l^* is defined as the following general formula for each $l \in \{\text{I, II, III, IV, V}\}$:

$$C_l(A_i) = \frac{E_l^*(A_i)}{E_l^*(A_i) + E_l^*(A_i)}, \quad \text{for } i = 1, 2, \ldots, m, \tag{27}$$

where $C_l(A_i)$ is the closeness coefficient of A_i and $0 \leqslant C_l(A_i) \leqslant 1$. If the criterion weights could be exactly assessed, the preference order of alternatives would be ranked according to the descending order of $C_l(A_i)$. Moreover, the alternative with the highest $C_l(A_i)$ value will be the best choice.

The optimal value of the closeness coefficient for the alternative A_i with respect to the positive and negative ideal solutions can be measured by a fractional programming model with weighted score functions. Because there are m alternatives in the set A, a total of m fractional programming models have to be solved to provide m optimal closeness coefficients. In a similar manner of the model with SAW, we assume that all non-dominated alternatives are of equal importance and then aggregate the m fractional programming models into one programming model. For each $l \in \{\text{I, II, III, IV, V}\}$, we establish the following fractional programming model:

$$\begin{aligned} &\max \left\{ \frac{1}{m} \sum_{i=1}^m C_l(A_i) = \frac{1}{m} \sum_{i=1}^m \frac{E_l^*(A_i)}{E_l^*(A_i) + E_l^*(A_i)} \right\} \\ &\text{s.t.} \begin{cases} w_j^l \leqslant w_j \leqslant w_j^u \ (j = 1, 2, \ldots, n), \\ \sum_{j=1}^n w_j = 1. \end{cases} \end{aligned} \tag{28}$$

In order to determine a consistent weight vector based on the models in (6) and (28), the following integrated multi-objective programming model is established for each $l \in \{\text{I, II, III, IV, V}\}$:

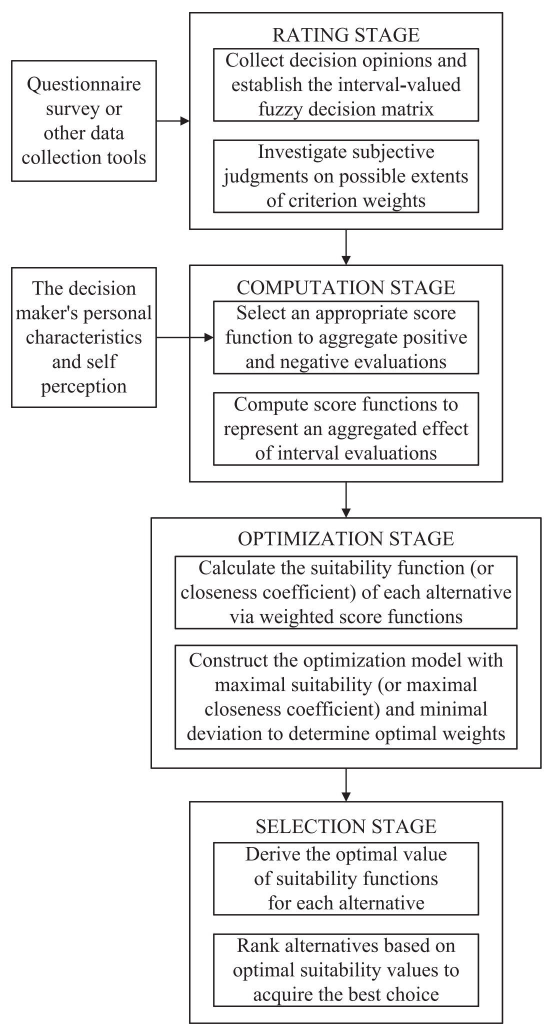
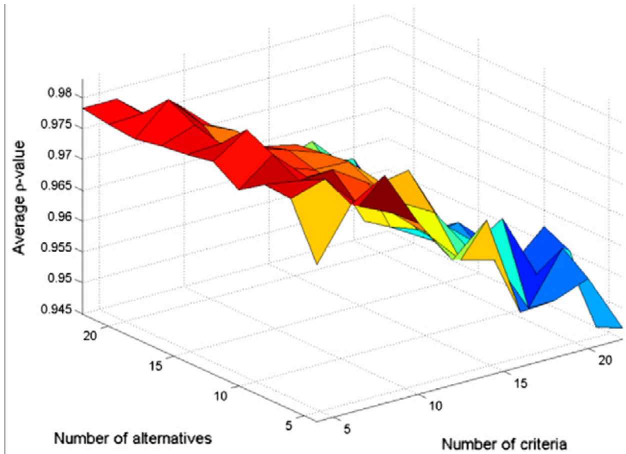
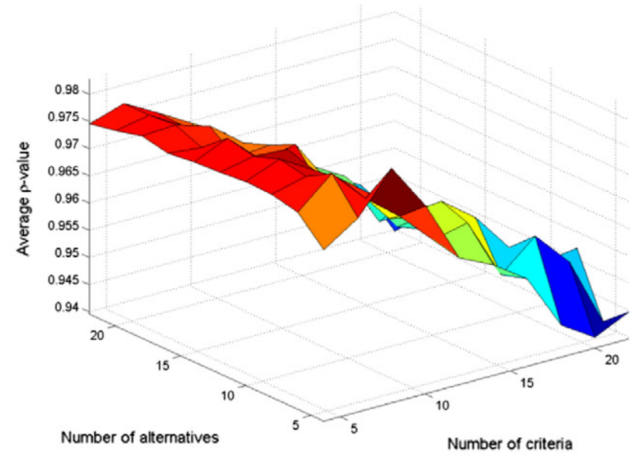


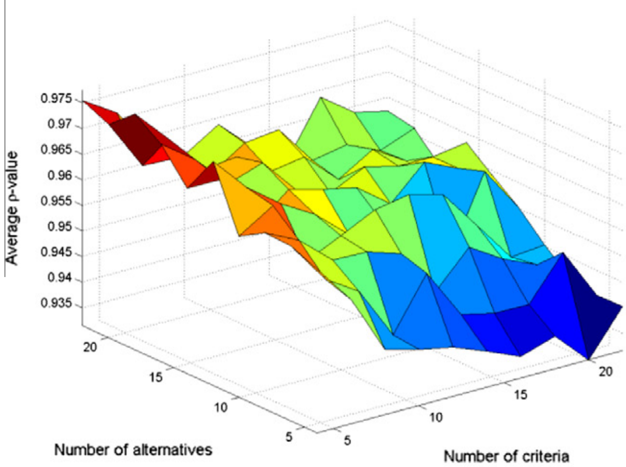
Fig. 1. The steps of the interval-valued fuzzy SAW and TOPSIS methods.



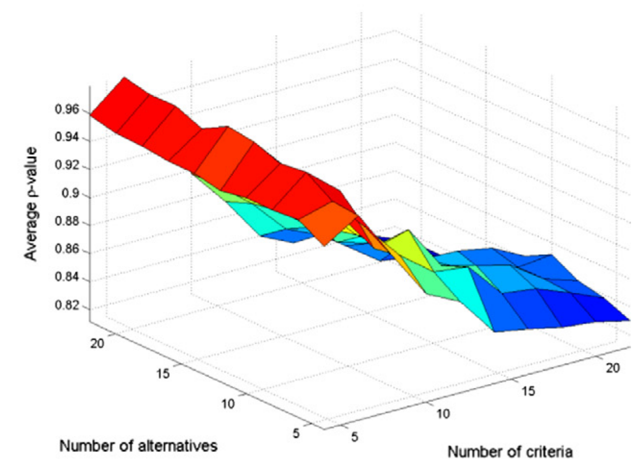
(a) Results obtained using $S_{\mathrm{I}}(X_{ij})$



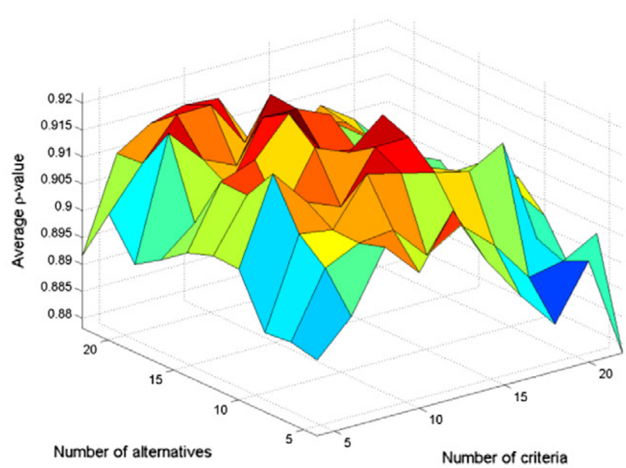
(b) Results obtained using $S_{\mathrm{II}}(X_{ij})$



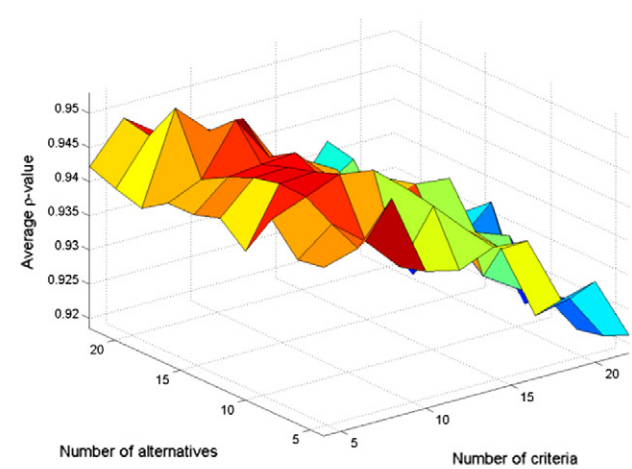
(c) Results obtained using $S_{\mathrm{III}}(X_{ij})$



(d) Results obtained using $S_{\mathrm{IV}}(X_{ij})$



(e) Results obtained using $S_{\mathrm{V'}}(X_{ij})$



(f) Results obtained using $S_{\mathrm{V''}}(X_{ij})$

Fig. 2. Experimental results in the condition (i): average Spearman correlation coefficients for different net predispositions.

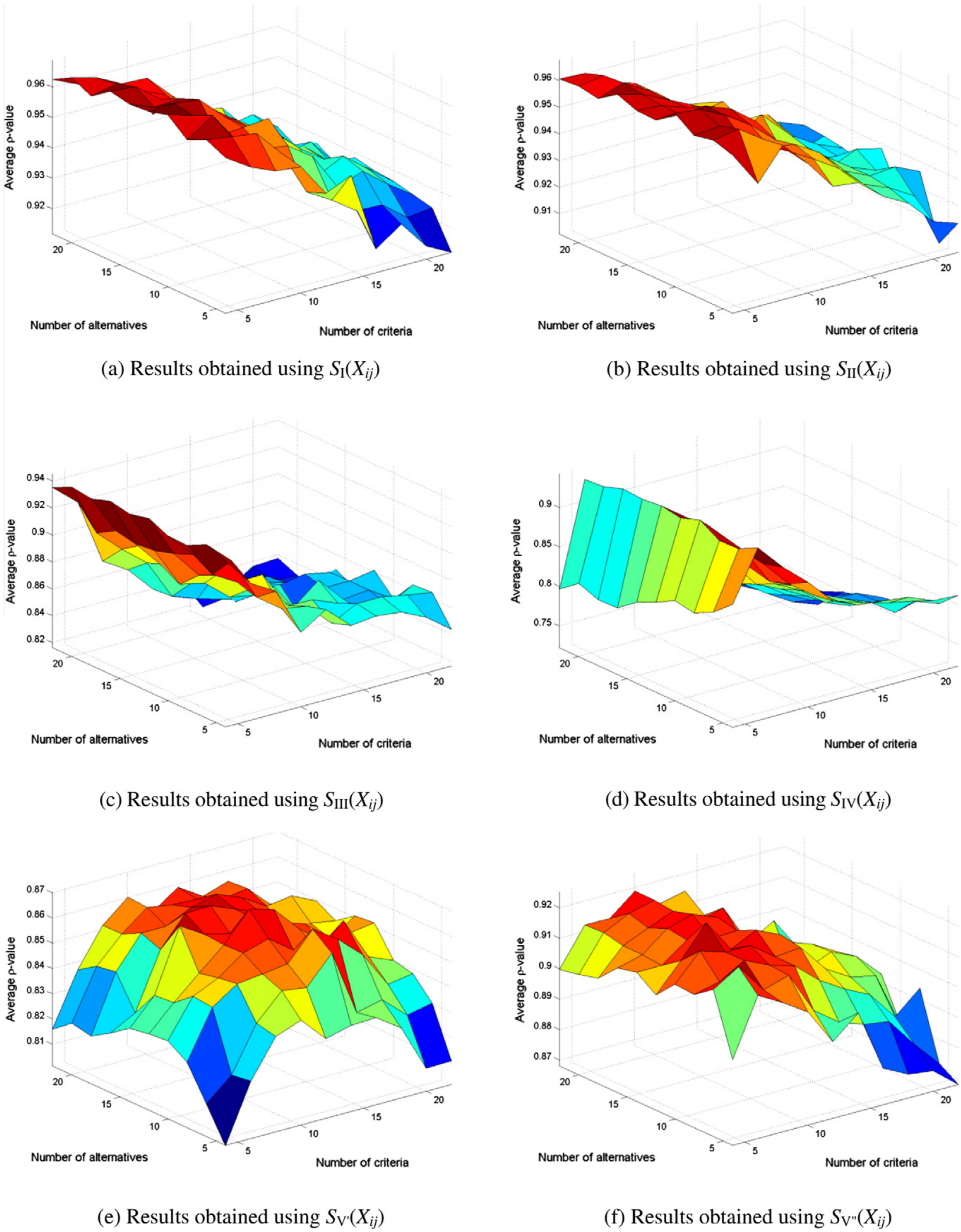
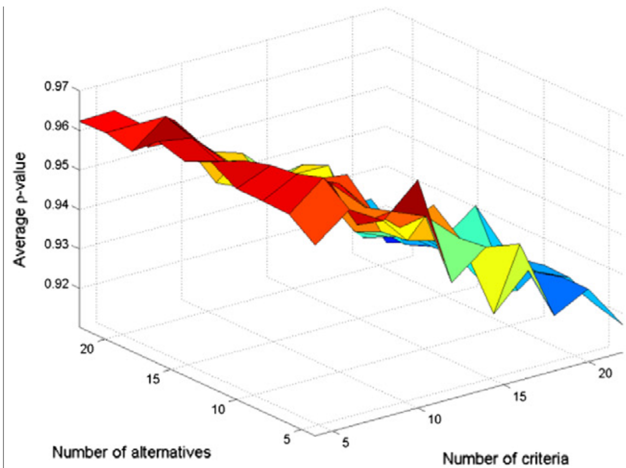
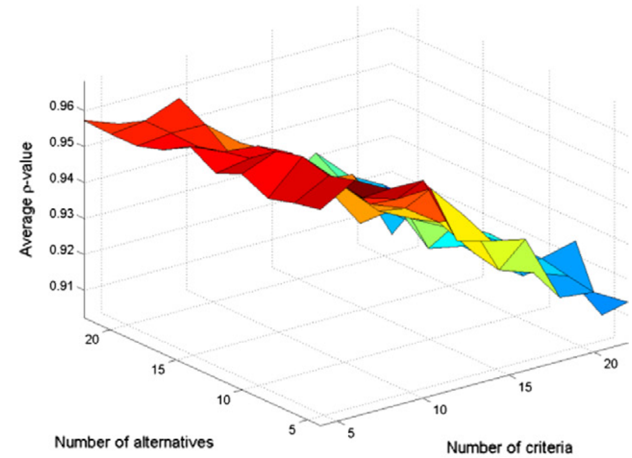


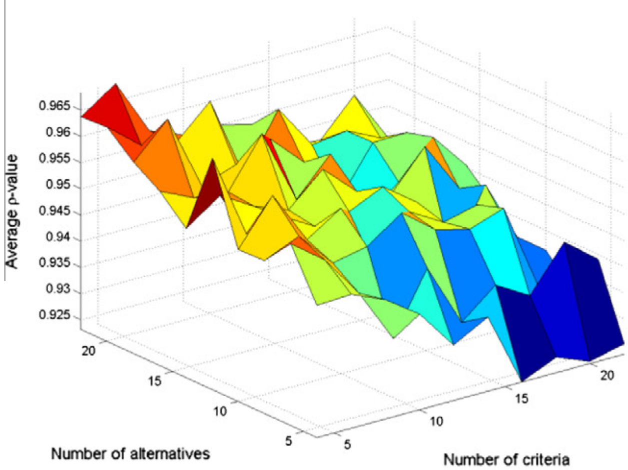
Fig. 3. Experimental results in the condition (ii): average Spearman correlation coefficients for different net predispositions.



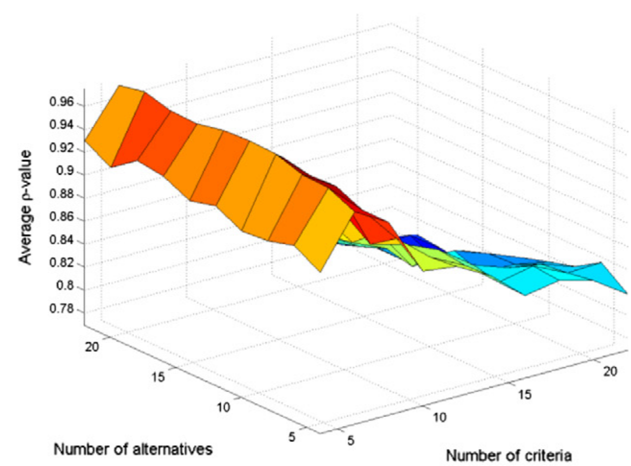
(a) Results obtained using $S_{\mathrm{I}}(X_{ij})$



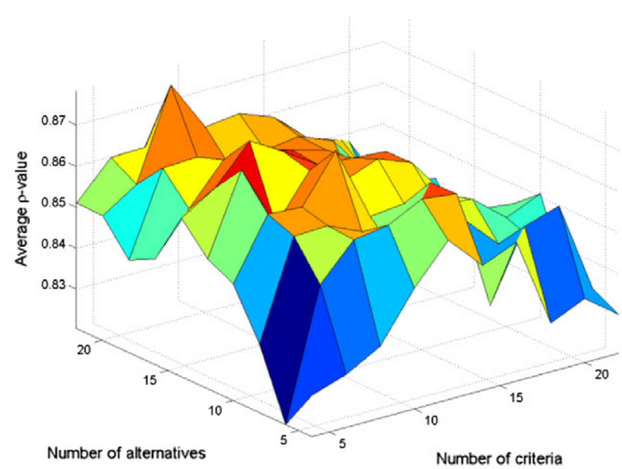
(b) Results obtained using $S_{\mathrm{II}}(X_{ij})$



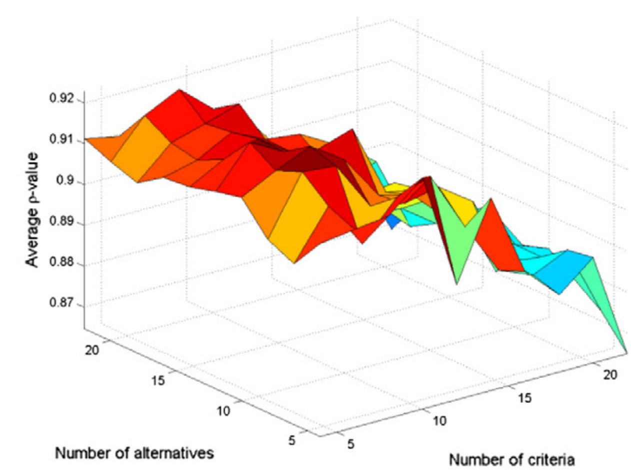
(c) Results obtained using $S_{\mathrm{III}}(X_{ij})$



(d) Results obtained using $S_{\mathrm{IV}}(X_{ij})$



(e) Results obtained using $S_{\mathrm{V'}}(X_{ij})$



(f) Results obtained using $S_{\mathrm{V''}}(X_{ij})$

Fig. 4. Experimental results in the condition (iii): average Spearman correlation coefficients for different net predispositions.

Table 1
Summarized results of average ρ-values from the computational experiments.

		$S_{\text{I}}(X_{ij})$	$S_{\text{II}}(X_{ij})$	$S_{\text{III}}(X_{ij})$	$S_{\text{IV}}(X_{ij})$	$S_{\text{V}'}(X_{ij})$	$S_{\text{V}''}(X_{ij})$	Total
(i)[a]	Mean	0.9665	0.9631	0.9565	0.8886	0.9060	0.9378	0.9364
	(STD)[b]	(0.0105)	(0.0116)	(0.0094)	(0.0500)	(0.0085)	(0.0092)	(0.0165)
(ii)	Mean	0.9459	0.9431	0.8750	0.8252	0.8464	0.9041	0.8899
	(STD)	(0.0150)	(0.0155)	(0.0330)	(0.0599)	(0.0142)	(0.0124)	(0.0250)
(iii)	Mean	0.9437	0.9397	0.9485	0.8689	0.8533	0.9013	0.9092
	(STD)	(0.0167)	(0.0177)	(0.0093)	(0.0582)	(0.0117)	(0.0146)	(0.0214)
Total	Mean	0.9521	0.9486	0.9267	0.8609	0.8686	0.9144	0.9119
	(STD)	(0.0141)	(0.0149)	(0.0173)	(0.0560)	(0.0114)	(0.0120)	(0.0210)

[a] (i) $\sum w_j^l < 1$ and $\sum w_j^u > 1$; (ii) $\sum w_j^l > 1$ and $\sum w_j^u > 1$; (iii) $\sum w_j^l < 1$ and $\sum w_j^u < 1$.
[b] Standard deviations are in parentheses.

$$\max\left\{\frac{1}{m}\sum_{i=1}^{m} C_l(A_i) = \frac{1}{m}\sum_{i=1}^{m}\frac{E_l^*(A_i)}{E_l(A_i)+E_l^*(A_i)}\right\}$$
$$\min\left\{\sum_{j=1}^{n}(d_j^- + d_j^+)\right\}$$
$$\text{s.t.}\begin{cases} w_j + d_j^- \geqslant w_j^l & (j=1,2,\ldots,n),\\ w_j - d_j^+ \leqslant w_j^u & (j=1,2,\ldots,n),\\ d_j^-, d_j^+, w_j \geqslant 0 & (j=1,2,\ldots,n),\\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{29}$$

Let the weighted closeness coefficient be $C_l(A_i)/m = d_i^{\prime l}$ for $i = 1, 2, \ldots, m$, where $l \in \{\text{I, II, III, IV, V}\}$. We construct the following single objective optimization model:

$$\max\left\{\sum_{i=1}^{m} d_i^{\prime l} - \sum_{j=1}^{n}(d_j^- + d_j^+)\right\}$$
$$\text{s.t.}\begin{cases} \frac{1}{m}C_l(A_i) = d_i^{\prime l} & (i=1,2,\ldots,m),\\ w_j + d_j^- \geqslant w_j^l & (j=1,2,\ldots,n),\\ w_j - d_j^+ \leqslant w_j^u & (j=1,2,\ldots,n),\\ d_j^-, d_j^+, w_j \geqslant 0 & (j=1,2,\ldots,n),\\ \sum_{j=1}^{n} w_j = 1. \end{cases} \tag{30}$$

By solving (30), we can obtain the optimal values of the closeness coefficients $\overline{C}_l(A_i)(= m \cdot \overline{d}_i^{\prime l})$ $(i = 1, 2, \ldots, m)$, the optimal weight vector, and the optimal deviation values. The ranking order of all alternatives is preceded by the closeness coefficients for individual alternatives. The best alternative $\overline{A}^{\prime l} \in A$ can be acquired such that

$$\overline{A}^{\prime l} = \{A_i \in A \mid \max_i \overline{C}_l(A_i)\}. \tag{31}$$

4.3. The procedural steps

As stated above, the proposed multiple-criteria decision-making methods based on SAW and TOPSIS can be summarized as series of successive steps, as shown in Fig. 1. The steps for the proposed method will be developed in the following four major stages: (1) rating stage, (2) computation stage, (3) optimization stage, and (4) selection stage. Fig. 1 illustrates procedural steps of the proposed interval-valued fuzzy SAW and TOPSIS methods. The purpose of the rating stage is to construct an interval-valued fuzzy decision environment comprising the decision matrix and the possible extents of criterion weights. According to the decision maker's personal characteristics and own perception of self, select an appropriate form of score functions in the computation stage and derive score functions to aggregate positive and negative evaluations into a net result for the next stage. The optimization stage is intended to acquire the optimal weight vector under the objective of maximal weighted suitability (or maximal closeness coefficient) and minimal deviation values. Finally, compute the optimal value of suitability functions (or closeness coefficients) for each alternative in the selection stage, and the alternatives are then ranked by the decreasing order of optimal suitability values.

5. Comparative study using experimental analysis

5.1. Design of computational experiments

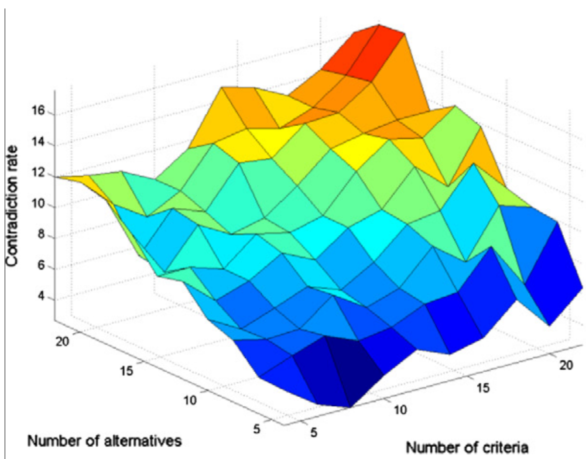
To compare the solution results yielded by the interval-valued fuzzy SAW and TOPSIS methods for different score functions and conditions of criterion importance, numerous random problems of different sizes were generated and computed. Then a comprehensive study was conducted to compare the ranking orders of the alternatives, including analysis of the average Spearman correlation coefficients and the contradiction rate of the best alternative.

Random data were generated to form MCDA problems with all possible combinations of 4, 6, 8, . . . , 22 alternatives and 4, 6, 8, . . . , 22 criteria. Hence, 100 (=10 $\times$ 10) different instances were examined in this study. For each instance, 1000 different interval-valued fuzzy decision matrices D were randomly produced under the preliminary condition of IVFSs. Therefore, a total of 100,000 (=100 $\times$ 1000) sets of experimental cases were generated. In addition, simulation data for criterion importance were randomly generated for each experimental case according to three conditions: (i) $\sum_{j=1}^{n} w_j^l \leqslant 1$ and $\sum_{j=1}^{n} w_j^u \geqslant 1$ (unbiased condition), (ii) $\sum_{j=1}^{n} w_j^l > 1$ and $\sum_{j=1}^{n} w_j^u > 1$ (positivity bias), and (iii) $\sum_{j=1}^{n} w_j^l < 1$ and $\sum_{j=1}^{n} w_j^u < 1$ (negativity bias).

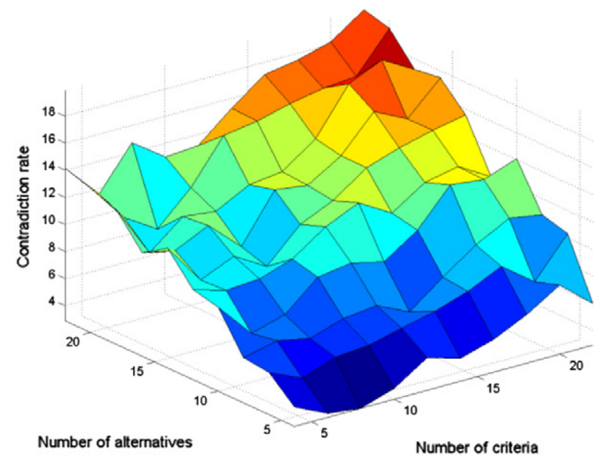
The computational experiments were conducted by following the steps of the interval-valued fuzzy SAW and TOPSIS methods in Fig. 1. For each instance, all possible conditions of criterion importance, including (i), (ii), and (iii), and different types of score functions were considered. Note that the parameter in $S_{\text{V}}(X_{ij})$ is designated as $\gamma = 0.2$ and $\gamma = 0.8$ for the corresponding score functions $S_{\text{V}'}(X_{ij})$ and $S_{\text{V}''}(X_{ij})$, respectively, where $S'_{\text{V}}(X_{ij}) = 0.2 \cdot M_{ij}^- + 0.8 \cdot M_{ij}^+$ and $S''_{\text{V}}(X_{ij}) = 0.8 \cdot M_{ij}^- + 0.2 \cdot M_{ij}^+$. Thus, six score functions, consisting of $S_{\text{I}}(X_{ij})$, $S_{\text{II}}(X_{ij})$, $S_{\text{III}}(X_{ij})$, $S_{\text{IV}}(X_{ij})$, $S_{\text{V}'}(X_{ij})$, and $S_{\text{V}''}(X_{ij})$, were considered in the computational experiments. More specifically, a comparative analysis of interval-valued fuzzy SAW and TOPSIS rankings was performed 18,000 (=6 score functions $\times$ 3 importance conditions $\times$ 1000) times for each combination of m and n values. In the following, we present the major computational results and comparative analysis.

5.2. Experimental results

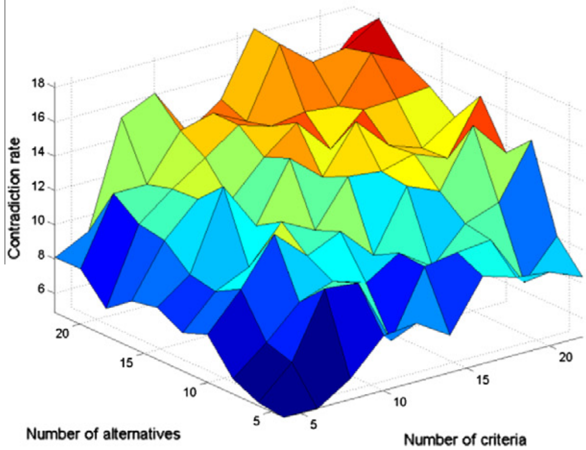
We first compare the average Spearman correlation coefficients (average ρ-values). The average ρ-value is the mean of 1000



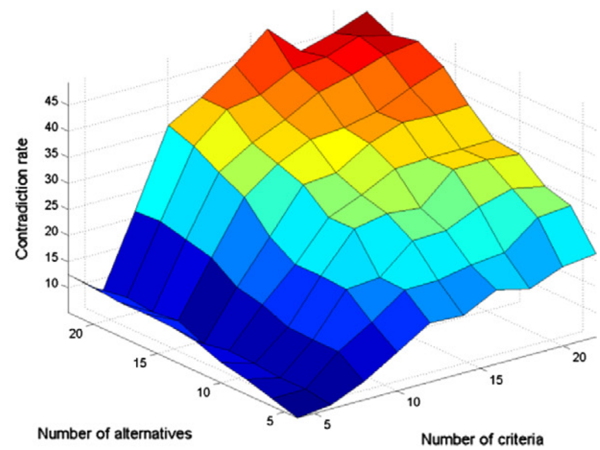
(a) Results obtained using $S_{\mathrm{I}}(X_{ij})$



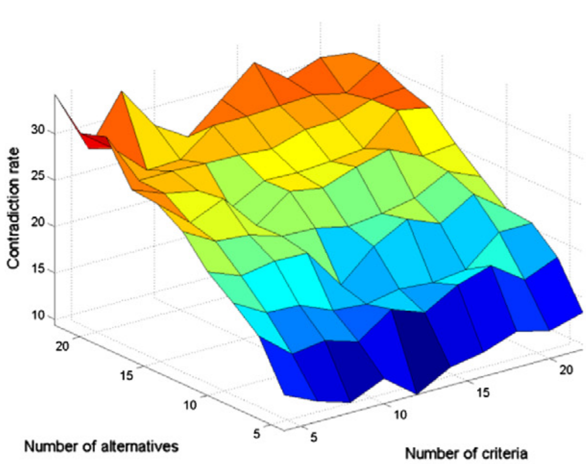
(b) Results obtained using $S_{\mathrm{II}}(X_{ij})$



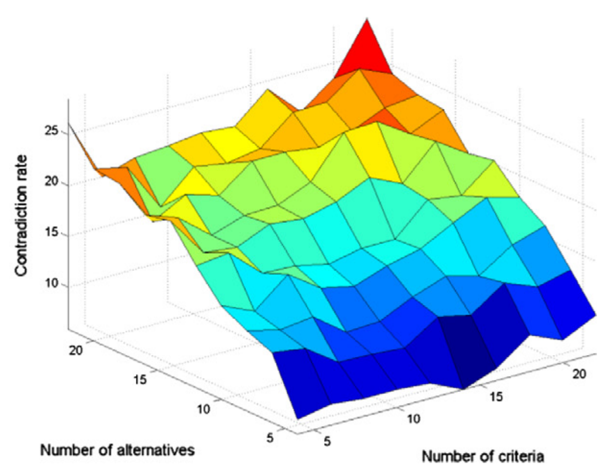
(c) Results obtained using $S_{\mathrm{III}}(X_{ij})$



(d) Results obtained using $S_{\mathrm{IV}}(X_{ij})$

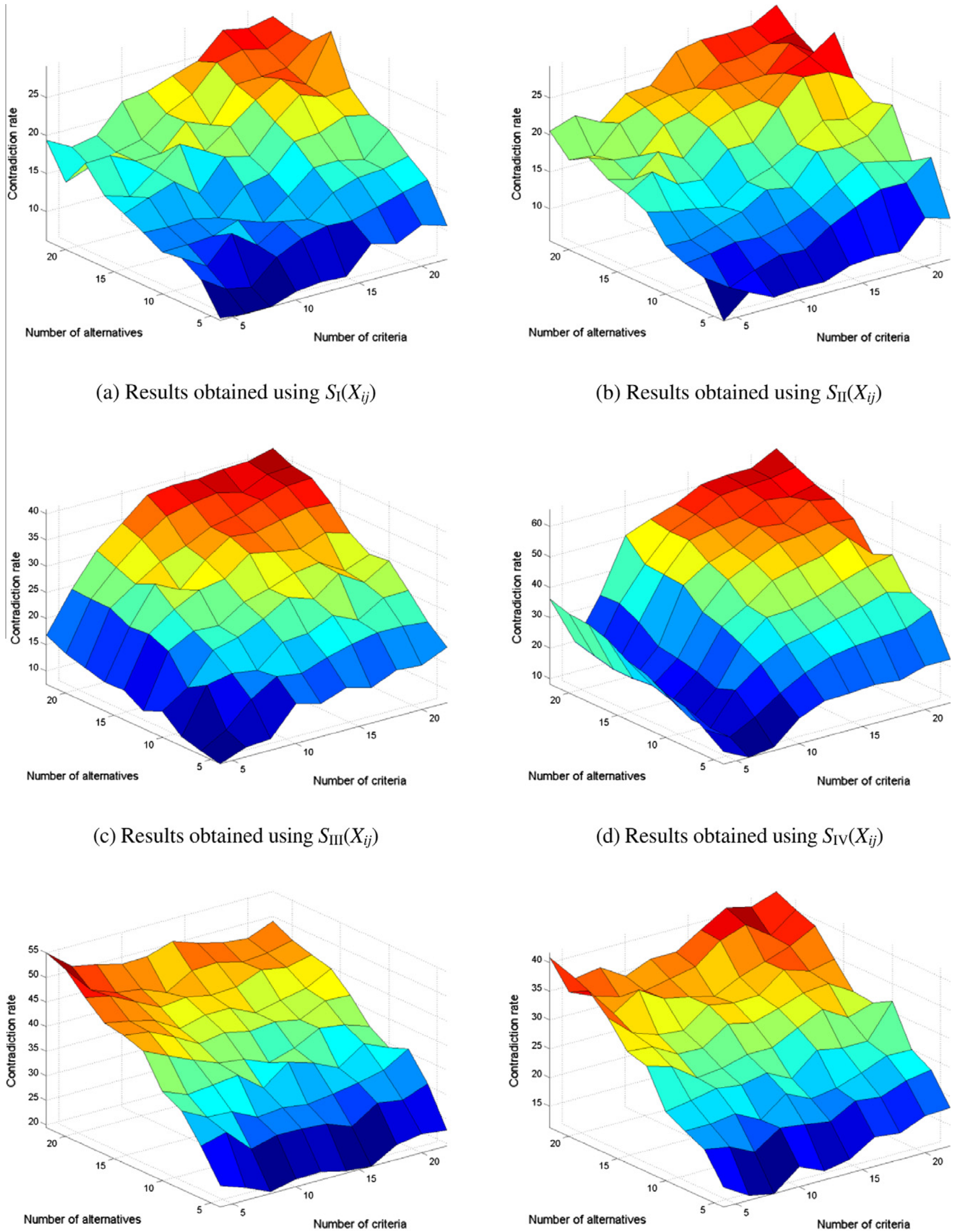


(e) Results obtained using $S_{\mathrm{V'}}(X_{ij})$



(f) Results obtained using $S_{\mathrm{V''}}(X_{ij})$

Fig. 5. Experimental results in the condition (i): contradiction rates (%) for different net predispositions.



(a) Results obtained using $S_{\mathrm{I}}(X_{ij})$

(b) Results obtained using $S_{\mathrm{II}}(X_{ij})$

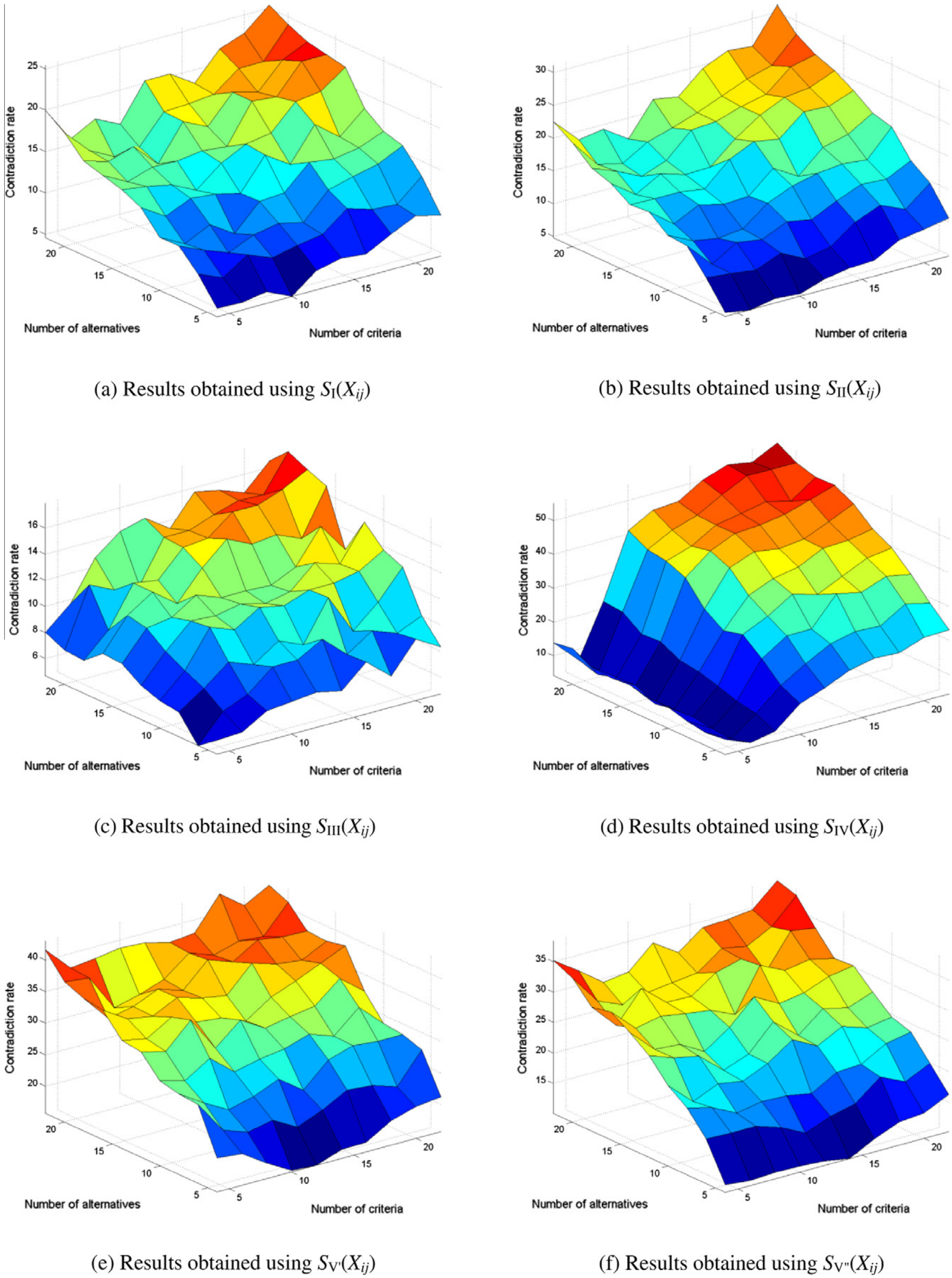
(c) Results obtained using $S_{\mathrm{III}}(X_{ij})$

(d) Results obtained using $S_{\mathrm{IV}}(X_{ij})$

(e) Results obtained using $S_{\mathrm{V'}}(X_{ij})$

(f) Results obtained using $S_{\mathrm{V''}}(X_{ij})$

Fig. 6. Experimental results in the condition (ii): contradiction rates (%) for different net predispositions.



(a) Results obtained using $S_{\mathrm{I}}(X_{ij})$

(b) Results obtained using $S_{\mathrm{II}}(X_{ij})$

(c) Results obtained using $S_{\mathrm{III}}(X_{ij})$

(d) Results obtained using $S_{\mathrm{IV}}(X_{ij})$

(e) Results obtained using $S_{\mathrm{V'}}(X_{ij})$

(f) Results obtained using $S_{\mathrm{V''}}(X_{ij})$

Fig. 7. Experimental results in the condition (iii): contradiction rates (%) for different net predispositions.

correlation coefficients for two rankings of alternatives derived from the application of interval-valued fuzzy SAW and TOPSIS methods in six score functions and three importance criterion conditions. The results are presented in Figs. 2–4. Each combination of alternatives and criteria was employed 1000 times with simulated data; each of the subordinate figures in Figs. 2–4 was obtained from 100,000 (10 alternatives × 10 criteria × 1000 times) different sets of simulated data.

Figs. 2–4 present the comparative results for three conditions of criterion importance (i), (ii), and (iii), respectively. The results obtained using $S_{I}(X_{ij})$ and $S_{II}(X_{ij})$ show a similar trend and shape, while other results have perturbation in some measure. In general, the average value of ρ decreases as the number of criteria increases but doesn't obviously change with the number of alternatives. However, the average ρ-value slight increases as the number of alternatives increases in condition (i) and using $S_{III}(X_{ij})$ (Fig. 2(c)) and in condition (iii) and using $S_{III}(X_{ij})$ (Fig. 4(c)). Furthermore, the comparative results using $S_{V'}(X_{ij})$ present significantly different patterns. In Figs. 2(e), 3(e), and 4(e), there is a slight increase as the number of alternatives or criteria increases when $m, n < 12$; in contrast, there is a gradual drop as the number of alternatives or criteria increases when $m, n > 12$. Except for the application of $S_{V'}(X_{ij})$, the average value of ρ reaches its maximum when the number of criteria is the largest on average. However, in Figs. 2(d), 3(d), and 4(d), it must be noted that the highest point of average ρ-values when using $S_{IV}(X_{ij})$ occurs at $n = 6$, not at $n = 4$. Among the six score function forms, $S_{I}(X_{ij})$ yields the largest average ρ-values and the relatively lower standard deviation of Spearman correlation coefficients. In contrast, the application of $S_{IV}(X_{ij})$ yields the lowest average ρ-values and the highest standard deviation of correlations. In addition, the condition (i) (i.e., unbiased condition) produces relatively larger ρ-values than the other conditions, and the condition (ii) (i.e., positivity bias) produces the smallest values. Nevertheless, the average ρ-value almost exceeds 0.80 as a whole.

The summary comparison results of interval-valued fuzzy SAW and TOPSIS rankings in the computational experiments are listed in Table 1. As the figures in Table 1 indicated, the highest total mean of the average value of ρ for all $m \times n$ combinations and three importance conditions is 0.9521 for $S_{I}(X_{ij})$, followed by 0.9486 for $S_{II}(X_{ij})$, 0.9267 for $S_{III}(X_{ij})$, 0.9144 for $S_{V''}(X_{ij})$, 0.8686 for $S_{V'}(X_{ij})$, and 0.8609 for $S_{IV}(X_{ij})$. In addition, the highest total mean of average ρ-values for all $m \times n$ combinations and six score functions is 0.9364 for the unbiased condition, then 0.9092 for the condition of negativity bias and 0.8899 for the condition of positivity bias. The total mean of average ρ-values for all experimental results is 0.9119 with the standard deviation of 0.0210. These results indicate that the interval-valued fuzzy SAW and TOPSIS methods produce very similar rankings of alternatives with consideration of several score functions and various conditions of criterion importance.

One type of ranking inconsistency deserves special attention: the contradiction rate of the best alternative. Because the decision-maker is always concerned with the best alternative, this study further observed the contradiction rate in the top rank of two ranking results. If the first-place alternatives of two rankings are different, then we count again. For example, if the ranking of a set of six alternatives was $\underline{A_5} \succ A_1 \succ A_6 \succ A_4 \succ A_3 \succ A_2$, as yielded from the interval-valued fuzzy SAW method, and $\underline{A_1} \succ A_5 \succ A_6 \succ A_4 \succ A_3 \succ A_2$, as yielded from the interval-valued fuzzy TOPSIS method, we denote this as a case of a ranking contradiction of the best alternative. The contradiction rate is calculated as the ratio of the accumulated count to the number of experimental cases, 1000, for each $m \times n$ combination. Figs. 5–7 show the contradiction rates for the top-ranked alternative.

The subordinate figures in Figs. 5–7 appear as roughly consistent shapes. The effects of the number of alternatives and the number of criteria are observable: the contradiction rates increase with an increase in the number of alternatives or criteria. In the condition (i), the contradiction rates of the interval-valued fuzzy SAW and TOPSIS rankings are very low when using $S_{I}(X_{ij})$, $S_{II}(X_{ij})$, and $S_{III}(X_{ij})$ (Figs. 5(a)–(c)), especially smaller than 10% at $m = 4$ or 6. It implies that the probability that the most preferred alternatives obtained from the interval-valued fuzzy SAW and TOPSIS methods are contradictory was estimated to be below 10%. On the whole, Figs. 5(a)–(c), 6(a)–(c), 7(a)–(c) show that the most of the contradiction rates are below 20%, which suggests that the top ranks obtained from the two methods using $S_{I}(X_{ij})$, $S_{II}(X_{ij})$, and $S_{III}(X_{ij})$ are very much alike. Conversely, the contradiction rate is moderately high if $S_{IV}(X_{ij})$ and $S_{V'}(X_{ij})$ are applied. Fig. 6(d) show that the extreme contradiction rates greater than 60% occur at very high values (20 and 22) of m and n. As illustrated in the plots in Figs. 5–7, the contradiction rates for the condition (i) are obviously lower than for the other conditions, followed by the condition (iii). Thus, the interval-valued fuzzy SAW and TOPSIS methods often yield the same best alternative in the unbiased condition and negativity bias condition. Among the six score functions, the application of $S_{I}(X_{ij})$ produces the lowest contradiction rates with the lowest standard deviations; conversely, the application of $S_{IV}(X_{ij})$ yields the largest contradiction rates with highest standard deviations.

The average contradiction rates of interval-valued fuzzy SAW and TOPSIS rankings are summarized in Table 2. The lowest total mean of the contradiction rates for all $m \times n$ combinations and three importance conditions is 13.7257% for $S_{I}(X_{ij})$, followed by 15.0983% for $S_{II}(X_{ij})$, 16.2100% for $S_{III}(X_{ij})$, 23.3973% for $S_{V''}(X_{ij})$, 30.7347% for $S_{V'}(X_{ij})$, and 30.7817% for $S_{IV}(X_{ij})$. Furthermore, the lowest total mean of the contradiction rates for all $m \times n$ combinations and six score functions is 16.4050% for the unbiased condition of criterion importance, then 21.1210% for the condition of negativity bias and 27.4478% for the condition of positivity bias. The total mean of average contradiction rates for all experimental results is 21.6579% with the standard deviation of 7.1015%. These

Table 2
Summarized results of contradiction rates (%) from the computational experiments.

		$S_{I}(X_{ij})$	$S_{II}(X_{ij})$	$S_{III}(X_{ij})$	$S_{IV}(X_{ij})$	$S_{V'}(X_{ij})$	$S_{V''}(X_{ij})$	Total
(i)[a]	Mean	9.7270	10.9530	11.8190	26.0250	22.8660	17.0400	16.4050
	(STD)[b]	(3.2142)	(3.9718)	(3.3252)	(12.7709)	(5.6752)	(5.0328)	(5.6650)
(ii)	Mean	16.9750	18.1280	25.5940	37.4370	38.2220	28.3310	27.4478
	(STD)	(5.5516)	(5.9038)	(8.8252)	(15.7283)	(8.6236)	(7.5058)	(8.6897)
(iii)	Mean	14.4750	16.2140	11.2170	28.8830	31.1160	24.8210	21.1210
	(STD)	(4.6635)	(5.6289)	(2.9341)	(15.2205)	(6.4906)	(6.7617)	(6.9499)
Total	Mean	13.7257	15.0983	16.2100	30.7817	30.7347	23.3973	21.6579
	(STD)	(4.4764)	(5.1682)	(5.0281)	(14.5732)	(6.9298)	(6.4334)	(7.1015)

[a] (i) $\sum w_j^l < 1$ and $\sum w_j^u > 1$; (ii) $\sum w_j^l > 1$ and $\sum w_j^u > 1$; (iii) $\sum w_j^l < 1$ and $\sum w_j^u < 1$.
[b] Standard deviations are in parentheses.

results suggest that the interval-valued fuzzy SAW and TOPSIS methods likely produce the same top-ranked alternative for the application of $S_{I}(X_{ij})$, $S_{II}(X_{ij})$, and $S_{III}(X_{ij})$ and for the unbiased condition of criterion importance. In contrast, the top ranks obtained from the application of $S_{IV}(X_{ij})$ and $S_{V'}(X_{ij})$ are more or less different with the probability of 30%.

6. Conclusions

This study took SAW and TOPSIS as the main structure to deal with MCDA problems in the decision context of IVFSs. Using an interval-valued fuzzy framework, a series of score functions for interval-valued evaluations was proposed to determine the mixed results of the outcome expectations. Based on the score functions, several optimization models with various suitability functions or closeness coefficients were established for incompletely-known membership grades. Then an integrated programming model designed to determine the optimal criterion weights was developed by utilizing both deviation variables and weighted suitability functions (or closeness coefficients). The SAW-based and TOPSIS-based methods were developed to acquire optimal multiple-criteria decisions. Furthermore, to clarify the relative differences in the ranking orders obtained from all $m \times n$ combinations, computational experiments were implemented that examined the relationship between the results yielded from the interval-valued fuzzy SAW and TOPSIS methods.

For further drawing a comprehensive comparison, the interval-valued fuzzy SAW and TOPSIS solution results obtained by employing different score functions and different conditions of criterion importance were examined in the computational experiments for all $m \times n$ combinations. Although the criterion weights do not necessarily present the same values between the SAW and TOPSIS results, the ranking results of alternatives yielded by the two methods are similar. Except for the application of $S_{IV}(X_{ij})$ and $S_{V'}(X_{ij})$, the solution results shows that interval-valued fuzzy SAW and TOPSIS methods produce the almost same preference structures because of the very similar ranking orders of the alternatives. Since the decision maker is always concerned with finding the best alternative, the top rank in priority orders of alternatives was further observed in the computational experiments. Regarding both the interval-valued fuzzy SAW and TOPSIS results, they often yield the same best alternative in most of the experimental cases, especially in the unbiased condition and negativity bias condition of criterion importance. In summary, the correlations and contradiction rates obtained suggest that evident similarities exist between the interval-valued fuzzy SAW and TOPSIS rankings, especially for the application of $S_{I}(X_{ij})$, $S_{II}(X_{ij})$, and $S_{III}(X_{ij})$ and for the unbiased condition of criterion importance.

The SAW method is probably the best known and widely-used method for MCDA. Although the interval-valued fuzzy TOPSIS method can add insight on the evaluating alternatives topic being studied, the SAW-based approach is easy to be understood why the method identifies the alternative priority. In addition, calculating the SAW-based method involves less complicated procedures and requires less effort to build preference priority of feasible alternatives using this technique. The experience of experimental analysis in this study reveals that the interval-valued fuzzy SAW method yields extremely close results to much more complicated nonlinear forms, integrated programming model by TOPSIS, while remaining far easier to be used and be understood. Because the interval-valued fuzzy SAW method has a simpler and faster computation process than TOPSIS, we suggest that it may be more suitable for applying SAW to develop MCDA methods in the context of IVFSs with respect to ease of understanding method, ease of employing method, and ease of applying method.

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