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A soft computing method of performance evaluation with MCDM based on interval-valued fuzzy numbers

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ABSTRACT

This study presented a new performance evaluation method for tackling fuzzy multicriteria decision-making (MCDM) problems based on combining VIKOR and interval-valued fuzzy sets. The performance evaluation problem often exists in complex administrative processes in which multiple evaluation criteria, subjective/objective assessments and fuzzy conditions have to be taken into consideration simultaneously in management. Here, the subjective, imprecise, inexact and uncertain evaluation processes are modeled as fuzzy numbers by means of linguistic terms, as fuzzy theory can provide an appropriate tool to deal with such uncertainties. However, the presentation of linguistic expressions in the form of ordinary fuzzy sets is not clear enough [15,21]. Interval-valued fuzzy sets can provide more flexibility [4,14] to represent the imprecise/vague information that results, and it can also provide a more accurate modeling. This paper presents the interval-valued fuzzy VIKOR, which aims to solve MCDM problems in which the weights and performances of criteria are unequal by using the concepts of interval-valued fuzzy sets. A case study for evaluating the performances of three major intercity bus companies from an intercity public transport system is conducted to illustrate the effectiveness of the method.

1. Introduction

The performance evaluation process of decision-making problems often involves a complex process in which multiple requirements and uncertain conditions have to be taken into consideration simultaneously [45]. In evaluating the suitability of alternatives, quantitative/qualitative assessments are often required to deal with uncertainty, subjectiveness and imprecise data, which are best represented with fuzzy numbers. Therefore, many precision-based multicriteria decision-making (MCDM) methods for evaluating performance and selecting alternatives have been developed, with most of the latter comparing all alternatives based on synthesized rankings. These studies concern the uncertainty/imprecise numeric values of decision/performance data and the subjective nature of human behavior. As a result, Chen [9] extended the TOPSIS method to solve MCDM problems with fuzzy/uncertain conditions: this method can deal with clear-cut and uncertain data simultaneously. Ding and Liang [13] used fuzzy MCDM to select partners for strategic alliances: this fuzzy decision method is based on the concepts of TOPSIS and entropy weighting. Liang [25] incorporated the fuzzy set theory based on the concepts of positive ideal and negative ideal solutions to expand MCDM in a fuzzy environment. Yeh et al. [45] proposed a new fuzzy MCDM method based on the concepts of positive ideal and negative ideal points to evaluate bus companies' performance. Kuo et al. [24] extended grey relational analysis to select locations for distribution centers. This method can simultaneously obtain the gap between the ideal alternative and each of the other alternatives, the preference relationship between two alternatives, and the ranking order of alternatives. Kuo et al. [23] used the concepts of positive ideal and negative ideal solutions to develop a novel fuzzy group MCDM method, and the results showed that this method can be implemented as an effective decision aid in MCDM problems. Yeh and Chang [43] presented a fuzzy group MCDM based on TOPSIS concepts with fuzzy sets to evaluate alternatives. Although the concepts of positive ideal and negative ideal points in the abovementioned studies are applicable to any decision-making problem of MCDM [19], and these concepts make it easy to find the suitable alternative, such MCDM methods only focus on selecting a best alternative from all alternatives based on the synthesized performance assessment value. In practice, decision-makers often evaluate their progress in attaining ideal aims and know the gaps

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between existing alternatives and the ideal alternative according to each criterion, and hope to achieve improvements in a way that will increase their competitive advantage.

Accordingly, Opricovic and Tzeng [28,30] proposed the new MCDM method of VIKOR based on the particular measure of "closeness" to the positive ideal solution; this method is suitable for certain situations in which the decision maker wants to obtain maximum profit and the risk associated with the decision is less important [31]. In addition, the VIKOR method can obtain the gap between the ideal alternative and each alternative, the rank order of alternatives, and the priority of improving of the weaknesses of each alternative. Therefore, some studies extend the VIKOR method to solve the problems of uncertain conditions, as this method can deal with clear-cut and uncertain data simultaneously. As a result, Opricovic and Tzeng [29] suggested using fuzzy logic for the VIKOR method by applying some defuzzification techniques, simply using fuzzy values to define the attributes' ratings and their importance in the first phase of their study. Büyüközkan and Ruan [2] also extended VIKOR to solve the problems of uncertain conditions by using fuzzy logic in the first phases, with all subsequent phases using fuzzy logic in order to avoid the loss of important information. Sanayei et al. [32] have also suggested using fuzzy VIKOR to select suitable suppliers of supply chain systems. The results showed that the model can deal with both qualitative and quantitative criteria, and can provide an outranking order of suppliers while rating the suppliers. Chen and Wang [6] proposed fuzzy VIKOR to select the optimal partner in IS/IT outsourcing projects. The results showed that this method can resolve the uncertainty and vagueness inherent in the group decision-making process, and can also obtain compromise solutions. Sayadi et al. [31] extended the VIKOR method to decision-making problems with interval numbers. The ranking is obtained through a comparison of interval numbers and comparisons between intervals can be achieved by using this method. These studies, under the uncertain condition of assessable performance with a weight for each criterion, proposed methods of fuzzy MCDM based on the concept of VIKOR.

Although the concepts in the abovementioned studies are applicable to any decision-making problem, typical fuzzy multiple criteria analysis requires the comparison of fuzzy numbers or the use of defuzzifying techniques to deal with fuzzy numbers. The results may be or may not be reliable, given that: (1) the decision makers often disagree on the method of defining linguistic variables based on the fuzzy sets theory, (2) the method may generate counter-intuitive ranking outcomes for similar fuzzy utilities [3,5,50]. Furthermore, linguistic expression in the form of ordinary fuzzy sets is not clear enough [15,21]. Bigand and Colot [4] and Gorzalczany [14] presented interval-valued fuzzy sets to represent the imprecise/vague information that results, as interval-valued fuzzy sets can provide more flexibility and can ensure that the presentation of a linguistic expression is sufficiently clear [1,37]. Therefore, Vahdani et al. [37] extended the VIKOR method to decision-making problems with interval-valued fuzzy numbers. Vahdani et al. [37] computed the distance between two interval-valued fuzzy numbers by using the Hamming distance. However, using the Hamming distance in the VIKOR method will result in certain issues, given that: (1) it does not calculate whether any data have negative values in any bounds of interval-valued fuzzy numbers by using arithmetic operations between two interval-valued fuzzy numbers (as shown in Definition 1), (2) by frequently calculating process of the interval-valued fuzzy numbers, it will result in the final interval-valued fuzzy numbers' form being changed. Then, the final ranking order results of all alternatives will also be changed [11,22,50].

In order to avoid the abovementioned issues, this study presents an extended form of the VIKOR method based on the interval-valued fuzzy numbers and Euclidean distance to analyze any decision-making problem or performance evaluation. Euclidean distance can easily calculate the distance between two interval-valued fuzzy numbers [1,10,12], and can also calculate whether any data have negative values in any bounds of interval-valued fuzzy numbers, and the Euclidean distance is close to the real distance [10]. In addition, this study uses the Euclidean distance to extend VIKOR, when one can reduce the frequently lengthy calculating process of the interval-valued fuzzy numbers. In this study, all performance rating values and weights of criteria can be expressed linguistics terms. The linguistic terms are characterized by using interval-valued fuzzy numbers. Through interval-valued fuzzy numbers, the proposed method can more efficiently address the ambiguity existing in the available information, as well as the essential fuzziness in human judgment and preference, as interval-valued fuzzy sets can provide more flexibility to represent the imprecise/vague information resulting from a lack of data [1,10]. Then, this proposed method is solved using an effective algorithm, which allows us not only to obtain the ranking order of all alternatives, but also to improve levels of performance quality to further the performance quality base by prioritizing the improvement of items of weakness. Finally, this paper will use a case study involving the performance evaluation of three major intercity bus companies from an intercity public transport system to illustrate the proposed method. Through this case study, this paper will demonstrate that the proposed fuzzy MCDM method for performance evaluation process is a good method, and that it appears to be more appropriate than other methods.

The remainder of this paper is organized as follows. The basic concepts, definitions and notations of VIKOR and interval-valued fuzzy numbers are introduced in Section 2. A new method of fuzzy MCDM based on the combined concepts of VIKOR and interval-valued fuzzy sets is proposed in Section 3. In Section 4, a case study applies the proposed interval-valued fuzzy VIKOR to evaluate the performance of three intercity bus companies, after which this study discusses and demonstrated how the new interval-valued fuzzy VIKOR method is effective in Section 5. Finally, conclusions are presented in Section 6.

2. Definitions and concepts of VIKOR and fuzzy numbers

In this section, the basic definition of VIKOR [26,29–32,36,38] and interval-valued fuzzy sets are briefly introduced [14,16,17,22,33–35,39,41]. Based on these basic concepts, a new fuzzy MCDM will be proposed.

2.1. VIKOR technique

The basic concepts of the VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje – a Serbian name) technique are based on the adoption of $L_{p\text{-}metric}$ concepts [46,49] to solve a decision-making problem with non-commensurable (different units) and conflicting criteria [26–28]. This method focuses on ranking and selecting from a set of alternatives and determines a compromise solution for a decision with conflicting criteria, and can help the decision makers to obtain a final solution and reach a decisive decision. This method assumes that the various alternatives will be denoted as $a_1, a_2, \ldots, a_m$. For an alternative a_i, the merit of the jth aspect is denoted by f_{ij}, i.e. f_{ij} is the value of jth criterion function for the alternative a_i. The compromise-ranking algorithm is briefly reviewed as follows:

(1) Determination of the best f_j^* and the worst f_j^- values of all criterion functions. Assuming that jth criterion function represents

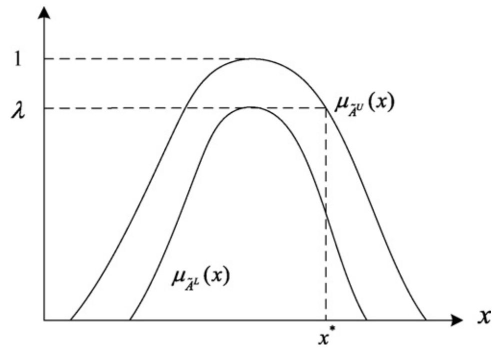

Fig. 1. Interval-valued fuzzy set $\tilde{A}$.

a benefit:

$$f_j^- = \min_i f_{ij}, \quad i = 1, 2, 3, \ldots, m, \quad f_j^* = \max_i f_{ij}, \quad i = 1, 2, 3, \ldots, m$$

(2) Compute the values S_i and R_i, $i = 1, 2, 3, \ldots, m$, by the relations

$$S_i = \left\{ \sum_{j=1}^{n} \left[\frac{w_j(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right]^p \right\}^{1/p}, \quad 1 \le p \le \infty; \quad i = 1, 2, \ldots, m.$$

$$R_i = \max_j \left[\frac{w_j(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right]^p, \quad 1 \le p \le \infty; \quad i = 1, 2, \ldots, m.$$

where w_j is the weight of jth criterion, expressing the DM's preference as the relative importance of the criteria.

(3) Compute the values Q_i for $i = 1, 2, 3 \ldots, m$, which are defined as

$$Q_i = v \left[\frac{S_i - S^*}{S^- - S^*} \right] + (1 - v) \left[\frac{R_i - R^*}{R^- - R^*} \right],$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, and v is a weighting reference. The variable v is introduced as the weight of the strategy of maximum group utility, whereas $(1 - v)$ is the weight of the individual risk. Thus, when the v reference is larger (>0.5), the index of Q_i will tend towards majority rule.

2.2. Interval-valued fuzzy numbers

In this subsection, this paper considers the fuzzy demand by using interval-valued fuzzy sets. Based on the definition of interval-valued fuzzy sets in Gorzalezany [14], an interval-valued fuzzy set (i–v fuzzy set for short) $\tilde{A}$ defined on $(-\infty, \infty)$ is given by

$$\tilde{A} = \{x, [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]\}, x \in (-\infty, \infty), \mu_{\tilde{A}^L}, \mu_{\tilde{A}^U} : (-\infty, \infty) \to [0, 1],$$

$$\mu_{\tilde{A}^L}(x) \le \mu_{\tilde{A}^U}(x), \quad \forall x \in (-\infty, \infty),$$

$$\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)], \quad x \in (-\infty, \infty),$$

where $\mu_{\tilde{A}^L}(x)$ is the lower limit of degree of membership and $\mu_{\tilde{A}^U}(x)$ is the upper limit of degree of membership.

In Fig. 1, the grade of membership at x^* of an interval-valued fuzzy set $\tilde{A}$ belonging to the interval $[\mu_{\tilde{A}^L}(x^*), \mu_{\tilde{A}^U}(x^*)]$, $\mu_{\tilde{A}^L}(x^*)$ is the minimum grade of membership and $\mu_{\tilde{A}^u}(x^*)$ is the maximum grade of membership.

Definition 1. Gorzalczany [14] proposed the concept of interval-valued fuzzy sets. According to Yao and Lin's [42] definition of triangular interval-valued fuzzy numbers, when one can be represented as $\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U] = [(a_1^L, a_2^L, a_3^L; \hat{w}_{\tilde{A}}^L), (a_1^U, a_2^U, a_3^U; \hat{w}_{\tilde{A}}^U)]$ (shown in Fig. 2), where $\tilde{A}^L$ and $\tilde{A}^U$ denote the lower and upper triangular interval-valued fuzzy numbers, $\tilde{A}^L \subset \tilde{A}^U$; $\mu_{\tilde{A}}(x)$ is the membership function, which denotes the degree to which an event x may be a member of $\tilde{A}$; $\mu_{\tilde{A}^L}(x) = \hat{w}_{\tilde{A}}^L$ and $\mu_{\tilde{A}^U}(x) = \hat{w}_{\tilde{A}}^U$ are the lower and upper membership functions, respectively. According to Fig. 2, the relations can be obtained as follows:

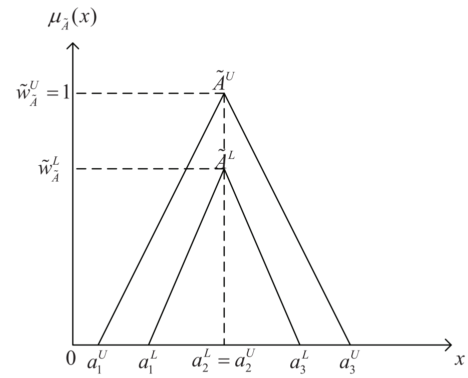

Fig. 2. A triangular interval-valued fuzzy number.

(1) If $\tilde{A}^L = \tilde{A}^U$, then the triangular interval-valued fuzzy number $\tilde{A}$ is a generalized triangular fuzzy number.
(2) If $a_1^L = a_1^U = a_2^L = a_2^U = a_3^L = a_3^U$ and $\hat{w}_{\tilde{A}}^L = \hat{w}_{\tilde{A}}^U$, then the triangular interval-valued fuzzy number $\tilde{A}$ is a crisp value.
(3) If $\hat{w}_{\tilde{A}}^L = \hat{w}_{\tilde{A}}^U = 1$ and $a_2^L = a_2^U$, then this study can denote the triangular interval-valued fuzzy number $\tilde{A}$ as $\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U] = [(a_1^U, a_1^L), (a_2^L = a_2^U), (a_3^L, a_3^U)]$.

According to Definition 1 (3), two triangular interval-valued fuzzy numbers can be represented as $\tilde{A} = [(a_1^U, a_1^L), a_2, (a_3^L, a_3^U)]$ and $\tilde{B} = [(b_1^U, b_1^L), b_2, (b_3^L, b_3^U)]$, respectively. Then, the arithmetic operations between $\tilde{A}$ and $\tilde{B}$ are proposed by [7,10,17,37] as follows:

(1) Addition of interval-valued fuzzy numbers $\oplus$:

$$\tilde{A} \oplus \tilde{B} = [(a_1^U, a_1^L), a_2, (a_3^L, a_3^U)] \oplus [(b_1^U, b_1^L), b_2, (b_3^L, b_3^U)] = [(a_1^U + b_1^U, a_1^L + b_1^L), a_2 + b_2, (a_3^L + b_3^L, a_3^U + b_3^U)].$$

(2) Subtraction of interval-valued fuzzy numbers Θ:

$$\tilde{A} \Theta \tilde{B} = [(a_1^U, a_1^L), a_2, (a_3^L, a_3^U)] \Theta [(b_1^U, b_1^L), b_2, (b_3^L, b_3^U)] = [(a_1^U - b_3^U, a_1^L - b_3^L), a_2 - b_2, (a_3^L - b_1^L, a_3^U - b_1^U)].$$

(3) Multiplication of interval-valued fuzzy numbers $\otimes$:

$$\tilde{A} \otimes \tilde{B} = [(a_1^U, a_1^L), a_2, (a_3^L, a_3^U)] \otimes [(b_1^U, b_1^L), b_2, (b_3^L, b_3^U)] = [(a_1^U \times b_1^U, a_1^L \times b_1^L), a_2 \times b_2, (a_3^L \times b_3^L, a_3^U \times b_3^U)]$$

for $a_i > 0$, $a_i^{\lambda} > 0$, $b_i^{\lambda} > 0$, where $i = 1, 2$; $\lambda = L, U$.

(4) Generalized division of fuzzy numbers $\varnothing$:

$$\tilde{A} \varnothing \tilde{B} = [(a_1^U, a_1^L), a_2, (a_3^L, a_3^U)] \varnothing [(b_1^U, b_1^L), b_2, (b_3^L, b_3^U)] = [(a_1^U \div b_3^U, a_1^L \div b_3^L), a_2 \div b_2, (a_3^L \div b_1^L, a_3^U \div b_1^U)]$$

for $a_i > 0$, $a_i^{\lambda} > 0$, $b_i^{\lambda} > 0$, where $i = 1, 2$; $\lambda = L, U$.

Table 1
Linguistic terms for the ratings (example).

Very poor (VP)	[(0, 0), 0, (1, 1.5)]
Poor (P)	[(0, 0.5), 1, (2.5, 3.5)]
Moderately poor (MP)	[(0, 1.5), 3, (4.5, 5.5)]
Fair (F)	[(2.5, 3.5), 5, (6.5, 7.5)]
Moderately good (MG)	[(4.5, 5.5), 7, (8, 9.5))]
Good (G)	[(5.5, 7.5), 9, (9.5, 10)]
Very good (VG)	[(8.5, 9.5), 10, (10, 10)]

Table 2
Linguistic terms for the importance weight of each criterion (example).

Very low (VL)	[(0, 0), 0, (0.1, 0.15)]
Low (L)	[(0, 0.05), 0.1, (0.25, 0.35)]
Medium low (ML)	[(0, 0.15), 0.3, (0.45, 0.55)]
Medium (M)	[(0.25, 0.35), 0.5, (0.65, 0.75)]
Medium high (MH)	[(0.45, 0.55), 0.7, (0.8, 0.95))
High (H)	[(0.55, 0.75), 0.9, (0.95, 1)]
Very high (VH)	[(0.85, 0.95), 1, (1, 1)]

Definition 2. Let $\tilde{A} = [\tilde{A}_x^L, \tilde{A}_x^U]$ and $\tilde{B} = [\tilde{B}_y^L, \tilde{B}_y^U]$ be two triangular interval-valued fuzzy numbers, then the Normalized Euclidean distance between $\tilde{A}$ and $\tilde{B}$ can be defined as follows [1,9,10].

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{6}\sum_{x,y=1}^{3}[(\tilde{A}_x^L - \tilde{B}_y^L)^2 + (\tilde{A}_x^U - \tilde{B}_y^U)^2]},$$

$$d(\tilde{A}, \tilde{B})^L = \sqrt{\frac{1}{3}\sum_{x,y=1}^{3}(\tilde{A}_x^L - \tilde{B}_y^L)^2}, \quad d(\tilde{A}, \tilde{B})^U = \sqrt{\frac{1}{3}\sum_{x,y=1}^{3}(\tilde{A}_x^U - \tilde{B}_y^U)^2}.$$

Definition 3. Let $\tilde{O} = [(0, 0), 0, (0, 0)]$ be original. If $d(\tilde{A}, \tilde{O}) < d(\tilde{B}, \tilde{O})$, then interval-valued fuzzy number $\tilde{A}$ is closer to the original than the other interval-valued fuzzy number $\tilde{B}$ [1].

Definition 4. A linguistic variable is a variable whose values are linguistic terms [20,47,48]. Linguistic terms – such as not important, somewhat important, important, very important, extremely important and very poor, poor, fair, good, very good – have been found to be intuitively easy to use in expressing the subjectiveness and/or qualitative imprecision of a decision maker's assessments [8,50]. Furthermore, Grattan-Guinness [15] and Karnik and Mendel [21] argued that the presentation of a linguistic expression in the form of ordinary fuzzy sets is not clear enough [4,14]; Gorzalczany [14] demonstrated that interval-valued fuzzy sets can provide more flexibility. These fuzzy sets can effective represent the imprecise/vague information that results, and then one is better able than decision-makers to ensure that the presentation of a linguistic expression is sufficiently clear [1,37,40,44].

3. A new fuzzy MCDM technique for interval-valued fuzzy VIKOR

In this section, this paper will present a new fuzzy MCDM technique based on the concepts of VIKOR, interval-valued fuzzy numbers and Euclidean distance. This method can handle complex decision processes, which often teem with vagueness: imprecise, indefinite, subjective and vague data and/or information. The method employs the major technique of interval-valued fuzzy sets to deal with vague information and/or data, as interval-valued fuzzy sets might provide the flexibility to represent the imprecise/vague information resulting from a lack of data [1].

Here, the importance weights of various criteria and the ratings (the imprecise assessment values) of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in triangular interval-valued fuzzy numbers as depicted in Tables 1 and 2 and illustrated in Fig. 3. The importance weight of each criterion can be obtained by either directly assigning or indirectly using pairwise comparisons [9,18]. Here, it is suggested that the decision makers use the linguistic terms (as shown in Tables 1 and 2) to evaluate the importance of the criteria and the ratings of alternatives with respect to each criterion. Through the questionnaires eliciting "linguistic terms for the level of importance" and "linguistic terms for the ratings", these linguistic terms can be questioned in real application (as shown in Appendices C and D). In general, the questionnaires can adopt a Likert-type five-point scale or seven-point scale. Here, this paper uses a seven-point Likert-type scale to analyze the supposed case study of the performance evaluation of three major intercity bus companies.

This paper assumes that an evaluation problem contains m possible alternatives and n criteria with which alternative performances are measured. If a decision group has K judges, then the importance weight of criteria can be calculated using Eq. (1), and the rating (performance value) of alternatives with respect to each criterion can be calculated using Eq. (2).

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1(+)\tilde{w}_j^2(+)\cdots(+)\tilde{w}_j^K] = \frac{1}{K}\sum_{t=1}^{K}\tilde{w}_j^t, \tag{1}$$

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1(+)\tilde{x}_{ij}^2(+)\cdots(+)\tilde{x}_{ij}^K] = \frac{1}{K}\sum_{t=1}^{K}\tilde{x}_{ij}^t, \tag{2}$$

where $\tilde{w}_j^t$ and $\tilde{x}_{ij}^t$ are the importance weight and rating by the tth judge, and then $\tilde{w}_{ij}$ and $\tilde{x}_{ij}$ are described by triangular interval-valued fuzzy numbers, $\tilde{w}_j = [(w_{j1}, w'_{j1}), w_{j2}, (w'_{j3}, w_{j3})] = (\tilde{w}_j^L, \tilde{w}_j^U)$ and $\tilde{x}_{ij} = [(l_{ij}, l'_{ij}), m_{ij}, (r'_{ij}, r_{ij})]$.

As stated above, we can express this concisely in matrix format. A MCDM problem for group decision-making is then considered in a fuzzy environment as follows:

$$\tilde{\boldsymbol{D}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} = [\tilde{x}_{ij}]_{m\times n},$$

$$\tilde{\boldsymbol{w}} = [\,\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_n\,], \tag{3}$$

where $\tilde{x}_{ij}$, $\forall i, j$ is the fuzzy rating (interval-valued fuzzy performance value) of possible alternative A_i, $i = 1, 2, \ldots, m$, with respect to criterion C_j, and $\tilde{w}_j$ is the fuzzy weight of criterion C_j, $j = 1, 2, \ldots, n$.

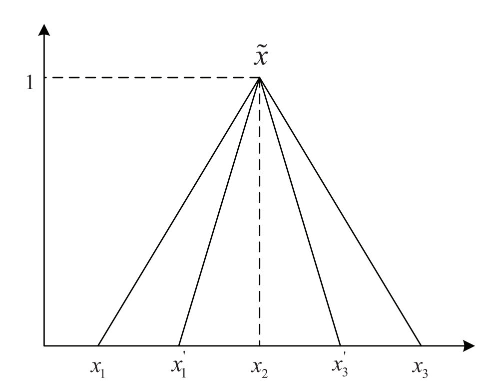

Fig. 3. Triangular interval-valued fuzzy number.

Considering the different importance of each criterion, we can construct the weighted performance interval-valued fuzzy decision matrix as

$$\tilde{\mathbf{V}} = [\tilde{\upsilon}_{ij}]_{m\times n}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \tag{4}$$

where $\tilde{\upsilon}_{ij} = \tilde{x}_{ij}(\cdot)\tilde{w}_j = ((a_{ij}, a'_{ij}), b_{ij}, (c'_{ij}, c_{ij}))$.

After completing the interval-valued fuzzy performance decision matrix and the weighted performance interval-valued fuzzy decision matrix, we can determine the positive ideal solution (A^{*1} and A^{*2}) and the negative ideal solution (A^{-1}), as follows:

$$A^{*1} = \{\tilde{x}_1^*, \tilde{x}_2^*, \ldots, \tilde{x}_n^*\} = \{(\max_i \tilde{x}_{ij} | j \in B) \text{ or } (\min_i \tilde{x}_{ij} | j \in C)\}$$

$$j = 1, 2, \ldots, n, = \{((l_1^*, l_1'^*), m_1^*, (r_1'^*, r_1^*), (l_2^*, l_2'^*), m_2^*,$$

$$(r_2'^*, r_2^*), \ldots, (l_n^*, l_n'^*), m_n^*, (r_n'^*, r_n^*))\},$$

$$A^{*2} = \{\tilde{\upsilon}_1^*, \tilde{\upsilon}_2^*, \ldots, \tilde{\upsilon}_n^*\} = \{(\max_i \tilde{\upsilon}_{ij} | j \in B) \text{ or } (\min_i \tilde{\upsilon}_{ij} | j \in C)\}$$

$$j = 1, 2, \ldots, n, = \{((a_1^*, a_1'^*), b_1^*, (c_1'^*, c_1^*), (a_2^*, a_2'^*), b_2^*, (c_2'^*,$$

$$c_2^*), \ldots, (a_n^*, a_n'^*), b_n^*, (c_n'^*, c_n^*))\},$$

$$A^{-1} = \{\tilde{x}_1^-, \tilde{x}_2^-, \ldots, \tilde{x}_n^-\} = \{(\min_i \tilde{x}_{ij} | j \in B) \text{ or } (\max_i \tilde{x}_{ij} | j \in C)\}$$

$$j = 1, 2, \ldots, n, = \{((l_1^-, l'^-_1), m_1^-, (r'^-_1, r_1^-), (l_2^-, l'^-_2), m_2^-,$$

$$(r'^-_2, r_2^-), \ldots, (l_n^-, l'^-_n), m_n^-, (r'^-_n, r_n^-))\}, \tag{5}$$

where B is associated with benefit criteria and C is associated with cost criteria.

Next, we can compute the values of $\tilde{S}_i$ and $\tilde{R}_i$, respectively, as below:

$$\tilde{S}_{ij}^U = \sum_{j\in B} \frac{\sqrt{\frac{1}{3}[(a_j^* - a_j)^2 + (b_j^* - b_j)^2 + (c_j^* - c_j)^2]}}{\sqrt{\frac{1}{3}[(l_j^* - l_j^-)^2 + (m_j^* - m_j^-)^2 + (r_j^* - r_j^-)^2]}} + \sum_{j\in C} \frac{\sqrt{\frac{1}{3}[(a_j - a_j^*)^2 + (b_j - b_j^*)^2 + (c_j - c_j^*)^2]}}{\sqrt{\frac{1}{3}[(l_j^- - l_j^*)^2 + (m_j^- - m_j^*)^2 + (r_j^- - r_j^*)^2]}},$$
$$i = 1, 2, \ldots, m,$$

$$\tilde{S}_{ij}^L = \sum_{j\in B} \frac{\sqrt{\frac{1}{3}[(a'^*_j - a'_j)^2 + (b_j^* - b_j)^2 + (c'^*_j - c'_j)^2]}}{\sqrt{\frac{1}{3}[(l'^*_j - l'^-_j)^2 + (m'^*_j - m'^-_j)^2 + (r'^*_j - r'^-_j)^2]}} + \sum_{j\in C} \frac{\sqrt{\frac{1}{3}[(a'_j - a'^*_j)^2 + (b_j - b_j^*)^2 + (c'_j - c'^*_j)^2]}}{\sqrt{\frac{1}{3}[(l'^-_j - l'^*_j)^2 + (m'^-_j - m'^*_j)^2 + (r'^-_j - r'^*_j)^2]}},$$
$$i = 1, 2, \ldots, m,$$

$$\tilde{S}_i = \sum_{j=1}^{n} \left(\frac{\tilde{S}_{ij}^U + \tilde{S}_{ij}^L}{2}\right), \quad i = 1, 2, \ldots, m, \quad \tilde{R}_i = \max_j \left(\frac{\tilde{S}_{ij}^U + \tilde{S}_{ij}^L}{2}\right),$$
$$i = 1, 2, \ldots, m, \tag{6}$$

According to the values of $\tilde{S}_i$ and $\tilde{R}_i$, we can compute $\tilde{Q}_i$ using these relationships:

$$\tilde{Q}_i = \nu \frac{(\tilde{S}_i - \tilde{S}^*)}{(\tilde{S}^- - \tilde{S}^*)} + (1-\nu)\frac{(\tilde{R}_i - \tilde{R}^*)}{(\tilde{R}^- - \tilde{R}^*)}, \tag{7}$$

where $\tilde{S}^* = \min_i \tilde{S}_i$, $\tilde{S}^- = \max_i \tilde{S}_i$, $\tilde{R}^* = \min_i \tilde{R}_i$, $\tilde{R}^- = \max_i \tilde{R}_i$, $\nu \in [0, 1]$ is introduced as the weight of the strategy of the "majority of criteria" (or "maximum group utility"), usually $\nu = 0.5$.

Next, a compromise solution can be determined according to the VIKOR process. Assuming that the two conditions given below are acceptable, the index $\tilde{Q}_i$ can be used to determine a compromise solution (a') as a single optimal solution.

[C1]. Acceptable advantage:

$$\tilde{Q}_{(a'')} - \tilde{Q}_{(a')} \geq DQ,$$

$$DQ = \frac{1}{m-1}, \tag{8}$$

where m is the number of alternatives.

[C2]. Acceptable stability in decision-making: alternative a' must also be the best ranked by $\tilde{S}_i$ and/or $\tilde{R}_i$.

If **[C1]** is not accepted and $\tilde{Q}_{(a^m)} - \tilde{Q}_{(a')} < DQ$, then ($a^m$) and ($a'$) are the same compromise solution. However, alternative a' does not have a comparative advantage, so the compromise solutions a', a'', ..., a^m are the same. If **[C2]** is not accepted, the stability in decision-making is deficient, although a' has a comparative advantage. Hence, the compromise solutions of a' and a'' are the same. According to the VIKOR method, the alternative that has the lowest $\tilde{Q}_i$ is the best alternative and it is selected as the compromise solution.

4. A case study for evaluating the performance of an intercity bus company

In this section, to illustrate how the present fuzzy MCDM method works, we present a case study that involves evaluating the performance of three major intercity bus companies (A_1, A_2 and A_3) from an intercity public transport system to demonstrate that the proposed fuzzy MCDM method is appropriate. The performance evaluation can ensure that effective and efficient services are maintained by every intercity bus company, allowing the traffic authorities at each country government to conduct regular assessment of the intercity bus companies' performance against specific criteria. In general, the criteria include safety (C_1), comfort (C_2),

Table 3
Criteria used for the performance evaluation of intercity bus companies.

Evaluation criteria	Descriptions
Safety (C_1)	This criterion refers to common indicators that can be used to measure the safety level of the vehicle, driving and traveling
Comfort (C_2)	Comfort is measured by cleanliness, lighting and congestion level of waiting areas/lounges, atmosphere/comfort, temperature/air conditioning, seat comfort, on-board information, driver's appearance and overall friendliness
Convenience (C_3)	The convenience criterion is mainly concerned with the punctuality of the bus service, route transferability, terminal space, and service reliability
Operation (C_4)	The operation efficiency criterion includes cost efficiency, cost effectiveness and service efficiency
Social duty (C_5)	The social duty criterion includes the air pollution level of the vehicle and the vehicle's noise level

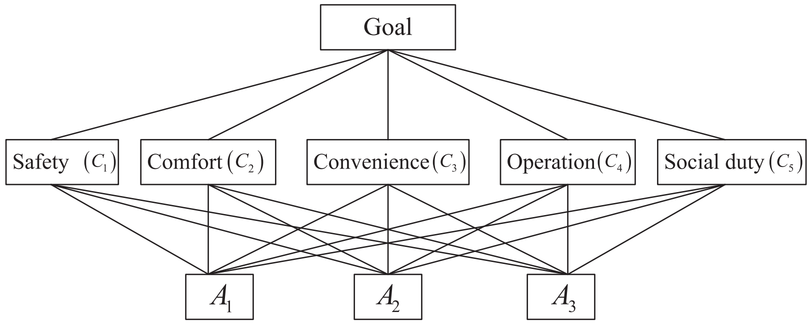

Fig. 4. The hierarchical structure.

Table 4
The importance weights of the criteria.

	D_1	D_2	D_3
C_1	MH	H	MH
C_2	M	ML	MH
C_3	H	H	VH
C_4	H	H	MH
C_5	VH	VH	VH

convenience (C_3), operation (C_4) and social duty (C_5) [45], and these criteria are selected in this study for assessing the performance of three intercity bus companies. The definitions of the relevant criteria are listed in Table 3. Here, we suppose that a committee of three judges, D_1, D_2 and D_3, has been formed to conduct the review and to evaluate the performance of the three bus companies.

The hierarchical structure of this performance evaluation problem is shown in Fig. 4. The proposed method is applied to solve this problem and the computational procedure is summarized as follows:

Step 1: The judges' subjective judgments use the linguistic terms for the importance weights (as shown in Table 1) to assess the importance of each criterion, as presented in Table 4.

Step 2: Judges use the linguistic terms shown in Table 2 to evaluate the rating of alternatives versus each criterion. The results are shown in Table 5.

Step 3: The linguistic evaluation (as shown in Tables 4 and 5) is converted into triangular interval-valued fuzzy numbers to construct the fuzzy decision matrix and determine the fuzzy weight of each criterion by using Eqs. (1)–(3) as Table 6.

Table 5
The ratings of the three intercity bus companies by judges under all criteria.

Criteria	Candidates	Judges		
		D_1	D_2	D_3
C_1	A_1	MG	G	G
	A_2	F	MG	VG
	A_3	G	VG	F
C_2	A_1	P	F	F
	A_2	F	MP	F
	A_3	F	P	F
C_3	A_1	MG	MG	F
	A_2	F	MG	G
	A_3	G	G	G
C_4	A_1	G	MG	MG
	A_2	G	G	MG
	A_3	F	G	MG
C_5	A_1	VG	G	G
	A_2	G	G	MG
	A_3	G	G	G

Step 4: Construct the weighted performance interval-valued fuzzy decision matrix by using Eq. (4) as shown in Table 7.

Step 5: Determine the positive ideal solution (A^{*1} and A^{*2}) and negative ideal solution (A^{-1}) by using Eq. (5).

$$A^{*1} = \{[(5.17, 6.83), 8.33, (9, 9.83)], [(1.67, 2.83), 4.33, (5.83, 6.83)], [(5.5, 7.5), 9, (9.5, 10)], [(5.17, 6.83), 8.33, (9, 9.83)], [(6.50, 8.17), 9.33, (9, 68.10)]\}$$

$$A^{*2} = \{[(2.50, 4.21), 6.39, (7.65, 9.51)], [(0.39, 0.99), 2.17, (3.69, 5.13)], [(3.58, 6.13), 8.4, (9.18, 10)], [(2.67, 4.67), 6.94, (8.1, 9.67)], [(5.53, 7.76), 9.33, (9.67, 10)]\}$$

$$A^{-1} = \{[(5.17, 6.17), 7.33, (8.17, 9)], [(1.67, 2.5), 3.67, (5.17, 6.17)], [(3.83, 4.83), 6.33, (7.5, 8.83)], [(4.17, 5.5), 7, (8, 9)], [(5.17, 6.83), 8.33, (9, 9.83)]\}$$

Step 6: Compute the values of $\tilde{S}_i$ and $\tilde{R}_i$ of each candidate by using Eq. (6). After the values of $\tilde{S}_i$ and $\tilde{R}_i$ have been computed, we can obtain the $\tilde{Q}_i$ values by using Eq. (7), as shown in Table 8. Here, the $\tilde{Q}_i$ value of each candidate is calculated, then this case will use each ν value as $\nu = 0$, $\nu = 0.5$ and $\nu = 1$ to calculate one.

Step 7: Rank and improve the candidates, sort by the values $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$, in decreasing order, and reduce the gaps in the criteria. The results are three ranking lists, with the best candidates having the lowest value. The decision maker can thus obtain a suitable alternative and items (criteria) that must be improved.

From the $\tilde{R}$ value as shown in Eq. (6), we can obtain the worst performance on a given criterion for each alternative. We can then find the criteria in which performance must be improved by each alternative company. Therefore, if the worst performance on some criterion is improved, it will make a great contribution to the integral performances of all criteria for that alternative. According to Table 8, it is possible to identify the poor performance on the key criterion/item for each alternative by using $\tilde{R}$ values as shown: the key criterion requiring improved performance is the convenience criterion (C_3) in alternative A_1; alternative A_2 must improve its performance on the social duty criterion (C_5); alternative A_3 must improve its performance on the operation criterion (C_4). This paper postulates that the ν value is $\nu = 1$ while the $\tilde{Q}$ values of each alternative A_1, A_2, and A_3 are 0, 1, and 0.7249, respectively. Therefore, the ranking order of the three alternatives is

Table 6
The interval-valued fuzzy decision matrix and fuzzy weights of the three candidates.

	C_1	C_2	C_3	C_4	C_5
Weight	[(0.48, 0.62), 0.77, (0.85, 0.97)]	[(0.23, 0.35), 0.50, (0.63, 0.75)]	[(0.65, 0.82), 0.93, (0.97, 1)]	[(0.52, 0.68), 0.83, (0.90, 0.98)]	[(0.85, 0.95), 1, (1, 1)]
A_1	[(5.17, 6.83), 8.33, (9, 9.83)]	[(1.67, 2.5), 3.67, (5.17, 6.17)]	[(3.83, 4.83), 6.33, (7.5, 8.83)]	[(4.83, 6.17), 7.67, (8.5, 9.67)]	[(6.5, 8.17), 9.33, (9.67, 10)]
A_2	[(5.17, 6.17), 7.33, (8.17, 9)]	[(1.67, 2.83), 4.33, (5.83, 6.83)]	[(4.17, 5.5), 7, (8, 9)]	[(5.17, 6.83), 8.33, (9, 9.83)]	[(5.17, 6.83), 8.33, (9, 9.83)]
A_3	[(5.5, 6.83), 8, (8.67, 9.17)]	[(1.67, 2.5), 3.67, (5.17, 6.17)]	[(5.5, 7.5), 9, (9.5, 10)]	[(4.17, 5.5), 7, (8, 9)]	[(5.50, 7.5), 9, (9.5, 10)]

Table 7
The weighted performance interval-valued fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	[(2.50, 4.21), 6.39, (7.65, 9.51)]	[(0.39, 0.88), 1.83, (3.27, 4.63)]	[(2.49, 3.95), 5.91, (7.25, 8.83)]	[(2.50, 4.21), 6.39, (7.65, 9.51)]	[(5.53, 7.76), 9.33, (9.67, 10)]
A_2	[(2.50, 3.80), 5.62, (6.94, 8.7)]	[(0.39, 0.99), 2.17, (3.69, 5.13)]	[(2.71, 4.49), 6.53, (7.73, 9)]	[(2.67, 4.67), 6.94, (8.1, 9.67)]	[(4.39, 6.49), 8.33, (9, 9.83)]
A_3	[(2.66, 4.21), 6.13, (7.37, 8.86)]	[(0.39, 0.88), 1.83, (3.27, 4.63)]	[(3.58, 6.13), 8.4, (9.18, 10)]	[(2.15, 3.76), 5.83, (7.2, 8.85)]	[(4.68, 7.13), 9, (9.5, 10)]

$A_1 \succ A_3 \succ A_2$. If $\nu = 0.5$, the $\tilde{Q}$ value of each alternative A_1, A_2, and A_3 would be 0.3267, 1, and 0.3624, respectively. The ranking order would thus be $A_1 \succ A_3 \succ A_2$. Finally, if $\nu = 0$, then the ranking order of the $\tilde{Q}$ values would be $A_3 \succ A_1 \succ A_2$; the $\tilde{Q}$ values would be 0, 0.6534, 1, respectively. Finally, if $\nu = 0$, then the $\tilde{Q}$ values would be $\tilde{Q} = (\tilde{R}_i - \tilde{R}_i^* / \tilde{R}_i^- - \tilde{R}_i^*)$. We then find that the $\tilde{R}$ values can affect the $\tilde{Q}$ values significantly. This information tells us how much attention must be paid to improving the performance of alternatives for a particular item. Therefore, we can obtain information for improving performance from $\tilde{R}$ values and $\tilde{Q}$ ($\nu = 0$) values. This result provides a guideline for intercity bus companies to maintain passenger service standards and the operation performance of each item and to identify areas for improvement in specific aspects of their service operations. In addition, if we use Eq. (9) to search for criteria whose improvement is less important, then we can identify some criteria that do not require improvement. For example, safety (C_1) and social duty (C_5) do not need to be improved in alternative A_1; alternative A_2 is not in urgent need of improvement in terms of comfort (C_2) and operation (C_4); alternative A_3 does not need improvement in terms of convenience (C_3).

$$\tilde{R}_i = \min_j \left(\frac{\tilde{S}_{ij}^U + \tilde{S}_{ij}^L}{2} \right), \quad i = 1, 2, \ldots, m. \tag{9}$$

5. Comparative study and discussions

As mentioned above, we know that the proposed model can be applied to entire/individual evaluators according to their own preferences to effectively select their ideal alternative and to calculate the gap between the ideal alternative and each other alternative, the ranking order of alternatives, and the priority of improving weaknesses for each alternative. Through a case study (numerical example), it is shown that the proposed method can be efficiently utilized to grasp the ambiguity existing in the available information as well as the essential fuzziness in human judgment and preference, and is able to tackle fuzzy MCDM problems in a fuzzy environment very well.

In this section, the concepts of comparative analysis between methods are illustrated to show that the proposed method is suitable for fuzzy MCDM problems. This paper uses the above case study to analyze comparable methods, which include interval-valued fuzzy SAW (shown in Appendix A), interval-valued fuzzy TOPSIS [1] and interval-valued fuzzy VIKOR [37]: the results from the comparable methods and the proposed method will identify the advantages that the proposed method has over other methods. First, the study used interval-valued fuzzy SAW to evaluate the performances of three major intercity bus companies; then the ranking order of the three alternatives was obtained as $A_3 \succ A_1 \succ A_2$. The final synthetic performance values of the three alternatives were $A_1 = 2.9877$, $A_2 = 2.9676$ and $A_3 = 3.0653$. When the study used interval-valued fuzzy TOPSIS to evaluate the performances of three major intercity bus companies, then the evaluation values of alternatives were $A_1 = 0.6001$, $A_2 = 0.5533$ and $A_3 = 0.5854$, respectively. The ranking order of the three alternatives was $A_1 \succ A_3 \succ A_2$. Next, this paper used Vahdani et al.'s interval-valued fuzzy VIKOR method [37] to evaluate this case. Here, we postulated that the ν value was $\nu = 1$ while the Q values of each alternative A_1, A_2, and A_3 were 0, 1, and 0.3536, respectively. The ranking order of the three alternatives was $A_1 \succ A_3 \succ A_2$. The ν value was $\nu = 0.5$, and the Q values of each alternative A_1, A_2, and A_3 were 0.26, 1, and 0.1768, respectively. The ranking order of the three alternatives was $A_3 \succ A_1 \succ A_2$. If the ν value was $\nu = 0$, then the ranking order was $A_3 \succ A_1 \succ A_2$. The Q value of each alternative A_1, A_2, and A_3 were 0.52, 1, and 0, respectively. Finally, this study found that the ranking orders of all alternatives were the same when using interval-valued fuzzy TOPSIS and the proposed interval-valued fuzzy VIKOR method with a ν value of 0.5, because some aspects of interval-valued fuzzy TOPSIS and the proposed interval-valued fuzzy VIKOR method are the same. The TOPSIS method is based on the concept that the selected alternatives will have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative-ideal solution (NIS) when solving a MCDM problem, and the proposed interval-valued fuzzy VIKOR method's concept is an aggregating function representing the distance from the ideal solution [28]. The ranking index is an aggregation of all criteria, the relative importance of the criteria, and a balance between total and individual satisfaction [28]. The VIKOR method's other concepts are based on selecting alternatives' degrees of risk. The SAW method is suitable for situations in which the decision-maker wants to obtain maximum profit. However, in what ways are the results of ranking orders between the proposed interval-valued fuzzy VIKOR method and Vahdani et al.'s method [37] are dissimilar? Because the VIKOR method's equations lead to frequently calculating

Table 8
The ranking of three major intercity bus companies by using the proposed method.

	$\tilde{S}$	$\tilde{R}$	$\tilde{Q}\,(V=1)$	Rank	$\tilde{Q}\,(V=0.5)$	Rank	$\tilde{Q}\,(V=0)$	Rank
A_1	1.8438	0.8891	0	1	0.3267	1	0.6534	2
A_2	2.4286	0.9402	1	3	1	3	1	3
A_3	2.2677	0.7927	0.7249	2	0.3624	2	0	1

process of the interval-valued fuzzy numbers in Vahdani et al.'s method [37], the form of the final interval-valued fuzzy numbers will be changed. Then, the final ranking order of all alternatives will also be changed. Furthermore, Vahdani et al.'s method [37] uses the usual arithmetic operations between interval-valued fuzzy numbers to extend the VIKOR method into the interval-valued fuzzy environment, and thus cannot be used with any decision data for an MCDM problem. According to Definition 1, this study finds that if the lowest limit is smaller than zero, then the two interval-valued fuzzy numbers cannot be calculated. In this paper, the decision data from the case study displayed the above-mentioned characteristics, so this study extended and defined the usual arithmetic operations between interval-valued fuzzy numbers into the arithmetic operations between *L*–*R* interval-valued fuzzy numbers (as shown in Appendix B and Definition 5). Therefore, while the proposed interval-valued fuzzy VIKOR method can avoid the above-mentioned issues, the proposed method can easily obtain a suitable alternative. The proposed method uses each ν value to evaluate all alternatives, and then the ranking orders of all alternatives are different. Through the ν value, this method enables the decision-maker to select/evaluate a suitable alternative through the characteristic of selecting the degree of risk of each alternative and the ideal alternative. The degree of risk is the concept of improving the gap between the ideal solution and each alternative under each criterion. The results of the present case inform us that the proposed method is a good means of evaluation and appears to be more appropriate than other methods. The result from the proposed method enables the decision-maker to obtain a suitable decision outcome.

6. Conclusion

An effective fuzzy multi-criteria analysis method incorporating the concepts of VKIOR and interval-valued fuzzy numbers is presented to solve fuzzy MCDM problems. A case study (numerical example) involving the evaluation of the performance of three intercity bus companies is conducted to examine the applicability of the proposed method. According to this case study, this paper finds that the proposed method can simultaneously obtain the gap between the ideal alternative and each of the other alternatives, the ranking order of alternatives, and the priority of improvements to weaknesses for the performance evaluation of each alternative. In addition, this method is easy to code into a computer program, and one can conduct fuzzy linguistic assessments (in which criteria values are interval-valued fuzzy numbers) as well as fuzzy and non-fuzzy objective assessments. Through the comparative study, this study has clearly identified the proposed method's advantages and each method's characteristic. Here, the proposed method is constructed under the two conditions of independent criteria and interval-valued fuzzy numbers. A subsequent method should extend the proposed method to consider the conditions of independent, dependent and/or interdependent criteria.

Although the method presented in this paper is illustrated by a problem involving the evaluation of the performance of intercity bus companies, it could also be applied to problems such as location selection, information project selection, material selection and many other areas of management decision problems or strategy selection problems.

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Appendix A.

According to the definition of interval-valued fuzzy numbers in Section 2.2, we can extend the SAW method into interval-valued fuzzy numbers in a fuzzy environment. The computational procedure is summarized as follows:

Step 1: The interval-valued fuzzy decision matrix and the interval-valued fuzzy weight of each criterion are obtained by using Eqs. (1)–(3). We must then normalize the performance rating using Chen [9] and Ashtiani et al. [1] giving $\tilde{x}_{ij} = [(\ell_{ij}, \ell'_{ij}), m_{ij}, (u'_{ij}, u_{ij})]$, and the normalized performance are expressed as:

$$\tilde{r}_{ij} = \left[\left(\frac{\ell_{ij}}{u_j^*}, \frac{\ell'_{ij}}{u_j^*}\right), \frac{m_{ij}}{u_j^*}, \left(\frac{u_{ij}}{u_j^*}, \frac{u'_{ij}}{u_j^*}\right)\right], \quad j \in B,$$

$$\tilde{r}_{ij} = \left[\left(\frac{\ell_j^-}{\ell'_{ij}}, \frac{\ell_j^-}{\ell_{ij}}\right), \frac{\ell_j^-}{m_{ij}}, \left(\frac{\ell_j^-}{u_{ij}}, \frac{\ell_j^-}{u'_{ij}}\right)\right], \quad j \in C,$$

$$u_j^* = \max_i u_{ij}, \quad \text{if} \quad j \in B,$$

$$\ell_j^- = \min_i \ell'_{ij}, \quad \text{if} \quad j \in C, \tag{10}$$

where B is the benefit criteria set and C is the cost criteria set. The normalization method mentioned above is used to preserve the property that the ranges of normalized triangular interval-valued fuzzy numbers belong to [0, 1].

Step 2: Construct the weighted normalized performance interval-valued fuzzy decision matrix by using Eq. (11).

$$\tilde{\boldsymbol{\nu}} = [\tilde{\nu}_{ij}]_{m \times n}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \tag{11}$$

where $\tilde{\nu}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j = ((\ell w_{ij}, \ell w'_{ij}), m w_{ij}, (u w'_{ij}, u w_{ij}))$.

Step 3: Next, we can rank order all alternatives according to the weighted performance interval-valued fuzzy decision matrix by using Definition 3. Therefore, we must let the $\tilde{O} = [(0, 0), 0, (0, 0)]$ be original. The two alternatives are expressed as $\tilde{A}$ and $\tilde{B}$. If $d(\tilde{A}, \tilde{O}) < d(\tilde{B}, \tilde{O})$, then the interval-valued fuzzy number $\tilde{A}$ is closer to the original than the other interval-valued fuzzy number $\tilde{B}$. Finally, we can obtain a ranking order of all alternatives.

Appendix B.

In the following, this study briefly reviews and describes some basic definitions of triangular interval-valued fuzzy numbers. These basic definitions and notations will be used for some interval-valued fuzzy MCDM methods in Section 5.

Definition 5. According to Definition 1 (3), Chen and Hwang [11] and Zimmermann [50], *L*–*R* triangular interval-valued fuzzy numbers can denoted by

$\tilde{A} = [m, (\alpha^U, \alpha^L), (\beta^U, \beta^L)]_{LR} = [a_2, ((a_2 - a_1^U), (a_2 - a_1^L)), ((a_3^L - a_2), (a_3^U - a_2))]_{LR}$ and $\tilde{B} = [n, (\gamma^U, \gamma^L), (\delta^U, \delta^L)]_{LR} = [b_2, ((b_2 - b_1^U), (b_2 - b_1^L)), ((b_3^L - b_2), (b_3^U - b_2))]_{LR}$. With this notation, the extended fuzzy operation can be calculated as follows:

Table 9
The seven standard importance level scales or self-created importance level scales.

	Standard importance level scales					Self-created importance level scales				
	Very low	Low	Medium	High	Very high	Very low	Low	Medium	High	Very high
Very low (VL)	(0)	(0)	(0)	(0.1)	(0.15)	( )	( )	( )	( )	( )
Low (L)	(0)	(0.05)	(0.1)	(0.25)	(0.35)	( )	( )	( )	( )	( )
Medium low (ML)	(0)	(0.15)	(0.3)	(0.45)	(0.55)	( )	( )	( )	( )	( )
Medium (M)	(0.25)	(0.35)	(0.5)	(0.65)	(0.75)	( )	( )	( )	( )	( )
Medium high (MH)	(0.45)	(0.55)	(0.7)	(0.8)	(0.95)	( )	( )	( )	( )	( )
High (H)	(0.55)	(0.75)	(0.9)	(0.95)	(1)	( )	( )	( )	( )	( )
Very high (VH)	(0.85)	(0.95)	(1)	(1)	(1)	( )	( )	( )	( )	( )

Table 10
The seven standard satisfaction level scales or self-created satisfaction level scales.

	Standard satisfaction level scales					Self-created satisfaction level scales				
	Very low	Low	Medium	High	Very high	Very low	Low	Medium	High	Very high
Very poor (VP)	(0)	(0)	(0)	(1)	(1.5)	( )	( )	( )	( )	( )
Poor (P)	(0)	(5)	(1)	(2.5)	(3.5)	( )	( )	( )	( )	( )
Moderately poor (MP)	(0)	(1.5)	(3)	(4.5)	(5.5)	( )	( )	( )	( )	( )
Fair (F)	(2.5)	(3.5)	(5)	(6.5)	(7.5)	( )	( )	( )	( )	( )
Moderately good (MG)	(4.5)	(5.5)	(7)	(8)	(9.5)	( )	( )	( )	( )	( )
Good (G)	(5.5)	(7.5)	(9)	(9.5)	(10)	( )	( )	( )	( )	( )
Very good (VG)	(8.5)	(9.5)	(10)	(10)	(10)	( )	( )	( )	( )	( )

(1) Minus L–R interval-valued fuzzy number:

$$-[m,(\alpha^U,\alpha^L),(\beta^U,\beta^L)]_{LR}=[-m,(\alpha^U,\alpha^L),(\beta^U,\beta^L)]_{LR}.$$

(2) Addition of L–R interval-valued fuzzy number $\oplus$:

$$\tilde{A}\oplus\tilde{B}=[(m+n),((\alpha^U+\gamma^U),(\alpha^L+\gamma^L)),((\beta^U+\delta^U),(\beta^L+\delta^L))]_{LR}.$$

(3) Subtraction of L–R interval-valued fuzzy numbers Θ:

$$\tilde{A}\,\Theta\,\tilde{B}=[(m-n),((\alpha^U+\delta^U),(\alpha^L+\delta^L)),((\beta^U+\gamma^U),(\beta^L+\gamma^L))]_{LR}.$$

(4) Multiplication of L–R interval-valued fuzzy numbers $\otimes$:

$$\tilde{A}\otimes\tilde{B}=\begin{cases}[(mn),((m\gamma^U+n\alpha^U),(m\gamma^L+n\alpha^L)),((m\delta^U+n\beta^U),(m\delta^L+n\beta^L))]_{LR}, & \text{when}\quad m>0,\quad n>0,\\ [(mn),((n\alpha^U-m\delta^U),(n\alpha^L-m\delta^L)),((n\beta^U-m\gamma^U),(n\beta^L-m\gamma^L))]_{LR}, & \text{when}\quad m<0,\quad n>0,\\ [(mn),((-n\beta^U-m\delta^U),(-n\beta^L-m\delta^L)),((-n\alpha^U-m\gamma^U),(-n\alpha^L-m\gamma^L))]_{LR}, & \text{when}\quad m<0,\quad n<0.\end{cases}$$

(5) Generalized division of L–R interval-valued fuzzy numbers $\varnothing$:

$$\tilde{A}\varnothing\tilde{B}=\begin{cases}\left[\left(\frac{m}{n}\right),\left(\left(\frac{m\delta^U+n\alpha^U}{n^2}\right),\left(\frac{m\delta^L+n\alpha^L}{n^2}\right)\right),\left(\left(\frac{m\gamma^U+n\beta^U}{n^2}\right),\left(\frac{m\gamma^L+n\beta^L}{n^2}\right)\right)\right]_{LR}, & \text{when}\quad m>0,\quad n>0,\\ \left[\left(\frac{m}{n}\right),\left(\left(\frac{n\alpha^U-m\gamma^U}{n^2}\right),\left(\frac{n\alpha^L-m\gamma^L}{n^2}\right)\right),\left(\left(\frac{n\beta^U-m\delta^U}{n^2}\right),\left(\frac{n\beta^L-m\delta^L}{n^2}\right)\right)\right]_{LR}, & \text{when}\quad m<0,\quad n>0,\\ \left[\left(\frac{m}{n}\right),\left(\left(\frac{-n\beta^U-m\gamma^U}{n^2}\right),\left(\frac{-n\beta^L-m\gamma^L}{n^2}\right)\right),\left(\left(\frac{-n\alpha^U-m\delta^U}{n^2}\right),\left(\frac{-n\alpha^L-m\delta^L}{n^2}\right)\right)\right]_{LR}, & \text{when}\quad m<0,\quad n<0.\end{cases}$$

Appendix C. Survey for the linguistic terms of importance level

In real application, the questionnaire can adopt a Likert-type seven-point scale. It has seven different levels, which are "Very low", "Low", "Medium low", "Medium", "Medium high", "High", and "Very high" – on an interval-valued fuzzy seven-level scale, through which its range is defined (the range lies between 0 and 1). Give a score between 0 and 1 to indicate the five different scales. Here, this study gives the seven standard importance level scales of linguistic terms to allow you to evaluate the level of relative importance of evaluation criteria, as shown in Table 9. If the standard importance level scales have not satisfied your need, then you can create your own suitable linguistic terms. For example, you might think that the importance level of a linguistic score of "High", would correspond to the interval-valued fuzzy number of [(0.65, 0.75), 0.85, (0.95, 1)], respectively. Please answer this questionnaire according to your perceptions. You must think that the linguistic term of your perceptions create low and high limit values that represent an acceptable range. Next, you must set the lowest enduring limit and the uppermost enduring limit values.

Appendix D. Survey for the linguistic terms of the ratings

In this section, the questionnaire also adopts a Likert-type seven-point scale, the points being "Very poor", "Poor", "Moderately poor", "Fair", "Moderately good", "Good", and "Very good", through which its range is defined (the range lies between 0 and 10). Give a score between 0 and 10 to indicate the five different scales. Here, we give the seven standard satisfaction level scales of linguistic terms to allow you to evaluate your level of satisfaction, as shown in Table 10. If the standard satisfaction level scales have not satisfied your need, then you can create your own linguistic terms. For example, you might think the satisfaction level for a linguistic score of "Very good" would correspond to the interval-valued fuzzy number of [(8, 9.5), 10, (10, 10)], respectively. Please answer this questionnaire according to your perceptions. You must think that the linguistic term of your perceptions create low and high limit values that represent an acceptable range. Next, you must set the lowest enduring limit and the uppermost enduring limit values.

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