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Fuzzy multi-objective project management decisions using two-phase fuzzy goal programming approach

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ABSTRACT

In practical project management (PM) decision problems, environmental coefficients and related parameters are frequently fuzzy in nature, and a decision maker (DM) must simultaneously consider various conflicting objectives in a framework of imprecise aspiration levels. This work focuses on developing a two-phase fuzzy mathematical programming (TPFGP) approach for solving the multi-objective PM decision problems in a fuzzy environment. The original fuzzy multi-objective programming model designed here attempts to simultaneously minimize total project costs, total completion time and total crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities and the constraint of available budget. An industrial case is used to demonstrate the feasibility of applying the proposed approach to real-world PM decisions. Consequently, the proposed approach yields an efficient solution and overall degree of decision maker (DM) satisfaction with the determined goal values. Several significant management implications relating to the practical application of the proposed approach are also presented. Overall, the main contribution of this work lies in presenting a two-phase fuzzy programming methodology for solving real-world PM decision problems with multiple objectives.

1. Introduction

Project management (PM) issues have long attracted interest from both practitioners and academics. Since the program evaluation and review technique (PERT) and the critical path method (CPM) were both developed in the 1950s, numerous models including mathematical programming techniques, algorithms and heuristics have been employed to solve PM decision problems, each with its own advantages and disadvantages (Davis & Patterson, 1975; Elsayed, 1982; Golenko-Ginzburz & Goink, 1998; Lin & Gen, 2007; Yin & Wang, 2008; Russell, 1986). However, most of the conventional PM models, such as Arikan and Gungor (2001), Buckley (1989), DePorter and Ellis (1990), Mjelde (1986), Wang and Fu (1998), consider only direct costs, neglecting relevant indirect and penalty costs. In practical situations, a project's total costs are the sum of direct costs (labor, materials, and other costs directly related to projected activities) and indirect costs (administration, depreciation, interest, contractual penalty and other variable overhead costs). The aim of evaluating time–cost trade-offs is to develop a suitable PM plan that will minimize the sum of direct and indirect costs.

When any of the conventional models are used to PM decisions, the goals and parameters are generally assumed to be deterministic/crisp (Deckor & Hebert, 1989; DePorter & Ellis, 1990; Kotiah & Wallace, 1973; MacCrimmon & Ryavec, 1964; Wiley, Deckro, & Jackson, 1998). In real-world PM decisions, the satisfying goal values should normally be imprecise/fuzzy owing to incomplete and unobtainable information over the project planning horizon. The decision maker (DM) must normally handle conflicting goals in term of the use of organizational resources, and these conflicting objectives are required to be optimized simultaneously by the project managers, often in the framework of fuzzy aspiration levels (Arikan & Gungor, 2001; Al-Fanzine & Haouari, 2005; Viana & Sousa, 2000; Wang & Liang, 2004a, 2006). Solutions to fuzzy multi-objective optimization problems benefit from assessing the imprecision of the DM's judgments, such as "the objective function of project duration should be substantially less than or equal to 120 days," and "total project costs should be substantially less than or equal to 5 million." Therefore, conventional deterministic techniques clearly cannot solve all fuzzy PM programming problems in fuzzy environments.

Fuzzy set theory, was presented by Zadeh (1965), has been found extensive applications in various fields (e.g. Abd El-Wahed & Lee, 2006; Carlsson & Korhonen, 1986; Klir & Yuan, 1995; Kumar, Vrat, & Shan, 2004; Lai & Hwang, 1992; Liang, 2008; Rommelfanger, 1996; Slowinski, 1986; Wang & Liang, 2004b; Weners, 1987; Zimmermann, 1996). Since Zimmermann (1976) first introduced fuzzy set theory into conventional LP problems, fuzzy mathematical programming techniques were developed to tackle problems

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encountered in real-world applications. This work introduced fuzzy set theory to develop a two-phase fuzzy goal programming approach for solving the PM decision problems with multiple goals to obtain an efficient solution in a fuzzy environment. The original multi-objective linear programming (MOLP) designed in this work attempts to minimize total project costs, total completion time and total crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities and the constraint of available budget.

The remainder of this work is organized as follows. Section 2 dedicates to a review of the literature. Section 3 describes the problem, details the assumptions and formulates the fuzzy MOLP model for PM decision problems. Subsequently, Section 4 develops the two-phase fuzzy programming approach to solve fuzzy multi-objective PM decision problems. Next, an industrial case is used to implement the feasibility of the proposed approach in Section 5. Finally, conclusions are drawn in Section 6.

2. Review of the literature

In practical PM decisions, environmental coefficients and parameters are frequently imprecise/fuzzy owing to some related information is incomplete or unavailable. Conventional deterministic PM techniques are unsuitable for yielding an effective solution in uncertain environments. To deal with imprecision, Golenko-Ginzburz and Goink (1997) introduced the techniques of probability theory to PM decisions for a PERT type project to minimize the expected project duration, and its contribution is the product of the average duration of the activity and its probability of being on the critical path in the course of the project's realization. Furthermore, Golenko-Ginzburz and Goink (1998) designed a heuristic for solving the resource-constrained network PM problems which each activity is of random duration depending on the resource amounts assigned to that activity. Rabbani, Ghomi, Jolai, and Lahiji (2007) developed a resource-constrained PM technique for stochastic networks resource allocation decisions having imprecise duration of each activity with a known distribution function in which the values of activities finish times were determined at decision points when at least one activity was ready to be operated and there were available resources. Related studies on stochastic PM decisions included Lukaszewicz (1965), Parks and Ramsing (1969) and Diaz and Hadipriono (1993).

In real-world situations, however, because of the vagueness of information regarding the environmental coefficients and parameters over the project planning horizon, stochastic programming techniques cannot yield an effective solution. Stochastic programming are primarily based on the concepts of randomness theory and can only take the limited form of a given probability distribution function such as normal, exponential and Beta, so it can do little to help practical PM decisions (Lai & Hwang, 1992; Lootsma, 1989; Rabbani et al., 2007). Buckley (1990) and Yazenin (1987) have made some comments regarding the comparison of fuzzy programming and stochastic programming. They noted that the critical problems of applying stochastic programming approaches to solve PM decision problems are lack of computational efficiency and inflexible probabilistic doctrines which might not be able to model the real imprecise meaning of DM because they can only take the limited form of a given probability distribution function. Obviously, complex non-linear PM decision problems are not easy to solve. Alternatively, fuzzy set theory has provided an appropriate methodology to deal quantitatively with decision problems that are formulated as mathematical programming models with imprecise parameters. Zimmermann (1976) first introduced fuzzy set theory into ordinary linear programming (LP) problems. That study considered LP problems with fuzzy goal and constraints. Following the fuzzy decision-making concept of Bellman and Zadeh (1970), that same study confirmed that an equivalent crisp LP problem exists. Subsequently, fuzzy set theory and Zimmermann's fuzzy programming techniques have developed into several fuzzy optimization approaches for solving the PM decision problems and avoiding unrealistic modeling.

Chanas and Kamburowsi (1981) originated the fuzzy PERT method that can derive the possibility distribution of the project completion time in the situation when particular activity duration times in the project network model were given in the form of fuzzy sets on the time space (as fuzzy variables). Mjelde (1986) formulated the special structure of fuzzy resource allocation problems and to define a dedicated algorithm for their solution which is based upon a LP formulation in terms of the resource allocation variables and a single additional variable describing the aspiration level of resource consumptions and activity returns. Chang, Tsujimuta, Gen, and Tozawa (1995) considered the activity times as fuzzy numbers (fuzzy intervals or time intervals) in the project network analysis and the fuzzy Delphi method was used to estimate a reliable time interval of each activity, and an efficient methodology for calculating the fuzzy project completion time and the degree of criticality for each path in a project was proposed based on these time estimates. Yao and Lin (2000) introduced a signed distance ranking method for fuzzy numbers in a CPM of activity-on-edge (AOE) networks, and used them to obtain fuzzy critical path. Liang (2006) designed an interactive FLP approach to solve PM decision problems with and fuzzy constraints in a fuzzy environment. That developed FLP approach attempts to minimize total project costs with reference to direct, indirect and penalty costs, durations of activities, specified project completion time and total allocated budget. More recently, Long and Ohsato (2008) presented a fuzzy critical approach for project scheduling problems under resource constraints and uncertainty, in which consisted of developing a desirable deterministic schedule under resource constraints, and adding a project buffer to the end of the schedule to deal with uncertainty. Additional works in which fuzzy mathematical programming was applied to PM decisions include Buckley (1989), Chanas and Zieliński (2001), Hussein and Abo-Sinna (1995), Hapke and Slowinski (1996), Wang and Fu (1998).

In practical situations, however, the PM plan generally have conflicting objectives regarding the use of organization's resources, and these conflicting objectives are required to be solved simultaneously by the decision maker in the framework of imprecise aspiration levels. Zimmermann (1978) first extended his FLP approach (1976) to an ordinary MOLP problem. For each of the objective functions of this problem, the DM is assumed to have a fuzzy goal such as, "the objective functions should be substantially less than or equal to some value." Then, the corresponding linear membership function is defined and the minimum operator is applied to aggregate fuzzy objective functions. Introducing an auxiliary variable can transform this problem into an equivalent conventional LP problem. Subsequent investigations on fuzzy goal programming (FGP) included those of Dubois and Fortemps (1999), Hannan (1981), Kuwano (1996), Leberling (1981), Luhandjula (1982). The main differences among these methods result from the types of aggregation operators and membership functions that they apply.

DePorter and Ellis (1990) presented a fuzzy programming technique for solving a multiple imprecise or unclear goal optimization problem in a way that compromised among the goals, and was solvable using ordinary LP computer software. Arikan and Gungor (2001) designed a practical application of FGP approach in a real-life project network problem with two fuzzy objectives as minimize completion time and crashing costs wanted to be optimized simultaneously, and comparisons between solutions of FGP, FLP and lexicographic maximization method were also presented. Wang and Liang (2004a) developed a multiple fuzzy goal programming (MFGP) model to PM decisions. That MFGP model can yield the DM's overall degree of satisfaction and the significant characteristics that differentiate the developed MFGP model from the traditional deterministic and stochastic programming models were presented. Chen and Huang (2006) proposed a fuzzy model by combining fuzzy set theory with PERT to calculate the total cycle time of a supply chain system. That designed fuzzy model adopted triangular fuzzy numbers to describe these uncertain variables and the promise delivery possibility index is defined to indicate the order fulfillment degree of a supply chain system based on the fuzzy completion time and fuzzy due date. Wang and Liang (2006) developed an interactive FGP model that offered a systematic framework that facilitates the fuzzy decision-making process to solve multi-objective PM problems. However, these simplified models described above neglect indirect costs, contractual penalty costs and the constraint of total budget; they are thus unrealistic in practical applications. In particular, although it had been justified that the minimum operator used extensively by above fuzzy programming methods to PM decisions possesses some good properties, the optimal solution yielded by the minimum operator may not be an efficient solution (Dubois & Fortemps, 1999; Guu & Wu, 1999; Lee & Li, 1993; Li, Zhang, & Li, 2006).

3. Problem formulation

3.1. Problem description, assumptions and notation

The fuzzy multi-objective PM decision problem examined in this work can be described as follows. Assume a project involves n interrelated activities that must be executed in a certain order before the entire task can be completed. In real-world PM decisions, the values of the objective functions cannot be accurately measured because some information regarding the environmental coefficients and related parameters is incomplete and/or unobtainable over the project planning horizon. Hence, this work focuses on developing a fuzzy mathematical programming technique to solve multi-objective PM decision problems in a fuzzy environment. The fuzzy MOLP model formulated here attempts to simultaneously minimize total project costs, total completion time and total crashing costs associated with direct costs, indirect and contractual penalty costs, duration of activities and the constraint of available budget. These objective functions are required to be optimized simultaneously by the project managers in the framework of fuzzy aspiration levels.

The fuzzy mathematical programming model is based on the following assumptions.

(1) All of the objective functions are fuzzy with imprecise aspiration levels.
(2) All of the objective functions and constraints are linear equations.
(3) The normal time and shortest possible time for each activity and the cost of completing the activity in the normal time and crash time are certain.
(4) The available total budget is known over the duration of the project.
(5) The piecewise linear membership functions are specified for fuzzy goals, and the minimum operator and the average operator are sequentially used to aggregate fuzzy sets in two-phase solution procedure.
(6) The total indirect costs can be divided into two categories, fixed costs and variable costs, and the variable costs per unit time are the same regardless of project completion time.

Assumption 1 relates to the fuzziness of the objective functions in practical PM decision problems, and incorporates the variations in the DM judgments regarding the solutions of fuzzy optimization problems in a framework of imprecise aspiration levels. Assumptions 2–4 indicate that the linearity, proportionality and certainty properties must be technically satisfied as a standard LP form (Carlsson & Korhonen, 1986; Lai & Hwang, 1992). Assumption 5 is made to use the piecewise linear membership functions are specified for representing all fuzzy goals involved. The minimum operator is used to aggregate fuzzy sets, and then the original fuzzy MOLP problem can be converted into an equivalent ordinary LP model that can be solved efficiently using the simplex method (Hannan, 1981; Wang & Liang, 2006; Zimmermann, 1978). Assumption 6 represents that the indirect costs can be divided into fixed costs and variable costs. Fixed costs represent the indirect costs under normal conditions and remain constant regardless of project duration. Meanwhile, variable costs, which are used to measure savings or increases in variable indirect costs, vary directly with the difference between actual completion and normal duration of the project (Liang, 2006; Liang, 2008; Wang & Liang, 2004a).

The following notation is used.

(i,j)	activity between events i and j
z_1	total project costs
z_2	total completion time
z_3	total crashing costs
D_{ij}	normal time for activity (i, j)
d_{ij}	minimum crashed time for activity (i, j)
$C_{D_{ij}}$	normal (direct) cost for activity (i, j)
$C_{d_{ij}}$	minimum crashed (direct) cost for activity (i, j)
k_{ij}	incremental crashing costs for activity (i, j)
Y_{ij}	crash time for activity (i, j)
t_{ij}	crashed duration time for activity (i, j)
E_i	earliest time for event i
E_j	earliest time for event j
E_1	project start time
E_n	project completion time
T_{nc}	project completion time under normal conditions
C_I	fixed indirect costs under normal conditions
m	variable indirect costs per unit time
B	total budget

3.2. Fuzzy multi-objective linear programming model

3.2.1. Objective functions

This work chose multiple objectives for solving the PM decision problems by reviewing the literature and considering practical situations. In real-world situations, most practical decisions made to solve PM problems must consider total project costs (direct costs, indirect costs, and contractual penalty costs), total completion time and/or total crashing costs (Al-Fanzine & Haouari, 2005; Arikan & Gungor, 2001; DePorter & Ellis, 1990; Lin & Gen, 2007; Wang & Liang, 2004a; Wang & Liang, 2006; Yin & Wang, 2008). Notably, these objectives are normally fuzzy owing to incomplete and unavailable information. Accordingly, three fuzzy objective functions are simultaneously considered during the formulation of the fuzzy MOLP model, as follows:

- Minimize total project costs

$$Min\ z_1 \cong \sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_{nc})] \tag{1}$$

where the terms $\sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j k_{ij} Y_{ij}$ are used to calculate total direct costs. Total direct costs include total normal cost and total crashing cost, obtained using additional direct resources such as overtime, personnel and equipment. Generally, the major direct costs such as overtime, personnel and equipment, depend either on activity times or on project completion time, although materials costs are fixed during the planning horizon. Total direct costs increase with decreasing project duration. The terms $[C_I + m(E_n - T_{nc})]$ denote total indirect costs, including administration, contractual penalties, depreciation, financial and other variable overhead costs that can be avoided by reducing total completion time.

- Minimize total completion time

$$Min\ z_2 \cong E_n - E_1 \tag{2}$$

- Minimize total crashing costs

$$Min\ z_3 \cong \sum_i \sum_j k_{ij} Y_{ij} \tag{3}$$

The symbol '≅' is the fuzzified version of '=' and refers to the fuzzification of the aspiration levels. For each objective function in the original fuzzy MOLP model, this work assumes that the DM has such fuzzy objective as, "the objective function should be essentially equal to some value". In real-world PM decisions, Eqs. (1)–(3) are generally fuzzy and incorporate the variations in the DM's judgments relating to the solutions of the fuzzy optimization problem.

3.2.2. Constraints

- *Constraints on the time between events i and j*

$$E_i + t_{ij} - E_j \leqslant 0 \quad \forall i,\ \forall j \tag{4}$$
$$t_{ij} = D_{ij} - Y_{ij} \quad \forall i,\ \forall j \tag{5}$$

- *Constraints on the crashing time for activity (i, j)*

$$Y_{ij} \leqslant D_{ij} - d_{ij} \quad \forall i,\ \forall j \tag{6}$$

- *Constraint on the total budget*

$$z_1 \leqslant B \quad \forall i,\ \forall j \tag{7}$$

- *Non-negativity constraints on decision variables*

$$t_{ij}, Y_{ij}, E_i, E_j \geqslant 0 \quad \forall i,\ \forall j \tag{8}$$

4. Solution methodology

4.1. Two-phase fuzzy goal programming approach (TPFGP)

4.1.1. Phase I

In phase I, the original fuzzy MOLP problem designed above can be solved using the fuzzy decision-making concept of Bellman and Zadeh (1970), together with the FGP technique of Hannan (1981). The piecewise linear membership functions are specified for representing all the fuzzy goals involved, and the minimum operator is adopted to aggregate fuzzy sets. By introducing the auxiliary variable $L^{(1)}$, the original fuzzy MOLP problem can be converted into an equivalent ordinary LP model that can be solved efficiently using the simplex method. Appendix A detailed the derivation of the equivalent ordinary LP model. Furthermore, fuzzy decision-making concept of Bellman and Zadeh (1970), that uses the minimum operator to aggregate all fuzzy sets, is presented in the Appendix B. Consequently, the complete equivalent ordinary LP model is as follows.

$$\begin{aligned}
&Max \quad L^{(1)} \\
&\text{s.t.} \quad L^{(1)} \leqslant -\left(\frac{t_{g2}-t_{g1}}{2}\right)(d^-_{g1} - d^+_{g1}) - \left(\frac{t_{g3}-t_{g2}}{2}\right)(d^-_{g2} - d^+_{g2}) - \cdots \\
&\qquad - \left(\frac{t_{g,P_g+1}-t_{gP_g}}{2}\right)(d^-_{gP_g} - d^+_{gP_g}) + \left(\frac{t_{g,P_g+1}+t_{g1}}{2}\right) z_g \\
&\qquad + \frac{S_{g,P_g+1}+S_{g1}}{2} \quad g = 1,2,\ldots,K \\
&\quad z_g + d^-_{ge} - d^+_{ge} = X_{ge} \quad g = 1,2,\ldots,K,\ e = 1,2,\ldots,P_g \\
&\quad \text{Eqs. (4)–(9)} \\
&\quad 0 \leqslant L^{(1)} \leqslant 1 \\
&\quad t_{ij}, Y_{ij}, E_i, E_j, d^-_{ge}, d^+_{ge} \geqslant 0 \quad \forall i,\ \forall j, \forall g,\ \forall e
\end{aligned} \tag{9}$$

where the auxiliary variable $L^{(1)}$ represents overall DM satisfaction with the given goal values. However, the optimal solution yielded by the minimum operator in phase I may not be an efficient solution, and the computing efficiency of the solutions obtained by the minimum operator is not been assured (Guu & Wu, 1999; Li et al., 2006). Furthermore, a two-phase fuzzy programming technique is presented to overcome the main disadvantage of the minimum approach in phase I.

4.1.2. Phase II

In phase II of the proposed TPFGP approach, the initial solution is forced to improve from that obtained by the minimum operator by adding phase I satisfaction degrees, $L^{(1)}$ to phase II as a constraint, and then the compensatory weighted average operator is used for to obtain overall DM satisfaction degree $L^{(2)}$.

Consequently, Model (9) can be reformulated as follows.

$$\begin{aligned}
&Max \quad L^{(2)} = \sum_{g=1}^{K} w_g L_g \\
&\text{s.t.} \quad L^l_g \leqslant L_g \leqslant -\left(\frac{t_{g2}-t_{g1}}{2}\right)(d^-_{g1} - d^+_{g1}) - \left(\frac{t_{g3}-t_{g2}}{2}\right)(d^-_{g2} - d^+_{g2}) \\
&\qquad - \cdots - \left(\frac{t_{g,P_g+1}-t_{gP_g}}{2}\right)(d^-_{gp} - d^+_{gp}) + \left(\frac{t_{g,P_g+1}+t_{g1}}{2}\right) z_g \\
&\qquad + \frac{S_{g,P_g+1}+S_{g1}}{2} \quad g = 1,2,\ldots,K \\
&\quad z_g + d^-_{ge} - d^+_{ge} = X_{ge} \quad g = 1,2,\ldots,K, \quad e = 1,2,\ldots,P_g \\
&\quad \sum_{g=1}^{K} w_g = 1 \\
&\quad \text{Eqs. (4)–(7)} \\
&\quad 0 \leqslant L^{(2)} \leqslant 1 \\
&\quad 0 \leqslant L^l_g \leqslant 1 \\
&\quad t_{ij}, Y_{ij}, E_i, E_j, d^-_{ge}, d^+_{ge} \geqslant 0 \quad \forall i,\ \forall j,\ \forall g,\ \forall e
\end{aligned} \tag{10}$$

where L^l_g and w_g are respectively the minimum satisfaction degree and the corresponding weight of the gth objective function chosen by DM, and $L^{(2)}$ represents the improved overall DM satisfaction.

4.2. Solution procedure

Step 1: Formulate the original fuzzy MOLP model for the multi-objective PM decision problems according to Eqs. (1)–(8).
Step 2: Specify the degree of membership $f_g(z_g)$ for several values of each objective function z_g, $g = 1, 2, \ldots, K$.
Step 3: Formulate the piecewise linear equations for each $f_g(z_g)$ using Appendix Eq. (A4), $g = 1, 2, \ldots, K$.

Step 4: Introduce the auxiliary variable $L^{(1)}$, thus enabling aggregation of the original fuzzy MOLP problem into an equivalent ordinary LP form using the minimum operator, as Model (9).
Step 5: Solve the Model (9) to obtain an initial compromise solution.
Step 6: Specify the minimum satisfaction degree L_g^l and the corresponding weight w_g of the gth objective function based on the initial compromise solution, and then reformulates Model (9) into an equivalent ordinary single-goal LP model using the weighted average operator, as Model (10).
Step 7: Solve the Model (10) to delivery an improved solution. If the DM is dissatisfied with the compromise solutions, the model should be adjusted interactively until a preferred efficient solution is obtained.

5. Implementation

5.1. Data description

Daya Technologies Corporation was used as a case study to demonstrate the practicality of the developed model in Section 4. The Daya Technologies Corporation is the leading producer of precision machinery and transmission components in Taiwan. Daya is the world's first ballscrew manufacturer certified to ISO 9001, ISO 14001, and OHSAS 18001, and the main manufacturer producing the super precision ballscrew, linear stage, linear bearing, guideways, and aerospace parts. The PM decision examined here involves expanding a metal finishing plant owned by Daya. Currently, the deterministic CPM approach used by Daya suffers from the limitation owing to the fact that a DM does not have sufficient information over the project planning horizon. The case study focuses on developing an interactive two-phase FGP approach to develop a suitable PM plan for the metal finishing plant in uncertain environments. The PM decision of Daya aims to simultaneously minimize total project costs, total completion time and total crashing costs in terms of direct costs, indirect costs, activity duration and the constraint of total budget. Table 1 lists the basic data of the case.

Other relevant data are as follows: fixed indirect costs \$12,000, saved daily variable indirect costs \$150, total budget \$38,500 and project completion time under normal conditions 125 days. The project start time is set to zero. The critical path is 1–5–6–7–9–10–11. The fuzzy multi-objective PM decision presented here focuses on developing a two-phase fuzzy programming approach for optimizing the PM plan of Daya. Solving the PM decision problem is expected to provide an efficient satisfactory result for three fuzzy objectives.

5.2. Solution procedure for the case problem

The fuzzy multi-objective PM decision problem in the Daya case can be solved according to the solution procedure set out above. In phase I, the original fuzzy MOLP model for the PM decision is first formulated according to Eqs. (1)–(8). Moreover, the ordinary single-objective LP problem is solved to obtain the initial solutions for each of the objective functions, the results are z_1 = \$35,900, z_2 = 108 days, and z_3 = \$0. In practical situations, the ordinary single-goal LP optimal solutions were generally used as a starting point of positive ideal solution, and the negative ideal solution for each of the objective functions can be specified subjectively based on historical data and/or knowledge and experience of DM. Then, the degree of membership $f_g(z_g)$, $g = 1, 2, 3$, for several values for each of the objective functions in Model (9) can be identified, as listed in Table 2.

By introducing the auxiliary variable $L^{(1)}$, the fuzzy multi-objective PM decision problem in the Daya case can be transformed into an equivalent ordinary LP form, as Model (9). The complete equivalent single-goal LP model can be formulated as follows.

$$\begin{aligned}
&Max \quad L^{(1)}\\
&\text{s.t.} \quad L \leqslant -0.00001(d_{11}^- - d_{11}^+) - 0.000005(d_{12}^- - d_{12}^+)\\
&\qquad\quad - 0.000035z_1 + 2.5065\\
&\quad L \leqslant -0.00415(d_{21}^- - d_{21}^+) - 0.02915z_2 + 4.25\\
&\quad L \leqslant -0.00005(d_{21}^- - d_{21}^+) - 0.00035z_3 + 1.1\\
&\quad z_1 + d_{11}^- - d_{11}^+ = 55,900\\
&\quad z_1 + d_{12}^- - d_{12}^+ = 45,900\\
&\quad z_2 + d_{21}^- - d_{21}^+ = 132\\
&\quad z_3 + d_{31}^- - d_{31}^+ = 2000\\
&\quad \text{Eqs. (4)–(9)}\\
&\quad 0 \leqslant L^{(1)} \leqslant 1\\
&\quad t_{ij}, Y_{ij}, E_i, E_j, d_{ge}^-, d_{ge}^+ \geqslant 0 \quad \forall i,\ \forall j,\ \forall g,\ \forall e
\end{aligned} \tag{11}$$

LINDO computer software is used to run this ordinary LP model. The optimal solutions are z_1 = \$35,900, z_2 = 116.82 days, z_3 = \$727.43, $(L_1, L_2, L_3) = (1, 0.775, 0.782)$, and the overall DM satisfaction with the given objective values is 0.7805. Furthermore, entering the phase II, initial solutions in phase I forced to improve by adding satisfaction degrees as a constraint, and the compensatory weighted average operator is used to obtain overall DM satisfaction degree. If the DM specifies the minimum satisfaction degree $(L_1^l, L_2^l, L_3^l) = (0.95, 0.92, 0.92)$ and the corresponding weights $(w_1, w_2, w_3) = (0.4, 0.4, 0.2)$ for three fuzzy objectives by referring the historical data, then the equivalent ordinary single-goal LP model can be formulated according to Model (10). Consequently, the improved efficient solutions are z_1 = \$35,951.36, z_2 = 111.29 days, z_3 = \$1608.16, but the overall degree of DM satisfaction increases sharply to 0.8515. Table 3 presents initial and improved PM plans for the Daya case with the proposed approach based on current information.

5.3. Findings

Several significant management implications when practically applying the proposed TPFGP approach to fuzzy multi-objective PM decisions are as follows. First, the proposed approach yields

Table 1
Summarized data in the Daya case (in US dollar).

(i,j)	D_{ij} (days)	d_{ij} (days)	$C_{D_{ij}}$ (\$)	$C_{d_{ij}}$ (\$)	k_{ij} (\$/day)
1–2	14	10	1000	1600	150
1–5	18	15	4000	4540	180
2–3	19	19	1200	1200	–
2–4	15	13	200	440	120
4–7	8	8	600	600	–
4–10	19	16	2100	2490	130
5–6	22	20	4000	4600	300
5–8	24	24	1200	1200	–
6–7	27	24	5000	5450	150
7–9	20	16	2000	2200	50
8–9	22	18	1400	1900	125
9–10	18	15	700	1150	150
10–11	20	18	1000	1200	100

Table 2
Piecewise linear membership functions for the objectives.

z_1	>65,900	65,900	55,900	45,900	35,900	<35,900
$f_1(z_1)$	0	0	0.5	0.8	1.0	1.0
z_2	>144	144	132	120	108	<108
$f_2(z_2)$	0	0	0.4	0.7	1.0	1.0
z_3	>3000	3000	2000	1000	0	<0
$f_3(z_3)$	0	0	0.4	0.7	1.0	1.0

Table 3
Initial and improved PM plans for the Daya case.

Item	Initial solutions (phase I)	Improved solutions (phase II)
Goal values	$L^{(1)} = 0.7818, z_1 = \$35,900.00, z_2 = 116.82$ days, $z_3 = \$727.43$	$L^{(2)} = 0.8515, z_1 = \$35,951.36, z_2 = 111.29$ days, $z_3 = \$1608.16$
Y_{ij} (days)	$Y_{12} = 0, Y_{15} = 0, Y_{23} = 0, Y_{24} = 0, Y_{47} = 0,$ $Y_{410} = 0, Y_{56} = 0, Y_{58} = 0, Y_{67} = 0, Y_{79} = 4,$ $Y_{89} = 0, Y_{910} = 2.18, Y_{1011} = 2$	$Y_{12} = 0, Y_{15} = 1.71, Y_{23} = 0, Y_{24} = 0,$ $Y_{47} = 0, Y_{410} = 0, Y_{56} = 0, Y_{58} = 0, Y_{67} = 3,$ $Y_{79} = 4, Y_{89} = 0, Y_{910} = 3, Y_{1011} = 2$
t_{ij} (days)	$t_{12} = 14, t_{15} = 18, t_{23} = 19, t_{24} = 15,$ $t_{47} = 8, t_{410} = 19, t_{56} = 22, t_{58} = 24,$ $t_{67} = 27, t_{79} = 16, t_{89} = 22, t_{910} = 15.82,$ $t_{1011} = 18$	$t_{12} = 14, t_{15} = 16.29, t_{23} = 19, t_{24} = 15,$ $t_{47} = 8, t_{410} = 19, t_{56} = 22, t_{58} = 24,$ $t_{67} = 24, t_{79} = 16, t_{89} = 22, t_{910} = 15, t_{1011} = 18$
E_i (days)	$E_1 = 0, E_2 = 14, E_4 = 33, E_5 = 18,$ $E_6 = 40, E_7 = 67, E_8 = 42, E_9 = 83,$ $E_{10} = 98.82, E_{11} = 116.82$	$E_1 = 0, E_2 = 14, E_4 = 33, E_5 = 16.29,$ $E_6 = 38.29, E_7 = 62.29, E_8 = 39.57,$ $E_9 = 78.29, E_{10} = 93.29, E_{11} = 111.29$

an efficient solution. An extreme point (feasible solution) vector $\bar{x} \in S$ (S is the feasible region) with corresponding objective function $z_g(\bar{x})$ is said to be efficient if there exists no other feasible point $x \in S$ such that $z_g(x) \leqslant z_g(\bar{x})$ for all g and $z_g(x) < z_g(\bar{x})$ for at least one g, $g = 1, 2, \ldots, K$ (Hannan, 1981; Zimmermann, 1978). The proposed two-phase fuzzy programming technique can overcome the disadvantage of using the minimum operator by adding phase I satisfaction degrees to phase II as a constraint, and the compensatory weighted average operator is employed for to obtain overall DM satisfaction degree. From Table 3, the optimal results by the proposed approach are an efficient solution, because of the solutions obtained using the proposed two-phase fuzzy programming are obviously better than that of one-stage minimum operator approach. Related investigations, such as Hannan (1981), Lee and Li (1993), Guu and Wu (1999), Li et al. (2006) and Özgen, Önut, Gülsün, Tuzkaya, and Tuzkaya (2008), had proved why the output result by the two-phase fuzzy programming approach is always an efficient solution by adopting the average operator in phase II to aggregate fuzzy sets. As a result, an improved PM plan can be obtained by the proposed approach under an acceptable degree of DM satisfaction. Particularly, the proposed approach presents the overall DM satisfaction (L) with the determined goal values in a fuzzy multi-objective PM decision problem. If the solution is $L = 1$, then each goal is fully satisfied; if it is $0 < L < 1$, then all of the goals are satisfied at the level of L, and if it is $L = 0$, then none of the goals are satisfied. Additionally, the L value may be adjusted to identify a better PM plan if the DM did not accept the initial overall degree of DM satisfaction. For example, the improved solutions in the Daya case are z_1 = \$35,951.36, z_2 = 111.29 days, z_3 = \$1608.16, and overall degree of DM satisfaction was generated as 0.8515.

Second, the project DM generally must solve PM decision problems with multiple fuzzy objectives owing to some information being incomplete or unobtainable over the planning horizon, and these conflicting objectives must be optimized simultaneously by the DM in the framework of imprecise aspiration levels. The comparisons listed in Table 3 shows that the interaction of trade-offs and conflicts among dependent multiple objective functions. Due to conflicting and vagueness nature of the multiple objectives, the conventional techniques may not comply with the actual aims of modeling PM decisions and are unsuitable to yield an effective solution. Alternatively, applying fuzzy set theory to imprecise multi-objective PM decisions provides more efficient and flexible model formulation and arithmetic operations (Arikan & Gungor, 2001; Buckley, 1989; Chen & Huang, 2006; DePorter & Ellis, 1990; Long & Ohsato, 2008). Analytical results obtained by implementing Daya case indicate that the developed TPFGP approach satisfies the requirement for the practical application since it attempts to simultaneously minimize the total project costs, total completion time and total crashing costs in a fuzzy environment.

Sensitivity analysis results for varying project duration indicate that minimizing completion time conflicts with minimizing the total project costs and the total crashing costs. The results of analyzing sensitivity for varying project duration indicate that minimizing completion time conflicts with minimizing the total project costs and the total crashing costs, as depicted in Fig. 1. From Table 4, as the duration of the project falls below 116.5 days, the total project costs and the total crashing costs increase. Notably, the solution is infeasible when the duration of the project is far below 108 days, because the cumulative crashing time for all activities on the critical path exceeds the allowed upper limit (17 days). Conversely, when the duration of the project exceeds 116.5 days, the total project costs increase sharply because the relevant indirect and contractual penalties are incurred. Especially, if the project duration is extended beyond 139 days, the project becomes infeasible because the total project costs exceed the total allocated budget. Thus, a DM may be able to shorten project completion time, realizing savings on indirect costs, by increasing direct expenses to accelerate the project. If the DM faces costly indirect and contractual penalties for being late in completing a project, the use of additional resources to reduce the project completion time may be worthwhile.

The comparisons of initial and improved compromise solutions in Table 3 reveal that the changes in the weight and the minimum satisfaction degree of each fuzzy objective function in Model (10) influence both goal and L values. In real-world situations, the values of the relative weights among multiple objective functions can be adjusted subjectively based on the experience and knowledge of DM and/or experts. Related techniques for determining weights include the analytic hierarchy process (AHP), fuzzy AHP and direct

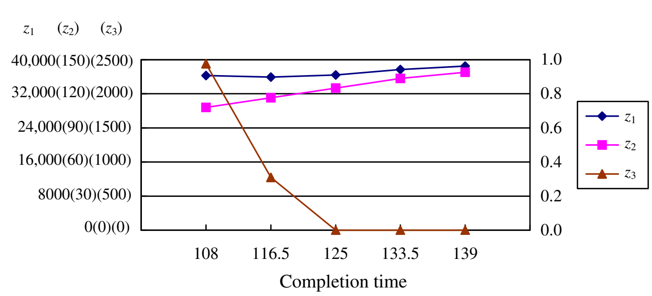

Fig. 1. Goal values of analyzing sensitivity for the total completion time.

Table 4
Results of sensitivity analysis for varying the project duration.

Item	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6
E_n (days)	<108.00	108.00	116.50	125.00	133.50	>139.00
z_1 (\$)	Infeasible	36,290	35,900	36,400	37,650	Infeasible
z_2 (days)		108.00	116.50	125.00	133.50	
z_3 (\$)		2440.00	775.00	0	0	

assessment methods. Several studies had presented related concepts and solution procedure of AHP and fuzzy AHP techniques (Klir & Yuan, 1995; Satty, 1980; Tam & Tummala, 2001; Özgen et al., 2008). Notably, if the minimum satisfaction degree for each of the objective functions is improperly given, it will make the solution procedure more complicated. If the DM increases the minimum satisfaction degree of one fuzzy objective function, it implies that the value of this fuzzy objective function is closer to the optimal value, but it may make other fuzzy objective values far from their optimal values. In particular, when the minimum satisfaction degree specified by DM is too great, Model (10) may have no solution (Li et al., 2006; Özgen et al., 2008). Generally, the derived degree of membership of each fuzzy objective function can be taken as its initial minimum satisfaction degree. As a result, the computing amounts of Model (10) will be decreased.

Additionally, this work uses the piecewise linear membership function to represent the fuzzy goals of the DM for the PM decision problems, and achieves more flexible doctrines via a fuzzy decision-making process. After the DM elicits a small finite number of membership values for each of the objective functions, the piecewise linear membership function enables the DM to approximate satisfactory levels for intermediate points located between these elicited points using line segments. The main advantage of the piecewise linear membership function is that it produces a computationally tractable membership function that closely reflects the real-world structure of the subjective concept of the DM regarding the objectives associated with maximizing the DM preferences (Hannan, 1981; Liang, 2008). In practice, ordinary single-goal LP optimal solutions were normally used as a starting point of positive ideal solution for specifying the degree of membership for several values for each of the objective functions in Model (9). The negative ideal solution for each of the objective functions can be specified subjectively based on historical data and/or knowledge and experience of DM.

Finally, the optimal solution yielded by using the minimum operator in phase I may not be an efficient solution, and the computing efficiency of the solution is not be assured. The minimum operator is preferable when the DM wishes to make the optimal membership function values approximately equal or when the DM feels that the minimum operator is an approximate representation. Aggregate operators can be roughly classified into three categories - intersection, union and averaging operators. Table 5 lists the comparisons of major types of aggregation operators in the existing literature (Klir & Yuan, 1995; Wang & Liang, 2004b; Zimmermann, 1996). Among the various types of aggregate operators, the minimum operator is used most often for solving fuzzy mathematical programming problems, through other patterns may be preferable in some applications. However, the primary drawback of the minimum operator is its lack of discriminatory power between solutions that strongly differ with respect to the fulfillment of membership to the various constraints (Dubois, Fargier, & Prade, 1996; Werner, 1987). For some practical situations, the application of the aggregate operator to draws maps above the maximum operator and below the minimum operator is important. Alternatively, average operators consider the relative importance of fuzzy sets and have the compensative property so that the result of combination will be medium. The proposed two-phase fuzzy programming approach is presented to overcome the main disadvantage of minimum operator by adding phase I satisfaction degrees to phase II as a constraint, and the compensatory weighted average operator is used for to obtain overall DM satisfaction degree. Zimmermann (1996) pointed out that the following eight criteria must be applied selecting an adequate aggregate operator– axiomatic strength, empirical fit, compensation, numerical efficiency, range of compensation, adaptability, aggregating behavior and required scale level of membership function.

5.4. Comparisons

Table 6 compares the results obtained by the ordinary single-goal LP model and the conventional one-stage FGP techniques (Arikan & Gungor, 2001; DePorter & Ellis, 1990; Wang & Liang, 2004a) with the proposed two-phase fuzzy programming approach based on current information for the Daya case. LP-1, LP-2 and LP-3 denote minimizing the total project costs, the total completion time and the total crashing costs by using ordinary single-goal LP model, respectively. As listed in Table 6, the results obtained using the conventional one-stage FGP techniques with linear membership functions are

Table 5
Comparisons of common aggregation operators.

Operator	Example	Brief description
Intersection (*t*-norms)	Minimum	An aggregation scheme is implemented where fuzzy sets are connected by a logical 'and'
	Algebraic product	The result of combination is high if and only if all values are high
	Bounded sum	The minimum operator is a greatest *t*-norm
	Drastic intersection	
Union (*t*-conorms)	Maximum	An aggregation scheme is implemented where fuzzy sets are connected by a logical 'or'
	Algebraic sum	The result of combination is high if some values are high
	Bounded difference	The minimum operator is a smallest *t*-conorm
	Drastic union	
Averaging (compensative)	Mean	Have the compensative property so that the result of combination will be medium
	Weighted	Consider the relative importance of the fuzzy sets
	γ	The γ-operator is the convex combination of the min-operator and the max-operator
	OWA (The ordered weighted averaging)	OWA enables a DM to specify linguistically his agenda for aggregating a collection of fuzzy sets

Table 6
Solution comparisons.

Item	LP-1	LP-2	LP-3	DePorter and Ellis (1990), Arikan and Gungor (2001), Wang and Liang (2004a)	The proposed TPFGP approach
Objective function	*Min* z_1	*Min* z_2	*Min* z_3	*Max L*	*Max L*
L (%)	100	100	100	75.60	85.15
$\tilde{z}_1$ ($)	35,900*	36,290	36,400	35,900.00	35,951.36
z_2 (days)	113	108*	125	116,79.00	111.29
z_3 ($)	1300	2440	0*	732.14	1608.16

* Denotes the optimal value by ordinary single-goal LP model.

z_1 = \$35,900, z_2 = 116.79 days, z_3 = \$732.14, and the overall degree of DM satisfaction is 0.7560. These figures indicate that the proposed approach yields an efficient compromise solution, compared to the ordinary single-goal LP and the conventional one-stage FGP techniques. In the real-world situations, most of the PM decisions problem in an environment takes place in which the goals and/or the constraints are not precisely known due to incomplete and unobtainable information over the project planning horizon. The concepts and techniques of probability theory and fuzzy theory are employed to deal quantitatively with uncertainty. This work introduced fuzzy set theory to develop a fuzzy goal programming approach for solving the multi-objective PM decision problems to obtain an efficient compromise solution in fuzzy environments.

Additionally, Table 7 presents the qualitative comparisons among the proposed TPFGP approach with those of the representative PM techniques, such as deterministic CPM/LP, stochastic programming (Rabbani et al., 2007), fuzzy linear programming (Wang & Fu, 1998) and FGP (Arikan & Gungor, 2001) models. Several important advantages of the proposed approach are summarized as follows. First, the proposed approach satisfies the practical application requirements because it simultaneously minimize total project costs, total completion time and total crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities and the constraint on available total budget. Second, the proposed approach provides a systematic decision-making framework that the DM adjusts interactively the search direction, until the efficient solution satisfies the DM's preferences and is considered to be the preferred satisfactory solution. Third, the proposed approach exhibits greater computational flexibility of the fuzzy arithmetic operations by employing the piecewise linear membership functions to represent fuzzy objectives, and the original fuzzy MOLP model can be converted into an equivalent ordinary single-goal LP form that is easily solved by the simplex method. Finally, computational methodology developed in this work can easily be extended to any other situations and can handle the realistic PM decision problems. Although only involves about 200 decision variables and related decision parameters, the industrial case illustrated in this work is sufficient to lay a strong foundation on which the DM can formulate additional applications of the proposed approach for solving large scale fuzzy/imprecise PM decision problems with multiple goals in a fuzzy environment.

6. Conclusions

In practical PM decisions, the project DM must simultaneously handle multiple conflicting objectives that govern the use of the constrained resources within organizations, and these conflicting objectives are normally imprecise because information is incomplete and/or unavailable over the planning horizon. This work aims to develop a two-phase fuzzy mathematical programming approach to imprecise multi-objective PM decisions. The proposed TPFGP approach attempts to minimize total project costs, total completion time and total crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities and the of constraint available budget. An industrial case is used to demonstrate the feasibility of applying the proposed approach to real PM decisions. Sensitivity analysis results for varying project duration indicate that minimizing completion time conflicts with minimizing the total project costs and the total crashing costs. It implies that if the project DM faces costly indirect and contractual penalties for being late in completing a project, the use of additional resources to reduce the project completion time may be worthwhile.

The main contribution of this work lies in presenting a two-phase fuzzy programming methodology for imprecise multi-objective PM decisions. The major limitations of the proposed approach concern the certain assumptions made for each of the unit cost/time coefficients in the objective functions and related available resources in the constraints. Future researchers may explore the fuzzy goals and fuzzy constraints that the properties of decision variables, unit cost/time coefficients and parameters in PM decision problems are fuzzy/imprecise. Moreover, the proposed approach implicitly assumes that the piecewise linear membership function is the proper representative fuzzy goals of the human DM for the PM decisions. Future works may also apply the linear and/or non-linear membership functions to develop a suitable PM plan. Additionally, in the Daya case implemented in this work, the corresponding weights for three fuzzy objectives are specified by referring the historical data and knowledge of project DM. Future researchers can adopt AHP, fuzzy AHP and other techniques for exploring the relative weights among multiple objective functions to make it better suited to practical applications.

Appendix A

The complete equivalent ordinary LP model for solving the fuzzy multi-objective PM decision problems in phase I is derived as follows (Hannan, 1981; Liang, 2008; Wang & Liang, 2004b)

Step 1: Specify the degree of membership $f_g(z_g)$ for several values for each of the objective functions z_g, $g = 1, 2, \ldots, K$, as Table A1.
Step 2: Convert the membership functions $f_g(z_g)$ into the following form

$$f_g(z_g) = \sum_{e=1}^{P_g} \alpha_{ge}|z_g - X_{ge}| + \beta_g z_g + \gamma_g \quad g = 1, 2, \ldots, K \tag{A1}$$

where, $\alpha_{ge} = -\frac{t_{g,e+1} - t_{ge}}{2}$, $\beta_g = \frac{t_{g,P_g+1} + t_{g1}}{2}$, $\gamma_g = \frac{S_{g,P_g+1} + S_{g1}}{2}$. Here it is assumed that $f_g(z_g) = t_{gr} z_g + S_{gr}$ for each segment $X_{g,r-1} \leqslant z_g \leqslant X_{gr}$, where t_{gr} denotes the slope and S_{gr} is the y-intercept of the sec-

Table 7
Comparisons of the major PM decision models.

Factor	Conventional deterministic CPM/LP	Stochastic programming (Rabbani et al., 2007)	Fuzzy linear programming (Wang & Fu, 1998)	FGP (Arikan & Gungor, 2001)	The proposed TPFGP approach
Objective function	Single, linear	Single, non-linear	Single, linear	Multiple, linear	Multiple, linear
Main consideration	Cost or time	Cost or time	Cost or time	Cost and time	Cost and time
Objective property	Crisp	Probabilistic	Fuzzy	Fuzzy	Fuzzy
Degree of satisfaction	Not presented	Not presented	Presented	Presented	Presented
Main consideration	Time or cost	Time or cost	Time or cost	Time and cost	Time and cost
Aggregate operator	–	–	Minimum	Minimum	Minimum and average
Output solution	Efficient	Not assured	Not assured	Not assured	Efficient
Decision parameter	Crisp	Probabilistic	Fuzzy	Crisp	Crisp
Revised flexibility	–	Low	Medium	Medium	High
Activity time	Crisp	Probabilistic	Fuzzy	Crisp	Crisp
Indirect cost	Not included	Not included	Not included	Not included	Included
Budget limit	Not included	Not included	Not included	Not included	Included

Table A1
Piecewise linear membership functions $f_g(z_g)$.

z_1	$> X_{10}$	X_{10}	X_{11}	X_{12}	...	X_{1P_1}	X_{1,P_1+1}	$< X_{1,P_1+1}$
$f_1(z_1)$	0	0	q_{11}	q_{12}	...	q_{1P_1}	1	1
z_2	$> X_{20}$	X_{20}	X_{21}	X_{22}	...	X_{2P_2}	X_{2,P_2+}	$< X_{2,P_2+1}$
$f_2(z_2)$	0	0	q_{21}	q_{22}	...	q_{2P_2}	1	1
...	...	...	...	...	...	...	...	...
z_K	$> X_{K0}$	X_{K0}	X_{K1}	X_{K2}	...	X_{KP_2}	X_{K,P_2+1}	$< X_{K,P_2+1}$
$f_K(z_K)$	0	0	q_{K1}	q_{K2}	...	q_{KP_2}	1	1

Note: $0 \leqslant q_{ge} \leqslant 1.0, q_{ge} \leqslant q_{g,e+1}, \quad g = 1,2,\ldots,K, \quad e = 1,2,\ldots,P_g$.

tion of the line segment initiated at $X_{g,r-1}$ and terminated at X_{gr} in the piecewise linear function. Hence,

$$f_g(z_g) = -\left(\frac{t_{g2}-t_{g1}}{2}\right)|z_g - X_{g1}| - \left(\frac{t_{g3}-t_{g2}}{2}\right)|z_g - X_{g2}| - \cdots - \left(\frac{t_{g,P_g+1}-t_{gP_g}}{2}\right)|z_g - X_{gP_g}| + \left(\frac{t_{g,P_g+1}+t_{g1}}{2}\right)z_g + \frac{S_{g,P_g+1}+S_{g1}}{2}\left(\frac{t_{g,e+1}-t_{ge}}{2}\right) \neq 0, \quad g = 1,2,\ldots,K, \quad e = 1,2,\ldots,P_g \tag{A2}$$

where $t_{g1} = \left(\frac{q_{g1}-0}{X_{g1}-X_{g0}}\right), t_{g2} = \left(\frac{q_{g2}-q_{g1}}{X_{g2}-X_{g1}}\right), \ldots, t_{g,P_g+1} = \left(\frac{1.0-q_{gP_g}}{X_{g,P_g+1}-X_{gP_g}}\right)$. P_g is the numbers of divided points of the *g*th objective function (piecewise linear), and S_{g,P_g+1} is the *y*-intercept of the section of the line segment initiated at X_{gP_g} and terminated at X_{g,P_g+1}.

Step 3: Introduce the nonnegative deviational variables

$$z_g + d_{ge}^- - d_{ge}^+ = X_{ge} \quad g = 1,2,\ldots,K, \; e = 1,2,\ldots,P_g \tag{A3}$$

where d_{ge}^+ and d_{ge}^- denote the deviational variables at the *e*th point and X_{ge} represents the values of the *g*th objective function at the *e*th point.

Step 4: Substituting expression (A3) into expression (A2), yields

$$f_g(z_g) = -\left(\frac{t_{g2}-t_{g1}}{2}\right)(d_{g1}^- - d_{g1}^+) - \left(\frac{t_{g3}-t_{g2}}{2}\right)(d_{g2}^- - d_{g2}^+) - \cdots - \left(\frac{t_{g,P_g+1}-t_{gP_g}}{2}\right)(d_{gP_g}^- - d_{gP_g}^+) + \left(\frac{t_{g,P_g+1}+t_{g1}}{2}\right)z_g + \frac{S_{g,P_g+1}+S_{g1}}{2} \quad g = 1,2,\ldots,K \tag{A4}$$

Step 5: By introducing the auxiliary variable $L^{(1)}$, and then the original fuzzy MOLP problem can be converted into the equivalent ordinary LP form using the minimum operator to aggregate fuzzy sets.

Appendix B

The fuzzy decision-making concept of Bellman and Zadeh (1970) is described as follows. Let X be a given set of all possible solutions to a decision problem. A fuzzy goal G is a fuzzy set on X characterized by its membership function.

$$\mu_G : X \to [0,1] \tag{B1}$$

A fuzzy constraint C is a fuzzy set on X characterized by its membership function

$$\mu_C : X \to [0,1] \tag{B2}$$

Then, G and C combine to generate a fuzzy decision D on X, which is a fuzzy set resulting from intersection of G and C, and is characterized by its membership function.

$$L = \mu_D(x) = \mu_G(x) \wedge \mu_C(x) = Min(\mu_G(x), \mu_C(x)) \tag{B3}$$

and the corresponding maximizing decision is defined by

$$Max\ L = Max\ \mu_D(x) = Max\ Min(\mu_G(x), \mu_C(x)) \tag{B4}$$

More generally, suppose the fuzzy decision D results from k fuzzy goals $G_1, \ldots, G_k$ and m constraints $C_1, \ldots, C_m$. Then the fuzzy decision D is the intersection of $G_1, \ldots, G_k$ and $C_1, \ldots, C_m$, and is characterized by its membership function.

$$L = \mu_D(x) = \mu_{G_1}(x) \wedge \mu_{G_2}(x) \wedge \ldots \wedge \mu_{G_k}(x) \wedge \mu_{C_1}(x) \wedge \mu_{C_2}(x) \wedge \ldots \wedge \mu_{C_m}(x) = Min\ (\mu_{G_1}(x), \mu_{G_2}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \mu_{C_2}(x), \ldots, \mu_{C_m}(x)) \tag{B5}$$

and the corresponding maximizing decision is defined by

$$Max\ L = Max\ \mu_D(x) = Max\ Min(\mu_{G_1}(x), \mu_{G_2}(x), \ldots, \mu_{G_k}(x), \mu_{C_1}(x), \ldots, \mu_{C_m}(x)). \tag{B6}$$

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