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A fuzzy TOPSIS model via chi-square test for information source selection

Jing Tian *, Dan Yu, Bing Yu, Shilong Ma

State Key Laboratory of Software Development Environment, Beihang University, No. 37, Xueyuan Road, Haidian District, 100191 Beijing, China

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ABSTRACT

The *Information Source* (*IS*) selection involves various aspects with different requirements under indeterminate conditions. It is such a complicated process pertaining to seeking for the most appropriate solution that how to resolve the constraint resources needs to be congruously considered. This paper proposes a *Multi-Criteria Group Decision Making* (*MCGDM*) model, which uniforms the quantitative and qualitative factual value of different attributes with trapezoidal fuzzy numbers. *Analytic Hierarchy Process* (*AHP*) and *Entropy Weights* (*EW*) are integrated to alleviate the conflicts by experts' intuitions and provide the accurate weight vector in this model. Besides, the *Euclidean Distance* (*ED*) is substituted by the *Value of Chi-Square Test* (*VCST*) to refine the *Relative Closeness* (*RC*), which theoretically excluded the potential bias arising from relative importance of the two types of distances, in a revised *Technique for Order Preference by Similarity to an Ideal Solution* (*TOPSIS*). The optimal recommendation compromises in a social decision making way. Finally, the software named "Evaluator", which is based on the presented model, is illustrated to show how it can be practically used for *IS* selection with comparative analysis.

1. Introduction

The regular function of modern organizations prominently relies on *IS* due to the increasing significance of information and intelligence. Hence, people begin to realize that it is crucial to evaluate a pool of *IS* providers comprehensively before they decide to achieve the most reasonable one to meet their practical demand from a finite set of alternatives [1]. For instance, a company needs to select one *IS* as their source access from several vendors for business reasons. Although the first party offers specific criteria on which it focuses, it has to leverage all the ingredients owing to financial state and source restriction with the difficulties of acquiring accurate data etc. Therefore, it is a challenging task to identify such a perfect candidate in terms of all indicators. Consequently, a systematic model accommodating goals to constraints seems essential to remove the barriers by a complex industrial environment.

Multi-Criteria Decision Making (*MCDM*) and *MCGDM* are provided to deal with the ranking and selecting the ideal *IS* under multiple influential criteria (attributes) by single expert or a group of professional members. Numerous methodologies and models have been studied on *MCDM* and *MCGDM*, yet the flaws of the current research are still obvious:

(a) Because of the vagueness of data, methods for evaluation of linguistic terms and qualitative information process are irrational or imprecise. (b) Many approaches of weights assignments are unilaterally subjective or simply depending on experts' preference ranking index. (c) In classic *TOPSIS*, an alternative might be erroneously judged as the best one via *RC*-value where *ED* metric does not consider the relative importance of distances to both the *Positive Ideal Solution* (*PIS*) and the *Negative Ideal Solution* (*NIS*).

The prime contributions of this paper are stated as follows:

(a) The fuzzy set theory is introduced to improve the accuracy in the presentation and processing of linguistic terms, while the triangular fuzzy number is replaced by the trapezoidal fuzzy number with modified value assignments for a broader range. (b) The subjective and objective methods are incorporated to determine the final weights which take both personal opinions of each individual *Decision Maker* (*DM*) and the information that the known data offers into consideration. (c) The chi-square test is used to calculate the degree of deviation between each alternative and its corresponding expected value, which mathematically ensures that the best alternative is the closest to *PIS* and the farthest from *NIS* simultaneously.

The rest of the paper is organized as follows: Section 2 overviews the recent related research works in this domain. Section 3 provides the preliminaries about trapezoidal fuzzy numbers and a social decision making method. Then the model is proposed through a novel approach particularly articulated in Section 4, while an illustrative example is given to apply the new fuzzy *MCGDM* model for *IS* selection through a software in Section 5.

* Corresponding author. Tel.: +86 10 82317643; fax: +86 10 82317657.
E-mail address: tianjing@nlsde.buaa.edu.cn (J. Tian).

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And Section 6 presents the comparative analysis with the existing research in five aspects. After the future work is outlined in Section 7, the conclusion is discussed in Section 8.

2. Related work

Many researchers have presented analytic models for *MCDM* including *Simple Additive Weighting* (*SAW*) [2,5], *Preference Ranking Organization Method for Enrichment Evaluation* (*PROMETHEE*) [3] and *ELimination and Choice Expressing REality* (*ELECTRE*) [4], etc. However, *TOPSIS* developed by Hwang and Yoon [5,11] has been prevalent because of the efficiency in identifying the best alternative. Rouhani et al. [6] designed an evaluation model for enterprise systems with considering *Business Intelligence* (*BI*) as a new customized fuzzy *TOPSIS* method with detailed stages. Five enterprise systems with those thirty-four criteria were assessed by a decision-making team after fuzzy *PIS* and *NIS* are determined. Then by computing final fuzzy score for each enterprise system and comparing them, the ranking of evaluating enterprise systems was presented. Olson [7] comparatively tested nine combinations of weight generation and distance metrics via both selections of the top-ranked alternative and matching rank at the end of the season. He highlighted that the key to accuracy in *TOPSIS* was to obtain an accurate weight. As methods merging *AHP*/*ANP* into *TOPSIS* had been exploited and applied in a variety of fields [8–11], Yu and Bai [12] proposed a methodology based on interval-valued *AHP* and triangular fuzzy number to facilitate the evaluation process. Wang and Lee [13] developed a novel approach that involves end-users into the whole decision making process which could be used for software outsourcing problem. In the fuzzy model, the subjective weights assigned by the end-users and objective weights based on *Shannon*'s entropy theory were hybrid with linguistic variables handled by fuzzy numbers. Moreover, Li et al. [14] generalized *Bernardo*'s method to *MCGDM* to get the final rankings by aggregating individual ordinal preference to obtain the rankings of alternatives under each criterion in the opinion of the group. However, Opricovic and Tzeng [15] comparatively analyzed *TOPSIS* and *VIKOR*. Four types of differences were clarified between them in procedural basis, normalization, aggregation and solution. It was pointed that *TOPSIS* introduced the ranking index when simply computing the *RC* using the distances from the *PIS* and the *NIS*, and the lack of the relative importance, which should be the major concern, made *TOPSIS* even infeasible for decision making. The "satisfactory level" proposed by Lai et al. [16] and weighted *ED* documented by Deng et al. [17] as well as *VIKOR* are all trials to conquer the drawback. Lu et al. [18] established a *New Product Development* (*NPD*) evaluation model under the theme of well-being design. It could be suitably used in many kinds of other products and/or with other themes. Moreover, a specific software tool was also developed to build the corresponding relation between human-sense and machine measurements.

3. Preliminaries

3.1. Trapezoidal fuzzy numbers

A generalized fuzzy number is a special fuzzy set satisfying $F = \{x \in R | \mu_F(x)\}$, where the value of x is in the domain of real number set R, while $\mu_F(x)$, named membership function, is a continuous mapping from R to the closed interval $[0,1]$. A generalized fuzzy number can be characterized as a tuple as $\widetilde{A} = (a_1, a_2, a_3, a_4; w)_{LR}$, where w is the weight of $\widetilde{A}$ and $a_1, a_2, a_3, a_4, w \geqslant 0$ with the restriction $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$ as well as L and R denote left and right bounded continuous functions respectively [19], so the membership function is

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ L(x) \text{ is monotonic increasing}, & a_1 \leqslant x \leqslant a_2 \\ w, & a_2 \leqslant x \leqslant a_3 \\ R(x) \text{ is monotonic decreasing}, & a_3 \leqslant x \leqslant a_4 \\ 0, & x > a_4 \end{cases} \quad (1)$$

when both $L(x)$ and $R(x)$ are straight lines with $w = 1$, $\widetilde{A}$ is a trapezoidal fuzzy number defined as a tetrad $\widetilde{A} = (a_1, a_2, a_3, a_4)$. In the quadruplet, a_1 and a_4 are called the lower bound and the upper bound of $\widetilde{A}$ respectively with the particular case that a trapezoidal fuzzy number is equivalently referred as a triangular fuzzy number if $a_2 = a_3$.

Then $\widetilde{A} = (a_1, a_2, a_3, a_4)$ obeys the following rules [11]:

$$\lambda \times \widetilde{A} = (\lambda \times a_1, \lambda \times a_2, \lambda \times a_3, \lambda \times a_4), \quad \lambda \in R \quad (2)$$

$$(\widetilde{A})^{-1} = (1/a_4, 1/a_3, 1/a_2, 1/a_1), \quad a_1, a_2, a_3, a_4 \neq 0 \quad (3)$$

when a fuzzy number $\widetilde{B} = (a, b, c, d)$ $a, b, c, d \in R$ operates with $\wedge$ and $\vee$, it regulates as

$$\begin{aligned} a \wedge b &= \min(a, b) \\ a \vee b &= \max(a, b) \end{aligned} \quad (4)$$

In this paper, the authors employ the uniform representation for both qualitative and quantitative numbers by expressing them with the trapezoidal fuzzy numbers. Take a quantitative number q for instance, it could be denoted as (q, q, q, q).

To produce a quantifiable result, the defuzzified value of $\widetilde{A}$, $e \in R$ yields [20]

$$e = \frac{1}{2} \int_0^1 (L^{-1}(x) + R^{-1}(x))dx = (a_1 + a_2 + a_3 + a_4)/4 \quad (5)$$

where $L^{-1}(x)$ and $R^{-1}(x)$ are respective inverse functions of $L(x)$ and $R(x)$ about $\widetilde{A}$.

3.2. Social decision making

A social decision function establishes a mapping from the subset of alternatives to personal preferences by receiving a series of individual preference ranks then outputting a single option. The *Borda Function* is a classic approach for social decision making proposed by *Jena Charles de Borda*. It is such a rating vote schema that voters elicit their preference orders on candidates by voting and the victory belongs to the one with the highest scores accumulated according to the poll.

The *Weighted Borda Function* (*WBF*) [21] is defined as:

$$BF_w(x) = \sum_{i=1}^{d} v_i N(x \succ_i y) \quad (6)$$

where $\succ$ is the partial relation and $x \succ_i y$ means x is better than y in the ith *DM*'s viewpoint $(1 \leqslant i \leqslant d)$, as N is the votes that the ith *DM* attained according to $x \succ_i y$ with the corresponding weight power V_i assigned by an expert. The individual who achieves the highest value of $BF_w(x)$ outperforms the other rivals.

4. Model proposed

The global process for information source selection is illustrated in Fig. 1.

4.1. Initialization

4.1.1. Initial conditions

The global *MCDM* process involves d *DMs*, m alternatives and n criteria.

Firstly the following initial data is prepared:

$$\widetilde{A^k} = \left(\widetilde{x_{ij}^k}\right)_{m\times n}\ (1 \leqslant k \leqslant d),\quad W_k = \left(w_1^k, w_2^k, \ldots, w_n^k\right)\ (1 \leqslant k \leqslant d),$$
$$V = (v_1, v_2, \ldots, v_d)$$

Here $\widetilde{A^k}$ represents the evaluation matrix with trapezoidal fuzzy numbers of the kth *DM* for all alternatives, where $\widetilde{x_{ij}^k}$ is the assessment of the kth *DM*s for the ith alternative on the jth criteria with $\widetilde{x_{ij}^k} = \left(a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k\right)$. W_k is the weight vector that the kth *DM* uses during the decision making process. Meanwhile, the element w_j^k denotes the weight that the kth *DM* offers on the jth criteria, satisfying $\sum_{j=1}^n W_j^k = 1$. Moreover, V is the voting power vector and v_k refers the voting weight of the kth *DM* during the group decision making process.

4.1.2. Normalization

Premise I: *The data values of the same criterion own the same data model or schema with the same representation forma.*

According to **Premise I**, all the data values from the same criterion should gain the same semantic interpretation with the same format. Furthermore, all the data values of the same attribute from different alternatives in this paper should be only either qualitative or quantitative simultaneously. And if it meets the latter situation, a uniform measurement unit has to be provided for all factual values towards the identical criterion.

In order to resist the disturbance by distinct physical measurement units and to preserve the property that the ranges of the fuzzy numbers belong to $[0,1]$, it is indispensable to transform A^k into a dimensionless matrix.

For the values on positive criteria (or benefit indicators),

$$\begin{aligned} y_{ij}^k &= \left(o_{ij}^k, p_{ij}^k, q_{ij}^k, r_{ij}^k\right) \\ &= \left(\left(a_{ij}^k/\max_i\left\{d_{ij}^k\right\}\right) \wedge 1\left(b_{ij}^k/\max_i\left\{c_{ij}^k\right\}\right) \wedge 1\left(c_{ij}^k/\max_i\left\{b_{ij}^k\right\}\right)\right. \\ &\left.\wedge 1\left(d_{ij}^k/\max_i\left\{a_{ij}^k\right\}\right) \wedge 1\right) \end{aligned} \tag{7}$$

For the values on negative criteria (or cost indicators),

$$\begin{aligned} y_{ij}^k &= \left(o_{ij}^k, p_{ij}^k, q_{ij}^k, r_{ij}^k\right) \\ &= \left(\left(\min_i\left\{a_{ij}^k\right\}/d_{ij}^k\right) \wedge 1\left(\min_i\left\{b_{ij}^k\right\}/c_{ij}^k\right) \wedge 1\left(\min_i\left\{c_{ij}^k\right\}/b_{ij}^k\right)\right. \\ &\left.\wedge 1\left(\min_i\left\{d_{ij}^k\right\}/a_{ij}^k\right) \wedge 1\right) \end{aligned} \tag{8}$$

Consequently, a normalized matrix with full trapezoidal fuzzy numbers is achieved:

$$\widetilde{B}^k = \left(y_{ij}^k\right)_{m\times n},\quad (1 \leqslant k \leqslant d)$$

4.2. Individual decision making

4.2.1. Weights determination

The evaluation of criteria entails diverse opinions and meanings, and there are two types of weighting methods: subjective methods and objective methods. Here a hybrid means from each of them is synthesized.

Step 1. The subjective weight is assigned via a preference elicitation technique: *AHP* [22]. In light of this method, every *DM* use "*the 1-to-9 scale strategy by trapezoidal fuzzy numbers*" to construct his own comparison matrix and get the subjective weight vector $\alpha^k = \left(\alpha_1^k, \ldots, \alpha_j^k, \ldots, \alpha_n^k\right)$ for each variable by computing the eigenvectors of that matrix.

Step 2. Get the objective weight vector $\beta^k = \left(\beta_1^k, \ldots, \beta_j^k, \ldots, \beta_n^k\right)$ by computing the amounts of objective information offered by each attribute via *EW* [1,23]:

$$w_j = d_j \Big/ \sum_{h=1}^n d_h,\quad d_j = 1 - e_j,\quad e_j = -k\sum_{j=1}^n y_{ij} \ln y_{ij},$$
$$k = 1/\ln(m) \tag{9}$$

Step 3. Integrate the above two approaches to make sure that the final result reflect both the judgment by experience and the discrepancy between the objective information and the alternatives. Thereupon, the aggregating function is designed by *Minimum-Information-Entropy Principle* in the additive way:

$$\min F = \sum_{j=1}^n w_j^k\left[\ln w_j^k - \ln \alpha_j^k\right] + \sum_{j=1}^n w_j^k\left[\ln w_j^k - \ln \beta_j^k\right]$$
$$s.t.\ \sum_{j=1}^n w_j^k = 1,\quad w_j^k > 0,\ j = 1, 2, \ldots, n \tag{10}$$

Step 4. Figure out $w_j^k = \sqrt{\alpha_j^k \beta_j^k}/\sum_{j=1}^n \sqrt{\alpha_j^k \beta_j^k}, j = 1, 2, \ldots, n$ by *Lagrangian Multiplier Method* [24]. Thus, for each *DM*, the weights vector W^k and his weighted matrix $\widetilde{C}^k = \left(Z_{ij}^k\right)_{m\times n}$ can be calculated, where $z_{ij}^k = \left(w_j^k o_{ij}^k, w_j^k p_{ij}^k, w_j^k q_{ij}^k, w_j^k r_{ij}^k\right)$, $1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n$.

4.2.2. Computing relative closeness

Finishing defuzzifications in matrix $\widetilde{C}^k$ with $f_{ij}^k = \left(w_j^k o_{ij}^k + w_j^k p_{ij}^k + w_j^k q_{ij}^k + w_j^k r_{ij}^k\right)/4$ by (5), a regular fusion matrix $F_k = \left(f_{ij}^k\right)_{m\times n}$ is subsequently attained.

The Set of *PIS* and the Set of *NIS* are respectively associated with:

$$\begin{aligned} f^{k+} &= \{f_1^{k+}, f_2^{k+}, \ldots, f_n^{k+}\} = \left\{\left(\max_i f_{ij}^k \middle| j \in J\right), \left(\min_i f_{ij}^k \middle| j \in J'\right)\right\} \\ f^{k-} &= \{f_1^{k-}, f_2^{k-}, \ldots, f_n^{k-}\} = \left\{\left(\min_i f_{ij}^k \middle| j \in J\right), \left(\max_i f_{ij}^k \middle| j \in J'\right)\right\} \end{aligned} \tag{11}$$

where J is the set of positive criteria, and J' is the set of negative criteria.

Premise II: *All the alternatives with PIS and NIS derive from the attributes of the identical population.*

Based on **Premise II**, the consistency of two cases is evaluated by degrees of deviation from the random variables to the respectively calculated expected values. The lower *VCST* [25] explicitly indicates higher probability to pass the hypothesis that both of them are from the same population, implying the higher degree of both consistency and closeness.

Moreover, it should be emphasized that the *VCST* does not scale the spatial distance between two cases as the (n-dimensional) *ED* does, instead, it measures the accumulated ratio sum of the square difference between the data value of any random variable and its expectation over the latter. As the *VCST* intrinsically illustrates the degree of dispersion between the factual observation of any random variable and its statistical expectation, it covers not only the interrelation of all the elements pertaining to a specific attribute, but also the correlation of distinct criteria. Nonetheless, *ED* neglects the relative importance which is the major concern in the decision making process. Consequently, in *TOPSIS*, *ED* has to be replaced by the *VCST* for distance metric. Then the separations of each alternative from *PIS* and *NIS* are

$$D_i^{k+} = \sum_{j=1}^{m}\left\{\left(f_{ij}^k - g_{ij}^{k+}\right)^2/g_{ij}^{k+} + \left(f_j^{k+} - g^k(\max,j)\right)^2/g^k(\max,j)\right\}$$
$$D_i^{k-} = \sum_{j=1}^{m}\left\{\left(f_{ij}^k - g_{ij}^{k-}\right)^2/g_{ij}^{k-} + \left(f_j^{k-} - g^k(\min,j)\right)^2/g^k(\min,j)\right\} \quad (12)$$

where

$$g_{ij}^{k+} = f_{ij}^k\sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k+}\right),\quad g_{ij}^{k-} = f_{ij}^k\sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k-}\right),$$
$$g^k(\max,j) = f_j^{k+}\sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k+}\right),$$
$$g^k(\min,j) = f_j^{k-}\sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k-}\right) \quad (13)$$

Proposition 1. $D_i^{k+} > D_i^{k-}$

Proof. Let

$$\theta = \sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k+}\right),\quad \eta = \sum_{i=1}^{m}f_{ij}^k\Big/\left(\sum_{j=1}^{n}f_{ij}^k + \sum_{j=1}^{n}f_j^{k-}\right)$$

Then $g_{ij}^{k+} = \theta f_{ij}^k$, $g_{ij}^{k-} = \eta f_{ij}^k$, $g^k(\max,j) = \theta f_j^{k+}$, $g^k(\min,j) = \eta f_j^{k-}$.

Therefore $\theta < \eta$, $1-\theta > 1-\eta$ and $(1-\theta)^2 > (1-\eta)^2$, due to $\sum_{j=1}^n f_j^{k+} > \sum_{j=1}^n f_j^{k-}$.

$$\begin{aligned}D_i^{k+} - D_i^{k-} &= \sum_{j=1}^{m}\left\{\left\{\left(f_{ij}^k - \theta f_{ij}^k\right)^2/\left(\theta f_{ij}^k\right) + \left(f_j^{k+} - \theta f_j^{k+}\right)^2/\left(\theta f_j^{k+}\right)\right\}\right.\\ &\quad \left. - \left\{\left(f_{ij}^k - \eta f_{ij}^k\right)^2/\left(\eta f_{ij}^k\right) + \left(f_j^{k-} - \eta f_j^{k-}\right)^2/\left(\eta f_j^{k-}\right)\right\}\right\}\\ &= \sum_{j=1}^{m}\left\{\left\{[(1-\theta)^2\eta - (1-\eta)^2\theta]f_{ij}^k/(\theta\eta)\right.\right.\\ &\quad \left.\left. + \left[(1-\theta)^2\eta f_j^{k+} - (1-\eta)^2\theta f_j^{k-}\right]/(\theta\eta)\right\}\right\}\end{aligned}$$

For $(1-\theta)^2\eta - (1-\eta)^2\theta > 0$ and $(1-\theta)^2\eta f_j^{k+} - (1-\eta)^2\theta f_j^{k-} > 0$

Thus $D_i^{k+} - D_i^{k-} > 0 \Rightarrow D_i^{k+} > D_i^{k-}$ is proved. □

The result ensures the unequal distances from an individual to either *PIS* or *NIS*, thus, the refined *RC* is formulated as

$$(RC)_i^k = D_i^{k-}/\left(D_i^{k+} + D_i^{k-}\right),\quad i = 1,2,\ldots,m \quad (14)$$

The higher value of $(RC)_i^k$ plausibly declares the better alternative, then the *i*th *DM* concludes his preference ranking. Actually, two conditions [15] impact the correctness of (14) when the pairwise superiority of alternatives a_r and a_s needs affirmation. If $\boldsymbol{a}_r \succ \boldsymbol{a}_s$, then $(RC)_r^k > (RC)_s^k$, i.e. $D_r^{k-}/\left(D_r^{k+} + D_r^{k-}\right) > D_s^{k-}/(D_s^{k+} + D_s^{k-})$, which will hold if

$$\begin{aligned}&(i)\ D_r^{k+} < D_s^{k+} \quad \text{and } D_r^{k-} > D_s^{k-};\quad \text{or}\\ &(ii)\ D_r^{k+} > D_s^{k+} \quad \text{and } D_r^{k-} < D_s^{k-},\quad \text{but } D_r^{k+} < D_s^{k+}D_r^{k-}/D_s^{k-}\end{aligned} \quad (15)$$

Condition (*i*) of (15) shows the "regular" situation, where a_r is superior to a_s because a_r is closer to *PIS* with a longer distance to *NIS* than a_s. However, condition (*ii*) in (15) claims an apparent defect of the original *TOPSIS*, namely mistaking a_r that is more distant from the *PIS* in fact as the better one instead of a_s.

Next, we would like to prove that condition (*ii*) could be mathematically excluded from our model. Suppose A and A' refer to the *PIS* and *NIS* respectively, with the aforementioned a_r and a_s ($a_r \succ a_s$), the coordinate graph is demonstrated in Fig. 2.

Whereas $a_r \succ a_s$ implies $D_r^{k+} < D_s^{k+}$, we focus on whether $D_r^{k-} > D_s^{k-}$ or not.

Proposition 2. $D_r^{k-} > D_s^{k-}$

Proof. Since the origin point depicts the midpoint of the distance from A to A' (termed as $\|AA'\|$) in Fig. 2, $0 \leqslant |\beta| < |\alpha| < \pi/4$ holds due to $D_r^{k+} > D_r^{k-}$ and $D_s^{k+} > D_s^{k-}$.

Based on the *Law of Cosines*,

$$\left(D_r^{k-}\right)^2 = \|AA'\|^2 + \left(D_r^{k+}\right)^2 - 2\|AA'\|D_r^{k+}\cos\alpha$$
$$\left(D_s^{k-}\right)^2 = \|AA'\|^2 + \left(D_s^{k+}\right)^2 - 2\|AA'\|D_s^{k+}\cos\beta$$

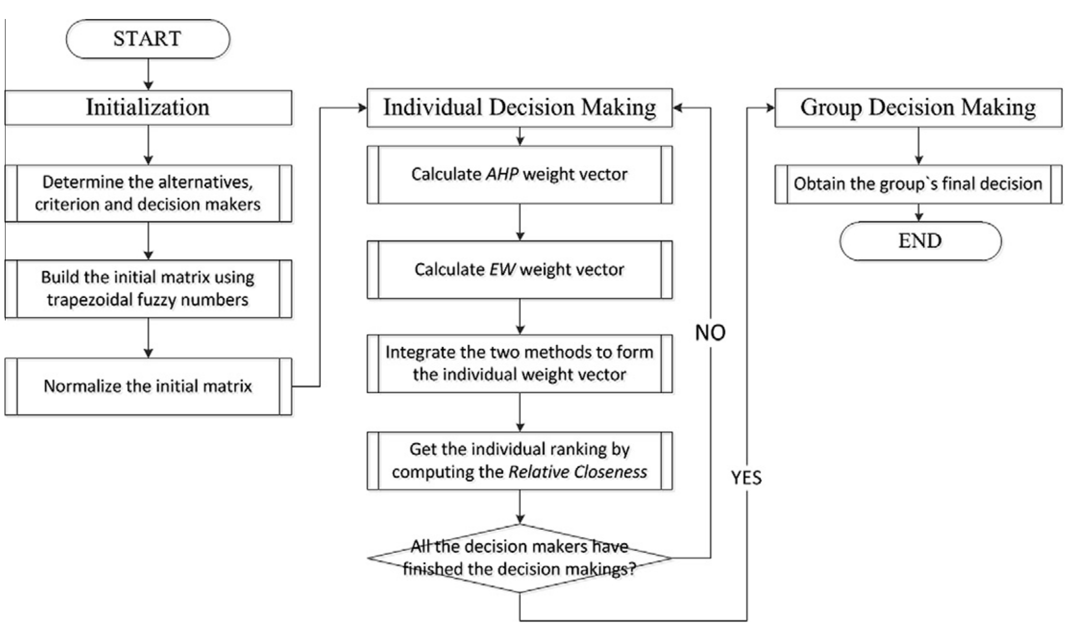

Fig. 1. The global process for IS selection.

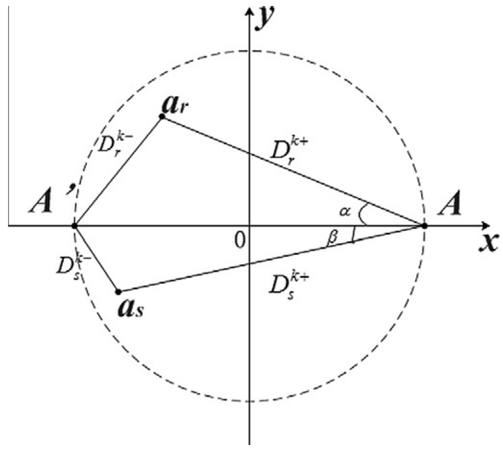

Fig. 2. The distances from two alternatives to PIS and NIS.

Table 1
The criteria involved in IS selection.

Criteria	Accuracy (C_1)	Flexibility (C_2)	Cost (C_3)	Complaint (C_4)
Type	Positive Quantitative	Positive Qualitative	Negative Quantitative	Negative Qualitative

Then $\left(D_r^{k-}\right)^2 - \left(D_s^{k-}\right)^2 = \left(D_r^{k+}\right)^2 - \left(D_s^{k+}\right)^2 - 2\|AA'\|\left(D_r^{k+}\cos\alpha - D_s^{k+}\cos\beta\right)$

$$\left.\begin{array}{l} D_r^{k+} > D_s^{k+} > 0 \\ 0 \leqslant |\beta| < |\alpha| < \pi/4 \Rightarrow \cos\beta > \cos\alpha \end{array}\right\} \Rightarrow D_r^{k+}\cos\alpha < D_s^{k+}\cos\beta$$

$$\Rightarrow \left(D_r^{k+}\right)^2 - \left(D_s^{k+}\right)^2 - 2\|AA'\|\left(D_r^{k+}\cos\alpha - D_s^{k+}\cos\beta\right) > 0$$

$$\Rightarrow \left(D_r^{k-}\right)^2 - \left(D_s^{k-}\right)^2 > 0$$

Hence $D_r^{k-} > D_s^{k-}$ is proved. □

Therefore, it is impossible that the global process would be trapped into condition (*ii*) of (15) or the relative importance of the parametric values in (14) would be omitted. It inherently states that the optimal candidate via *VCST* metric is definitely the closest from the *PIS* and farthest from the *NIS* simultaneously.

4.3. Group decision making

As the *WBF* is used to vote on the rankings from all *DMs*, the *IS* which wins the highest amount of votes is eventually regarded as the optimal choice.

5. Experimental study

To fairly assess multiple *ISs* and identify the best one for large institutes or enterprises, the software named "Evaluator" is developed. It is based on our model and has been deployed with good performance in *China Academy of Space Technology* (*CAST*) to support *MCGDM* for upper-layer applications. It is designed as "Client/Server" architecture so that it could be conveniently delivered and installed. This section demonstrates how the "Evaluator" performs to make the most suitable option from a finite number of *ISs* by several *DMs* for data integration in *CAST*.

The *IS* selection task consists of three alternatives, four criteria and five *DMs*. The criteria cover accuracy, flexibility, cost and complaint with a brief instruction in Table 1.

Trapezoidal fuzzy numbers are adopted due to both the higher generality and the boarder range than triangular fuzzy numbers. So *DMs* are provided more information to make more subtle decisions. The value for each term with a trapezoidal fuzzy number could be predesigned through the interface in Fig. 3 before the global process starts.

The mapping relationship between linguistic terms and their corresponding coverage of fuzzified values are displayed in Fig. 4.

The related initial and normalization information is described in Tables 2 and 3.

Traditional "*the 1-to-9 scale strategy*" [26] has some defects when describing linguistic terms with inadequate smoothness for empirical comparison. If *A* is assumed weakly more important than *B*, for instance, the proportion *A* over *B* is defined as "3:1", namely

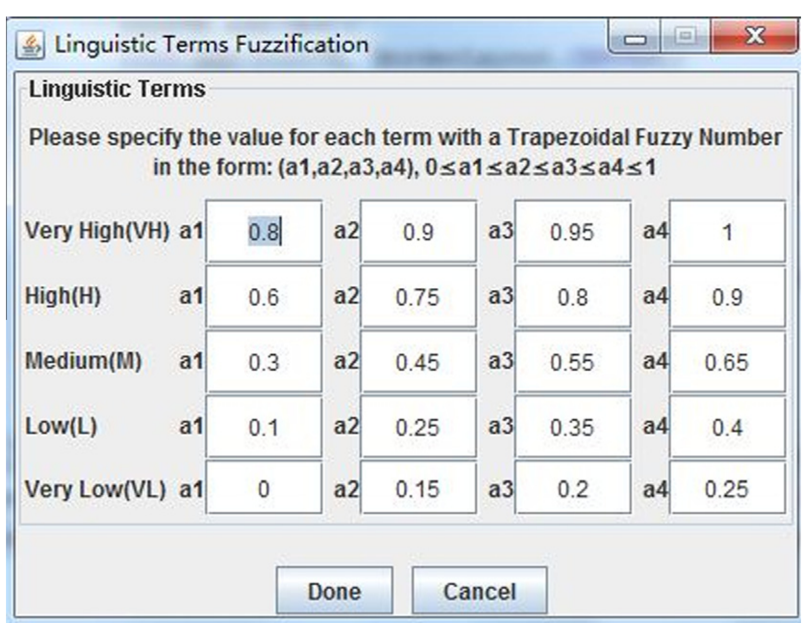

Fig. 3. Linguistic terms and their corresponding trapezoidal fuzzy numbers.

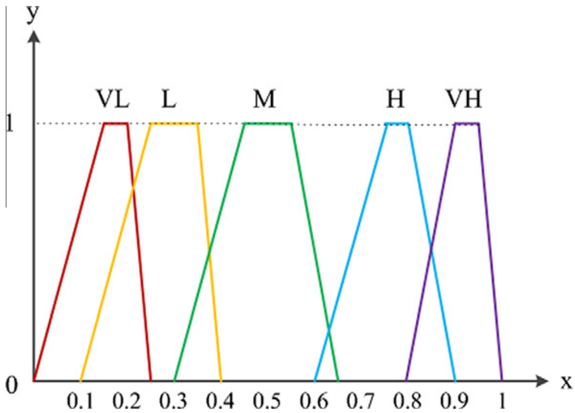

Fig. 4. The linguistic variables for each criterion.

Table 2
Initial information for each DMs.

DMs	Alternative	Criteria			
		C_1	C_2	C_3	C_4
D_1	A_1	0.99	M	5000	H
	A_2	0.95	L	3000	M
	A_3	0.90	H	4500	M
D_2	A_1	0.99	L	5000	M
	A_2	0.95	VL	3000	H
	A_3	0.90	M	4500	VH
D_3	A_1	0.99	H	5000	M
	A_2	0.95	M	3000	H
	A_3	0.90	VH	4500	L
D_4	A_1	0.99	M	5000	VL
	A_2	0.95	L	3000	H
	A_3	0.90	H	4500	M
D_5	A_1	0.99	VH	5000	L
	A_2	0.95	H	3000	VL
	A_3	0.90	H	4500	H

0.75:0.25, which is apparently unreasonable. So the crisp method is meliorated to "*1-to-9 scale strategy by trapezoidal fuzzy number*" via assigning the improved values about relative importance to the elements in comparison matrices. The tactic is illustrated in Tables 4 and 5 and Figs. 5–7.

Then a single-layer *AHP* is used during this scenario. Firstly, each *DM* should carefully judge the relative importance and make his decision according to their own experiences, knowledge and

Table 4
An expert-designed "1-to-9 scale strategy by trapezoidal fuzzy number".

Trapezoidal fuzzy number	Value of membership function
$\tilde{1}$	$(1, 1, \frac{3}{2}, 2)$
$\tilde{x}$	$(x-1, x-\frac{1}{2}, x+\frac{1}{2}, x+1),\ x = 2, 3, \ldots, 8$
$\tilde{9}$	$(8, \frac{17}{2}, 9, 9)$

Table 5
The relative importance and value assigned of pairwise comparison.

Relative importance of x_i over x_j	Traditional value assigned	Improved value assigned
Equally Important (EI)	1	$\tilde{5}/\tilde{5} = (1, 1, 1, 1)$
Weakly More Important (WMI)	3	$\tilde{6}/\tilde{4} = (1, 11/9, 13/7, 7/3)$
Obviously More Important (OMI)	5	$\tilde{7}/\tilde{3} = (3/2, 13/7, 3, 4)$
Strongly More Important (SMI)	7	$\tilde{8}/\tilde{2} = (7/3, 3, 17/3, 9)$
Extremely More Important (EMI)	9	$\tilde{9}/\tilde{1} = (4, 17/3, 9, 9)$

perception of the problem when comparing and weighing each pair of criteria. Fig. 5 exemplifies a single *DM*'s opinion. After all the individuals' results have been submitted and synchronized to the server via "Evaluator", the experts could see the details converged in a single form in Fig. 6.

As there are four criteria in the comparison, the value of the *Random Consistency Index* (*RI*) equals 0.90 [26]. The observations of *Consistency Index* (*CI*), *Consistency Ratio* (*CR*) and *Consistency* situation are computed by$CR = CI/RI$ in Fig. 7.

The weight vectors of *AHP* are calculated by computing the eigenvector of the comparison matrices offered in Fig. 6, while the weight vectors of *EW* are figured by Eq. 9. Then the composite weight vectors are aggregated according to the methods detailed in Section 4.2.1. The result is consequently collected in Fig. 8.

Multiplying the combined weight vector in Fig. 8 and the corresponding fuzzy normalized matrix in Table 3 for each individual *DM* respectively, the *VCST* between the alternatives and the ideal solutions is regarded as the distance to achieve the *RC* in Fig. 9 and individual preference ranking in Fig. 10.

The final decision is accomplished by a committee including five members with the predefined voting power $V = (10, 9, 8, 7, 6)$, the result by *WBF* is shown in Fig. 11.

Therefore, the top-ranked alternative A_3 is eventually identified as the optimal one.

Table 3
The normalized fuzzy information matrices for each DM.

DMs	Normalized fuzzy information matrices
D_1	$\begin{bmatrix} (1, 1, 1, 1), (0.3333, 0.5625, 0.7333, 1), (0.6, 0.6, 0.6, 0.6), (0.3333, 0.5625, 0.7333, 1.0) \\ (0.9596, 0.9596, 0.9596, 0.9596), (0.1111, 0.3125, 0.4667, 0.6667), (1.0, 1.0, 1.0, 1.0), (0.4615, 0.8182, 1.0, 1.0) \\ (0.9091, 0.9091, 0.9091, 0.9091), (0.6667, 0.9375, 1.0, 1.0), (0.6667, 0.6667, 0.6667, 0.6667), (0.4615, 0.8182, 1.0, 1.0) \end{bmatrix}$
D_2	$\begin{bmatrix} (1.0, 1.0, 1.0, 1.0), (0.1538, 0.4545, 0.7778, 1.0), (0.6, 0.6, 0.6, 0.6), (0.4615, 0.8182, 1.0, 1.0) \\ (0.9596, 0.9596, 0.9596, 0.9596), (0.0, 0.2727, 0.4444, 0.8333), (1.0, 1.0, 1.0, 1.0), (0.3333, 0.5625, 0.7333, 1.0) \\ (0.9091, 0.9091, 0.9091, 0.9091), (0.4615, 0.8182, 1.0, 1.0), (0.6667, 0.6667, 0.6667, 0.6667), (0.3, 0.4737, 0.6111, 0.8125) \end{bmatrix}$
D_3	$\begin{bmatrix} (1.0, 1.0, 1.0, 1.0), (0.6, 0.7895, 0.8889, 1.0), (0.6, 0.6, 0.6, 0.6), (0.154, 0.4545, 0.7778, 1.0) \\ (0.9596, 0.9596, 0.9596, 0.9596), (0.3, 0.4737, 0.6111, 0.8125), (1.0, 1.0, 1.0, 1.0), (0.1111, 0.3125, 0.4667, 0.6667) \\ (0.9091, 0.9091, 0.9091, 0.9091), (0.8, 0.9474, 1.0, 1.0), (0.6667, 0.6667, 0.6667, 0.6667), (0.25, 0.7143, 1.0, 1.0) \end{bmatrix}$
D_4	$\begin{bmatrix} (1.0, 1.0, 1.0, 1.0), (0.3333, 0.5625, 0.7333, 1.0), (0.6, 0.6, 0.6, 0.6), (0.0, 0.7499, 1.0, 1.0) \\ (0.9596, 0.9596, 0.9596, 0.9596), (0.1111, 0.3125, 0.4667, 0.6667), (1.0, 1.0, 1.0, 1.0), (0.0, 0.1875, 0.2667, 0.4167) \\ (0.9091, 0.9091, 0.9091, 0.9091), (0.6667, 0.9375, 1.0, 1.0), (0.6667, 0.6667, 0.6667, 0.6667), (0.0, 0.2727, 0.4444, 0.8333) \end{bmatrix}$
D_5	$\begin{bmatrix} (1.0, 1.0, 1.0, 1.0), (0.8, 0.9474, 1.0, 1.0), (0.6, 0.6, 0.6, 0.6), (0.0, 0.4286, 0.8, 1.0) \\ (0.9596, 0.9596, 0.9596, 0.9596), (0.6, 0.7895, 0.8889, 1.0), (1.0, 1.0, 1.0, 1.0), (0.0, 0.7499, 1.0, 1.0) \\ (0.9091, 0.9091, 0.9091, 0.9091), (0.6, 0.7895, 0.8889, 1.0), (0.6667, 0.6667, 0.6667, 0.6667), (0.0, 0.1875, 0.2667, 0.4167) \end{bmatrix}$

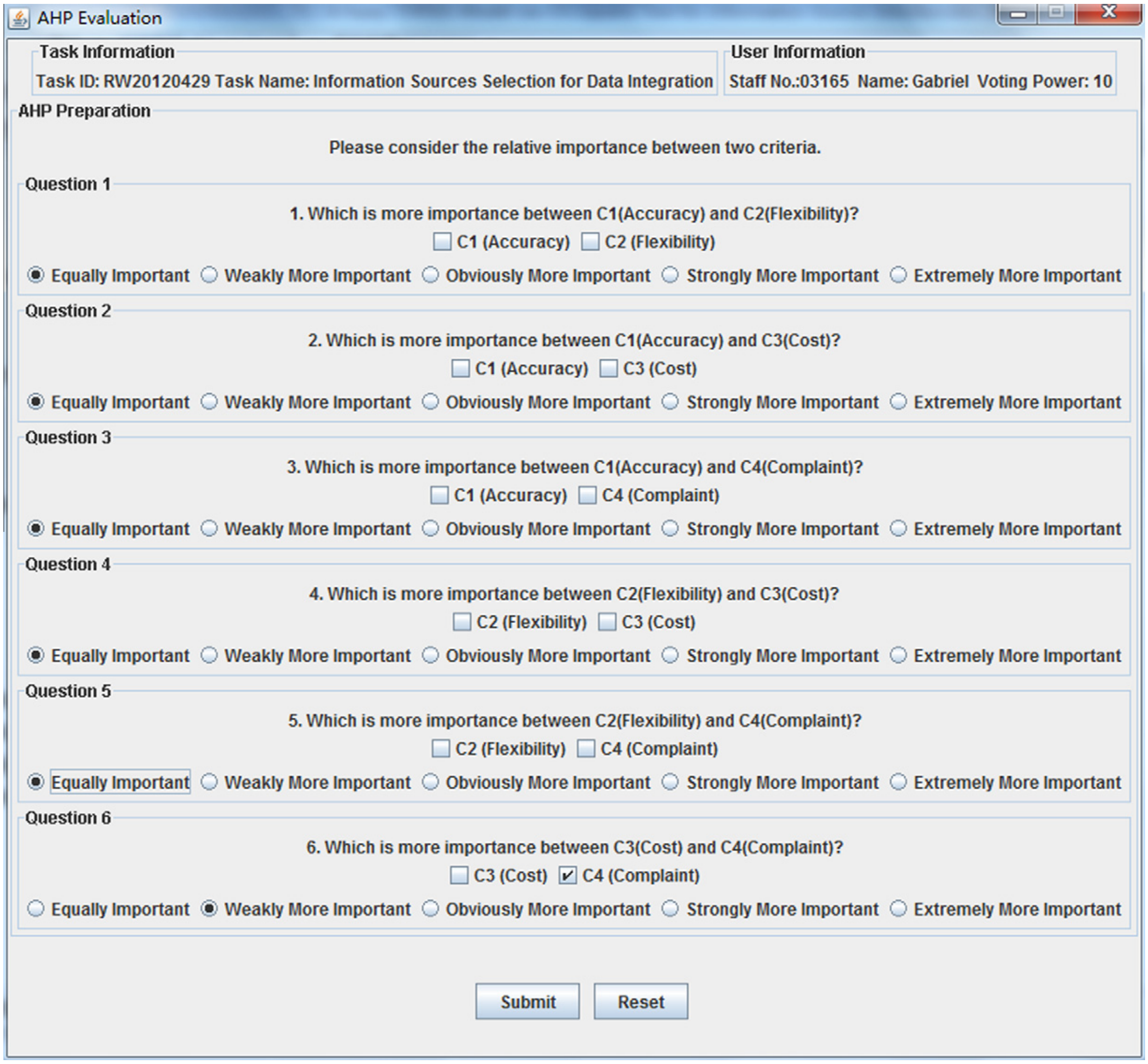

Fig. 5. An example of an individual AHP evaluation.

6. Comparative study and discussions

This section firstly elaborates the essentials of some classic *MCDM* methods, which are the prototypes of the related massive extensions, modifications and (or) advancements for both academic research and applications. Then the important procedural steps are compared with a number of recent approaches. Finally, two categories of group decision making methods are analyzed.

6.1. Rationale

ELECTRE is based on the study of outranking relations and exploitation notions of concordance by using pairwise comparisons among alternatives under each criterion separately. Concordance, discordance indexes and threshold values are used to analyze the outranking relations among the alternatives [27]. *PROMETHEE* also follows the outranking concept to rank the alternatives, combined with the ease of use and decreased complexity. Two complete preorders can be obtained by ranking the alternatives according to their incoming flow and their outgoing flow. The intersection of these two preorders yields the partial preorder of *PROMETHEE I* where incomparabilities are allowed. The ranking of the alternatives according to their net flow yields the complete preorder of *PROMETHEE II* [28]. Both of the two classic families of approaches intrinsically contain many outranking relations with heterogeneous expressions. Furthermore, the correlation between concordance and discordance indexes, as well as the ambiguousness on identifying the incoming flow and outgoing flow, complicates the decision making process on preference selection for *DMs*. *VIKOR* focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the ranking index based on the particular measure of "closeness" to the "ideal" solution [29]. Similarly, the basic principle of *TOPSIS* is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution. *TOPSIS* has been proved to perform well even when the number of alternatives and criteria is too many due to its simplicity in perception and use. However, this technique is often criticized because of its inability to deal adequately with uncertainty and imprecision inherent in the process of mapping the perceptions of decision-makers [30–32]. Bellman and Zadeh [33] first introduced the theory of fuzzy sets in problems of *MCDM* as an effective approach to treat vagueness, lack of knowledge and ambiguity inherent in the human decision making process. *TOPSIS* has been expanded to deal *MCDM* with an uncertain decision matrix resulting in fuzzy *TOPSIS*,

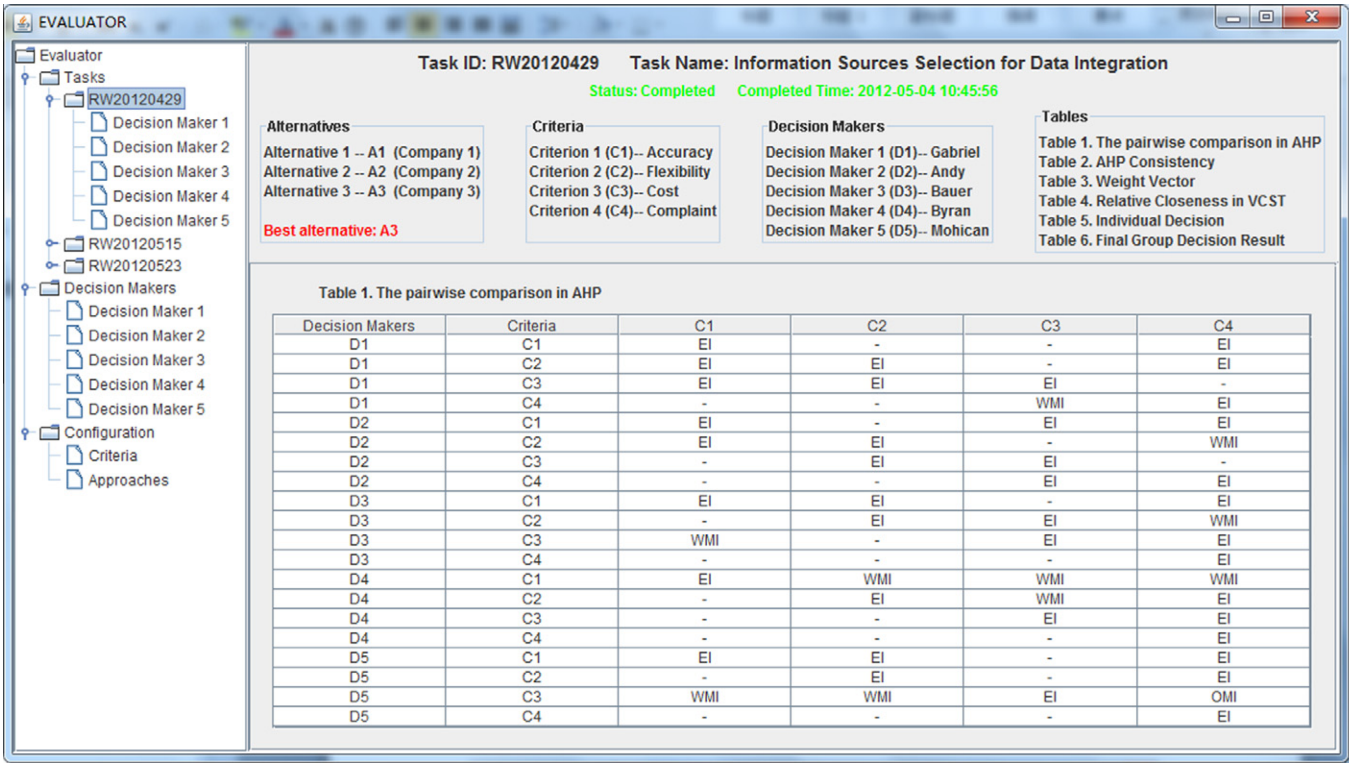

Fig. 6. The pairwise comparison in AHP.

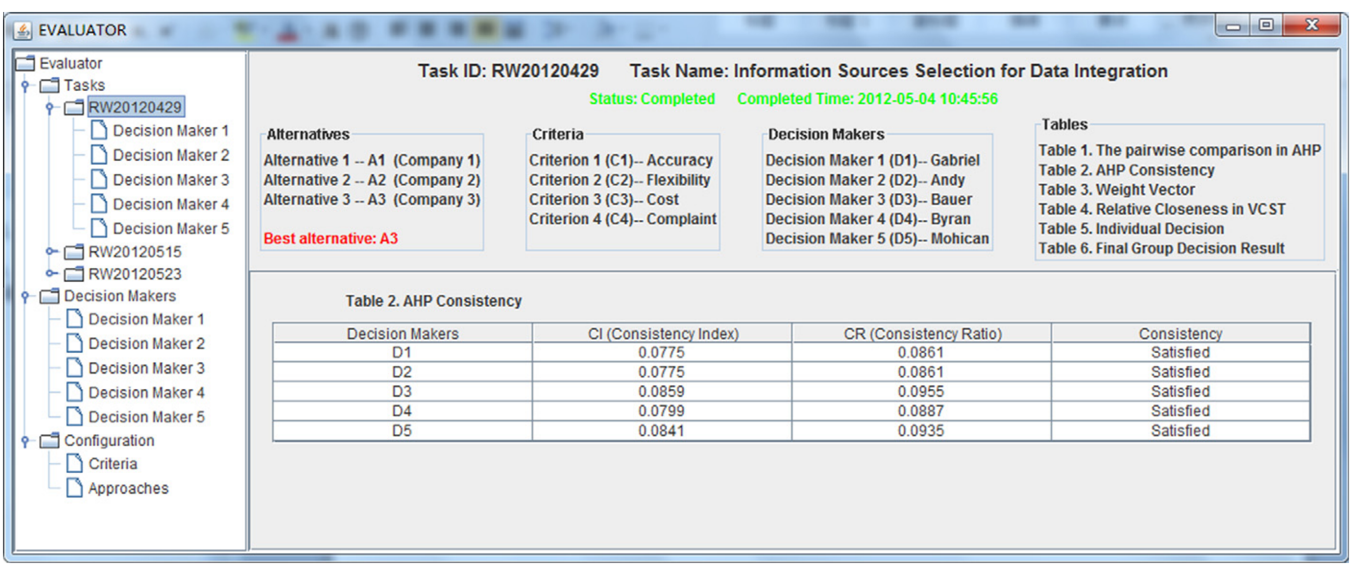

Fig. 7. The consistency check in AHP.

which has successfully been applied to solve various *MCDM* problems. Therefore, our model uniformly transforms the initial data into trapezoidal fuzzy numbers so that it could perform in the fuzzy environment. Meanwhile, it makes use of the generic framework of *TOPSIS*, which needs to calculate of the "proximity" to the *PIS* and the "remoteness" to the *NIS*, in terms of the sound logic that represents the rationale of human choice.

6.2. Normalization

Most of the classic models conduct the normalization in three ways: (i) vector normalization; (ii) linear normalization and its variants; (iii) non-monotonic normalization [34,35]. The vector normalization in original *TOPSIS* has been censured that the normalized value could be different for different evaluation unit of a particular criterion. Therefore, *VIKOR* uses the linear normalization and the normalized value does not depend on the assessment unit of a criterion [15,29]. All the three modes of normalization could be easily extended into the fuzzy scenario upon all the elements of each tuple for each fuzzy number. The specific linear normalization stated in Eqs. (7) and (8) is applied to our model in order to ensure compatibility between evaluation of objective criteria and linguistic ratings of subjective criteria as well as facilitating the computational problems where the different units of the attribute values present in the decision matrix.

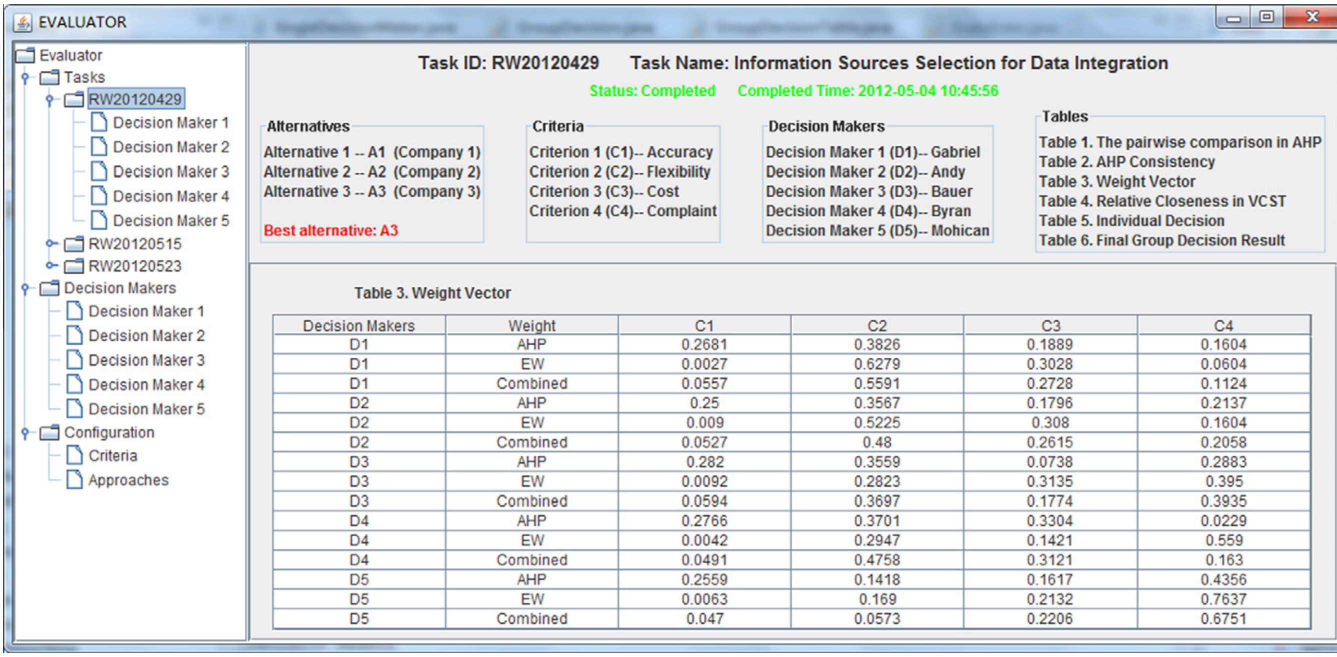

Fig. 8. The weight vectors.

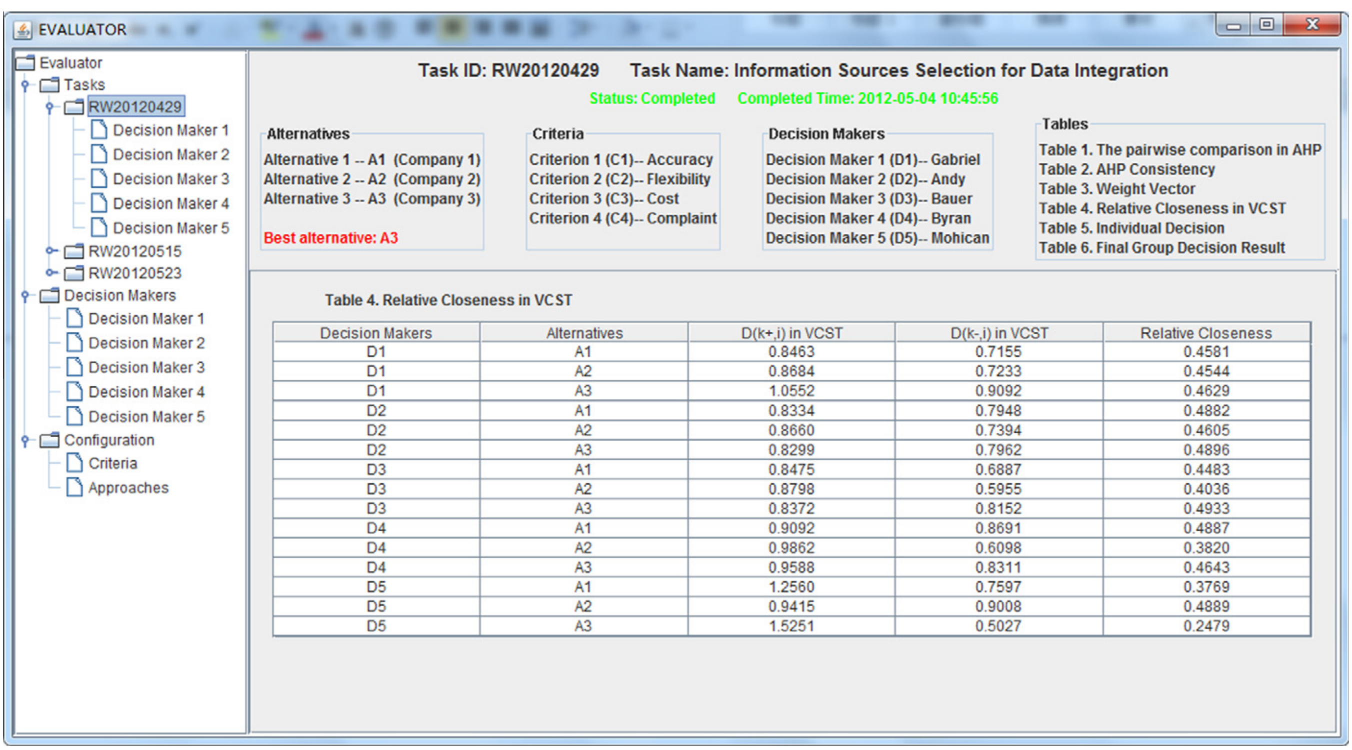

Fig. 9. Relative closeness in VCST.

6.3. Distance metric

Various distance metrics have been attempted in similarity measurements between alternatives in *MCDM* [36–39]. Literature [40] directly measured the distance between two trapezoidal fuzzy numbers by a vertex method resulting in a crisp distance value and used the ideal and anti-ideal solutions to define a crisp overall score for each alternative. Several aggregation operators have been proposed and utilized with high time complexity [41,42]. Thus, the fuzzied values have been defuzzified before the calculation of distances in our model. *Minkovski*'s L_p metric is the most prevalent one since it intuitively reflects the positional relations in n-dimensional space, including *Manhattan Distance* (*MD*, when $p = 1$), *ED* (when $p = 2$) and *Chebyshev Distance* (*CD*, when $p = \infty$) [43]. *Gray Theory* (*GT*) [44,45] is introduced to express the variation of situation for data sequences, scaling the similarity of the shapes about

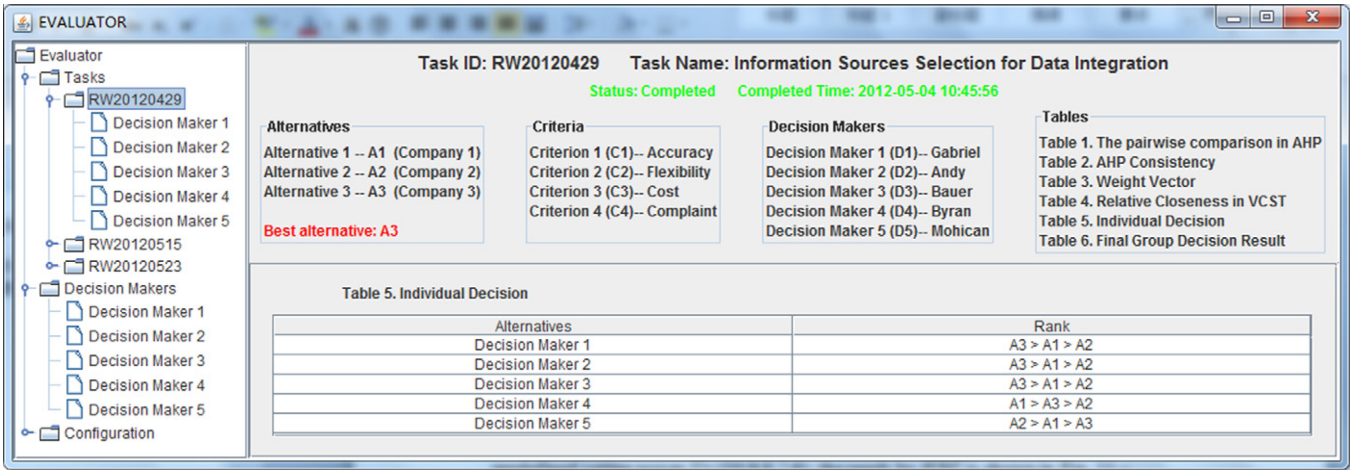

Fig. 10. Individual preference ranking.

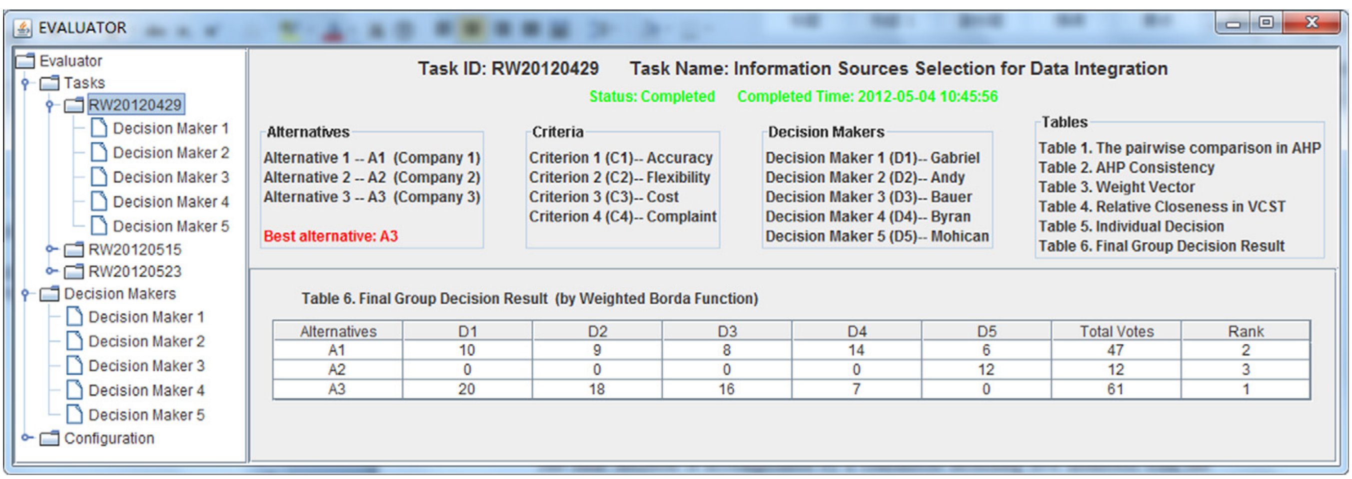

Fig. 11. Final group decision result by WBF.

the corresponding curves. If the distances between candidates and *NIS*|*PIS* are measured by *MD*, *ED*, *CD GT* and *VCST* respectively with other unchanged constraints in our model, the decision made by D_1 are exemplified in Table 6.

The result in Table 6 explicitly states that A_1 with the highest ranking (obtaining the maximum value of *RC*, 0.6404) in *ED* is merely the closest to *PIS* (0.0342) but not the farthest from the *NIS* (0.0610, lower than A_3's 0.0726) simultaneously. The phenomenon confirms the defect described in Section 4.2.2.

6.4. Determining of weight

In this section, α_j^k and β_j^k still indicate the subjective and the objective weight vector of the *k*th *DM* respectively as discussed earlier.

6.4.1. Subjective weight

In the *Simple Multiple Attribute Rating Technique* (*SMART*), participants are required to prioritize the importance of the changes in the criteria from the worst criteria levels to the best. Then 10 points are assigned to the least important criteria, and increasing numbers of points (without explicit upper limit) are assigned to the other criteria to address their relative importance to the least important criteria. The weights are calculated by normalizing the sum of the points to one. The idea of the improved version, that is *SMARTER*, exploits the centroid method [46].

In the *Pair-Wise Comparison* (*PWC*) method, *DM*s are presented a worksheet and are asked to score the relative importance of two

Table 6
Preference ranking affected by three distance metrics about D_1.

Metric	Alternative	Distance to PIS	Distance to NIS	RC	Preference ranking
MD	A_1	0.0699	0.1348	0.6584	$A_3 \succ A_1 \succ A_2$
	A_2	0.2037	0.0009	0.0048	
	A_3	0.0177	0.1870	0.9132	
ED	A_1	0.0342	0.0610	0.6404	$A_1 \succ A_3 \succ A_2$
	A_2	0.0865	0.0001	0.0009	
	A_3	0.0466	0.0726	0.6087	
CD	A_1	0.0699	0.0769	0.5237	$A_3 \succ A_1 \succ A_2$
	A_2	0.1468	0.0010	0.0066	
	A_3	0.0080	0.1468	0.9482	
GT	A_1	0.8780	0.6697	0.4327	$A_2 \succ A_3 \succ A_1$
	A_2	0.6314	0.9992	0.6127	
	A_3	0.8456	0.7806	0.48	
VCST	A_1	0.8463	0.7233	0.4608	$A_3 \succ A_1 \succ A_2$
	A_2	0.8684	0.7154	0.4516	
	A_3	0.8405	0.7243	0.4628	

Table 7
Preference ranking affected by three objective weight determination methods about D_1.

Methods	Weight	Alternative	Distance to PIS	Distance to NIS	RC	Preference ranking
SD	[0.0562, 0.4647, 0.3344, 0.1447]	A_1	0.7334	0.6381	0.4652	$A_3 \succ A_1 \succ A_2$
		A_2	0.7776	0.6745	0.4645	
		A_3	0.9275	0.8203	0.4693	
CRITIC	[0.0654, 0.528, 0.2846, 0.1221]	A_1	0.7531	0.6446	0.4612	$A_3 \succ A_1 \succ A_2$
		A_2	0.7882	0.6697	0.4593	
		A_3	0.9509	0.8289	0.4657	
EW	[0.0027, 0.6279, 0.3028, 0.0604]	A_1	0.8463	0.7233	0.4608	$A_3 \succ A_1 \succ A_2$
		A_2	0.8684	0.7154	0.4516	
		A_3	0.8405	0.7243	0.4628	

Table 8
Preference ranking affected by three weight integration methods about D_1.

Integration	Weight	Alternative	Distance to PIS	Distance to NIS	RC	Preference ranking
Multiplicative	[0.0077, 0.7762, 0.1848, 0.0313]	A_1	1.1191	0.9246	0.4524	$A_3 \succ A_1 \succ A_2$
		A_2	1.0980	0.8669	0.4411	
		A_3	1.3527	1.1324	0.4556	
Additive (q = 0.5)	[0.1385, 0.5052, 0.2459, 0.1104]	A_1	0.7853	0.6652	0.4585	$A_3 \succ A_1 \succ A_2$
		A_2	0.8133	0.6810	0.4557	
		A_3	0.9870	0.8522	0.4633	
Our method	[0.0557, 0.5591, 0.2728, 0.1124]	A_1	0.8463	0.7233	0.4608	$A_3 \succ A_1 \succ A_2$
		A_2	0.8684	0.7154	0.4516	
		A_3	0.8405	0.7243	0.4628	

criteria at a time. The scales can be various, for example, a scale of 0 (equal importance) to 3 (absolutely more important) is commonly adopted. The results are consolidated by adding up the scores obtained by each criterion when preferred to the criteria it is compared with. The results are then normalized to a total of 1. This weighting method provides a framework for comparing each criterion against all others, and helps to reveal the difference in importance between criteria. However, the consistency of participants' preferences, especially, their transitivity is not allowed to be checked [47].

Nevertheless, in the context, *AHP* is preferred, because the *IS* selection problem could be dissociated as criteria and alternatives and the effect of each subject is demanded for measuring. The hierarchical strategy with an improved version of "*the 1-to-9 scale strategy*" provides *DM*s more information to make more subtle and reasonable decisions rather than the normal methods based on exact numbers of points and scores.

6.4.2. Objective weight

The principle of weight determination by the *Standard Deviation* (*SD*) is that the criterion obtaining the larger value of *SD* weights more significantly due to the higher degree of data variation and more information revealed [17,48]. The weight is characterized as

$$\beta_j^k = \delta_j^k \Big/ \sum_{j=1}^{n} \delta_j^k \tag{16}$$

where δ_j^k is the *SD* of the *j*th criterion for *k*th *DM*.

In *CRiteria Importance Through Inter-criteria Correlation* (*CRITIC*) method, the weights derived incorporate both contrast intensity and conflict which are contained in the structure of the decision problem. The developed method is based on the analytical investigation of the evaluation matrix for extracting all information contained in the criteria. The amount of the information that the *j*th criterion is calculated by [48]

$$C_j^k = \delta_j^k \sum_{t=1, t\neq j}^{n} (1 - r_{tj}) \tag{17}$$

where r_{tj} is the relation coefficient between the *t*th and *j*th criterion.

And the weight formula is given as

$$\beta_j^k = C_j^k \Big/ \sum_{j=1}^{n} C_j^k \tag{18}$$

Table 7 demonstrates how different methods of objective weight assignments impact the preference rating for D_1 without any other changed conditions in our model.

The result of Table 7 seems to suggest that objective weights derived by the *EW* are more significantly different to each other. This reflects the capability of the *EW* in providing the average intrinsic information generated by the performance of *IS*s. This would help the *DM* discriminate the most important criterion.

6.4.3. Weighting integration methods

Weighting integration methods have been applied to the evaluation and comparison of complex systems. These methods could roughly be classified into two groups of operations: multiplicative integration and additive integration [47].

The multiplicative synthesis is expressed as

$$w_j^k = \alpha_j^k \beta_j^k \Big/ \sum_{j=1}^{n} \alpha_j^k \beta_j^k, \quad j = 1, 2, \ldots, n \tag{19}$$

While the additive synthesis is expressed as

$$w_j^k = q\alpha_j^k + (1 - q)\beta_j^k, \quad j = 1, 2, \ldots, n, \quad 0 \leqslant q \leqslant 1 \tag{20}$$

where q is the additive integration coefficient.

Then both of the patterns of integration are comparably illustrated with our method about D_1's preference order, for example, in Table 8.

6.5. Group decision methods

In extending *TOPSIS* to a group decision environment, the methods can be categorized into two varieties: mathematical methods and voting methods.

6.5.1. Mathematical methods

Most of the mathematical works aggregate the importance of the criteria and/or the rating of alternatives with respect to each criterion from individuals of the group via some specific operators [40–42,47]. Literature [49] endeavored to design a global *TOPSIS* after a number of individual fuzzy *TOPSIS* procedure, while others compared the effects of external aggregation and internal aggregation of group preferences [30].

6.5.2. Voting methods

Social preference functions are commonly based on voting rules. According to *Copeland* rule, the option with the largest number (i.e. with the highest ranking) in exhaustive pairwise comparison is the most recommended. The *Borda* rule is to select the option that on average stands highest in the voters' rankings [50]. Under most scenarios, the diverse knowledge or comprehensibility of each group member cause different levels of professional decisions. Thus the personal authority needs to be considered, so that the *Borda* function with the assigned weights which is described in Section 3.2 actually reflects the rationale of committee choice.

7. Future work

As the decision making is a dynamic process, further studies should focus on users' feedbacks and their influence on the next turn of decision, that is, how to develop a suitable strategy to manage the variable weights or voting power. Moreover, the difficulties of incomplete information resulting from various reasons, such as the inadequate knowledge or unintentional ignorance, are needed to consolidate the current research.

8. Conclusion

This paper primarily analyzes the characteristics of *IS* selection and points out the drawbacks of the current researches. To properly deal with these problems, a *MCGDM* model is elaborated. This new model relies on the chi-square test metric in a fuzzy *TOPSIS* fashion. It is composed of a committee-decision process with appropriate resolutions in linguistic terms, quantitative information, weight determination and *RC* computation using trapezoidal fuzzy numbers, and additive amalgamation of weight assignments respectively. In addition, the problem caused by the relative importance of distances to ideal/anti-ideal solutions in the original *TOPSIS* has been mathematically eliminated via *VCST* metric. After the illustrative example and comparative studies, it finally demonstrates that the model could provide an effective framework for ranking competing alternatives of *IS*. And it could be easily adaptable and extended to other applications.

Acknowledgements

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