A fuzzy TOPSIS model via chi-square test for information source selection

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1. Introduction

The regular function of modern organizations prominently relies on IS due to the increasing significance of information and intelligence. Hence, people begin to realize that it is crucial to evaluate a set of alternatives [1]. For instance, a company needs to select one IS as their source access from several vendors for business reasons. Although the first party offers specific criteria on which it focuses, it has to leverage all the ingredients owing to financial state and source restriction with the difficulties of acquiring accurate data etc. Therefore, it is a challenging task to identify such a perfect candidate in terms of all indicators. Consequently, a systematic model accommodating goals to constraints seems essential to remove the barriers by a complex industrial environment.

Multi-Criteria Decision Making (MCDM) and MCGDM are provided to deal with the ranking and selecting the ideal IS under multiple influential criteria (attributes) by single expert or a group of professional members. Numerous methodologies and models have been studied on MCDM and MCGDM, yet the flaws of the current research are still obvious:

(a) Because of the vagueness of data, methods for evaluation of linguistic terms and qualitative information process are irrational or imprecise. (b) Many approaches of weights assignments are unilaterally subjective or simply depending on experts’ preference ranking index. (c) In classic TOPSIS, an alternative might be erroneously judged as the best one via RC-value where ED metric does not consider the relative importance of distances to both the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). The prime contributions of this paper are stated as follows:

(a) The fuzzy set theory is introduced to improve the accuracy in the presentation and processing of linguistic terms, while the triangular fuzzy number is replaced by the trapezoidal fuzzy number with modified value assignments for a broader range. (b) The subjective and objective methods are incorporated to determine the final weights which take both personal opinions of each individual Decision Maker (DM) and the information that the known data offers into consideration. (c) The chi-square test is used to calculate the degree of deviation between each alternative and its corresponding expected value, which mathematically ensures that the best alternative is the closest to PIS and the farthest from NIS simultaneously.

The rest of the paper is organized as follows: Section 2 overviews the recent related research works in this domain. Section 3 provides the preliminaries about trapezoidal fuzzy numbers and a social decision making method. Then the model is proposed through a novel approach particularly articulated in Section 4, while an illustrative example is given to apply the new fuzzy MCGDM model for IS selection through a software in Section 5.
And Section 6 presents the comparative analysis with the existing research in five aspects. After the future work is outlined in Section 7, the conclusion is discussed in Section 8.

2. Related work

Many researchers have presented analytic models for MCDM including Simple Additive Weighting (SAW) [2,5], Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) [3] and Elimination and Choice Expressing Reality (ELECTRE) [4], etc. However, TOPSIS developed by Hwang and Yoon [5,11] has been prevalent because of the efficiency in identifying the best alternative. Rouhani et al. [6] designed an evaluation model for enterprise systems with considering Business Intelligence (BI) as a new customized fuzzy TOPSIS method with detailed stages. Five enterprise systems with those thirty-four criteria were assessed by a decision-making team after fuzzy PIS and NIS are determined. Then by computing final fuzzy score for each enterprise system and comparing them, the ranking of evaluating enterprise systems was presented. Olson [7] comparatively tested nine combinations of weight generation and distance metrics via both selections of the top-ranked alternative and matching rank at the end of the season. He highlighted that the key to accuracy in TOPSIS was to obtain an accurate weight. As methods merging AHP/ANP into TOPSIS had been exploited and applied in a variety of fields [8–11], Yu and Bai [12] proposed a methodology based on interval-valued AHP and triangular fuzzy number to facilitate the evaluation process. Wang and Lee [13] developed a novel approach that involves end-users into the whole decision making process which could be used for software outsourcing problem. In the fuzzy model, the subjective weights assigned by the end-users and objective weights based on Shannon's entropy theory were hybrid with linguistic variables handled by fuzzy numbers. Moreover, Li et al. [14] generalized Bernardo’s method to MCGDM to get the final rankings by aggregating individual ordinal preference to obtain the rankings of alternatives under each criterion in the opinion of the group. However, Opricovic and Tzeng [15] comparatively analyzed TOPSIS and VIKOR. Four types of differences were clarified between them in procedural basis, normalization, aggregation and solution. It was pointed that TOPSIS introduced the ranking index when simply computing the RC using the distances from the PIS and the NIS, and the lack of the relative importance, which should be the major concern, made TOPSIS even infeasible for decision making. The “satisfactory level” proposed by Lai et al. [16] and weighted ED documented by Deng et al. [17] as well as VIKOR are all trials to conquer the drawback. Lu et al. [18] established a New Product Development (NPD) evaluation model under the theme of well-being design. It could be suitably used in many kinds of other products and/or with other themes. Moreover, a specific software tool was also developed to build the corresponding relation between human-sense and machine measurements.

3. Preliminaries

3.1. Trapezoidal fuzzy numbers

A generalized fuzzy number is a special fuzzy set satisfying \( F = \{x \in R | \mu_F(x)\} \), where the value of \( x \) is in the domain of real number set \( R \), while \( \mu_F(x) \), named membership function, is a continuous mapping from \( R \) to the closed interval \([0, 1]\). A generalized fuzzy number can be characterized as a tuple as \( \tilde{A} = (a_1, a_2, a_3, a_4; w) \), where \( w \) is the weight of \( \tilde{A} \) and \( a_1, a_2, a_3, a_4, w \geq 0 \) with the restriction \( a_1 \leq a_2 \leq a_3 \leq a_4 \) as well as \( L \) and \( R \) denote left and right bounded continuous functions respectively [19], so the membership function is

\[
\mu\_{\tilde{A}}(x) = \begin{cases} 
0, & x < a_1 \\
L(x) & a_1 \leq x \leq a_2 \\
w, & a_2 \leq x \leq a_3 \\
R(x) & a_3 \leq x \leq a_4 \\
0, & x > a_4
\end{cases}
\]

when both \( L(x) \) and \( R(x) \) are straight lines with \( w = 1 \). \( \tilde{A} \) is a trapezoidal fuzzy number defined as a tetrad \( \tilde{A} = (a_1, a_2, a_3, a_4) \). In the quadruplet, \( a_1 \) and \( a_4 \) are called the lower bound and the upper bound of \( \tilde{A} \) respectively with the particular case that a trapezoidal fuzzy number is equivalently referred as a triangular fuzzy number if \( a_2 = a_3 \).

Then \( \tilde{A} = (a_1, a_2, a_3, a_4) \) obeys the following rules [11]:

\[
\begin{align*}
\lambda \times \tilde{A} &= (\lambda \times a_1, \lambda \times a_2, \lambda \times a_3, \lambda \times a_4), \quad \lambda \in R \\
(\tilde{A})^{-1} &= (1/a_4, 1/a_3, 1/a_2, 1/a_1), \quad a_1, a_2, a_3, a_4 \neq 0 \\
\text{when a fuzzy number } \tilde{B} = (a, b, c, d) \text{ a, b, c, d } &\in R \text{ operates with } \land \text{ and } \lor, \text{ it regulates as} \\
a \land b &= \min(a, b) \\
a \lor b &= \max(a, b)
\end{align*}
\]

In this paper, the authors employ the uniform representation for both qualitative and quantitative numbers by expressing them with the trapezoidal fuzzy numbers. Take a quantitative number \( q \) for instance, it could be denoted as \( [q, q, q] \).

To produce a quantifiable result, the defuzzified value of \( \tilde{A} \), \( e \in R \) yields [20]

\[
e = \frac{1}{2} \int_0^1 (L^{-1}(x) + R^{-1}(x)) dx = (a_1 + a_2 + a_3 + a_4)/4
\]

where \( L^{-1}(x) \) and \( R^{-1}(x) \) are respective inverse functions of \( L(x) \) and \( R(x) \) about \( \tilde{A} \).

3.2. Social decision making

A social decision function establishes a mapping from the subset of alternatives to personal preferences by receiving a series of individual preference ranks then outputting a single option. The Borda Function is a classic approach for social decision making proposed by Jena Charles de Borda. It is such a rating vote schema that voters elicit their preference orders on candidates by voting and the victory belongs to the one with the highest scores accumulated according to the poll.

The Weighted Borda Function (WBF) [21] is defined as:

\[
BF_w(x) = \sum_{i=1}^{d} \pi_i \cdot N(x > y)
\]

where \( \succ \) is the partial relation and \( x \succ y \) means \( x \) is better than \( y \) in the ith DM’s viewpoint \((1 \leq i \leq d)\), as \( N \) is the votes that the ith DM attained according to \( x \succ y \) with the corresponding weight power \( \pi_i \) assigned by an expert. The individual who achieves the highest value of \( BF_w(x) \) outperforms the other rivals.

4. Model proposed

The global process for information source selection is illustrated in Fig. 1.

4.1. Initialization

4.1.1. Initial conditions

The global MCDM process involves \( d \) DMs, \( m \) alternatives and \( n \) criteria.
Firstly the following initial data is prepared:

\[
\tilde{A}^k = (x_{ij}^k)_{m \times n}, \quad (1 \leq k \leq d), \quad W_k = (w_k^1, w_k^2, \ldots, w_k^d) \quad (1 \leq k \leq d),
\]

\[
V = (v_1, v_2, \ldots, v_m).
\]

Here \(\tilde{A}^k\) represents the evaluation matrix with trapezoidal fuzzy numbers of the kth DM for all alternatives, where \(x_{ij}^k\) is the assessment of the kth DM for the jth alternative on the jth criteria with

\[
x_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k). \quad W_k = \text{the weight vector that the kth DM uses during the decision making process. Meanwhile, the element } w_k^j \text{ denotes}.
\]

Here \(V\) represents the voting power vector.

4.1.2. Normalization

**Premise 1:** The data values of the same criterion own the same data model or schema with the same representation form.

According to **Premise 1**, all the data values from the same criterion should gain the same semantic interpretation with the same format. Furthermore, all the data values of the same attribute from different alternatives in this paper should be only either qualitative or quantitative simultaneously. And if it meets the latter situation, a uniform measurement unit has to be provided for all factual values towards the identical criterion.

In order to resist the disturbance by distinct physical measurement units and to preserve the property that the ranges of the fuzzy numbers belong to \([0,1]\), it is indispensable to transform \(A^k\) into a dimensionless matrix.

For the values on positive criteria (or benefit indicators),

\[
y_{ij}^+ = \left(\frac{a_{ij}^+}{b_{ij}^+}, \frac{b_{ij}^+}{c_{ij}^+}, \frac{c_{ij}^+}{d_{ij}^+}, r_{ij}^+\right)
\]

\[
= \left(\frac{\min\{a_{ij}\}}{\max\{d_{ij}\}} \wedge \frac{\min\{b_{ij}\}}{\max\{c_{ij}\}} \wedge \frac{\min\{c_{ij}\}}{\max\{b_{ij}\}} \wedge \frac{\min\{d_{ij}\}}{\max\{a_{ij}\}}\right)
\]

(7)

For the values on negative criteria (or cost indicators),

\[
y_{ij}^- = \left(\frac{a_{ij}^-}{b_{ij}^-}, \frac{b_{ij}^-}{c_{ij}^-}, \frac{c_{ij}^-}{d_{ij}^-}, r_{ij}^-\right)
\]

\[
= \left(\frac{\min\{a_{ij}\}}{\max\{d_{ij}\}} \wedge \frac{\min\{b_{ij}\}}{\max\{c_{ij}\}} \wedge \frac{\min\{c_{ij}\}}{\max\{b_{ij}\}} \wedge \frac{\min\{d_{ij}\}}{\max\{a_{ij}\}}\right)
\]

(8)

Consequently, a normalized matrix with full trapezoidal fuzzy numbers is achieved:

\[
\tilde{B}^k = (y_{ij}^k)_{m \times n}, \quad (1 \leq k \leq d)
\]

4.2. Individual decision making

4.2.1. Weights determination

The evaluation of criteria entails diverse opinions and meanings, and there are two types of weighting methods: subjective methods and objective methods. Here a hybrid means from each of them is synthesized.

Step 1. The subjective weight is assigned via a preference elicitation technique: AHP [22]. In light of this method, every DM uses “the 1-to-9 scale strategy by trapezoidal fuzzy numbers” to construct his own comparison matrix and get the subjective weight vector \(\tilde{w}^k = (x_1^k, x_2^k, \ldots, x_n^k)\) for each variable by computing the eigenvectors of that matrix.

Step 2. Get the objective weight vector \(w^k = (\beta_1^k, \beta_2^k, \ldots, \beta_n^k)\) by computing the amounts of objective information offered by each attribute via EW [1,23]:

\[
w_j = d_j = \frac{d_j}{\sum_{b=1}^{n} d_b}, \quad d_j = 1-e_j, \quad e_j = -k \sum_{j=1}^{n} y_{ij} \ln y_{ij},
\]

(9)

Step 3. Integrate the above two approaches to make sure that the final result reflect both the judgment by experience and the discrepancy between the objective information and the alternatives. Thereupon, the aggregating function is designed by Minimum-Information-Entropy Principle in the additive way:

\[
\min F = \sum_{j=1}^{n} w_j^k \left[ \ln w_j^k - \ln x_j^k \right] + \sum_{l=1}^{n} w_l^k \left[ \ln w_l^k - \ln \beta_l^k \right]
\]

s.t., \(\sum_{j=1}^{n} w_j^k = 1, \quad w_j^k > 0, \quad j = 1, 2, \ldots, n\)

(10)

4.2.2. Computing relative closeness

Finishing defuzzifications in matrix \(\tilde{B}^k\) with \(f_{ij}^k = \left(\hat{x}_{ij}^k, \hat{y}_{ij}^k, \hat{z}_{ij}^k, \hat{w}_{ij}^k\right) / 4\) by (5), a regular fusion matrix \(F_k = (f_{ij}^k)_{m \times n}\) is subsequently attained.

The Set of PIS and the Set of NIS are respectively associated with:

\[
f^{k+} = \{f_{ij}^k, f_{ij}^{k+}, \ldots, f_{ij}^{k+}\} = \{\min f_{ij}^k, \max f_{ij}^k\}
\]

(11)

where \(J\) is the set of positive criteria, and \(F\) is the set of negative criteria.

**Premise II:** All the alternatives with PIS and NIS derive from the attributes of the identical population.

Based on **Premise II**, the consistency of two cases is evaluated by degrees of deviation from the random variables to the respectively calculated expected values. The lower VCST [25] explicitly indicates higher probability to pass the hypothesis that both of them are from the same population, implying the higher degree of both consistency and closeness.

Moreover, it should be emphasized that the VCST does not scale the spatial distance between two cases as the \((n\text{-dimensional }) ED\) does, instead, it measures the accumulated ratio sum of the square difference between the data value of any random variable and its expectation over the latter. As the VCST intrinsically illustrates the degree of dispersion between the factual observation of any random variable and its statistical expectation, it covers not only the interrelation of all the elements pertaining to a specific attribute, but also the correlation of distinct criteria. Nonetheless, \(ED\) neglects the relative importance which is the major concern in the decision making process. Consequently, in TOPSIS, \(ED\) has to be replaced by the VCST for distance metric. Then the separations of each alternative from PIS and NIS are
Thus \( D_i^{k+} - D_i^{k-} > 0 \Rightarrow D_i^{k+} > D_i^{k-} \) is proved. □

The result ensures the unequal distances from an individual to either PIS or NIS, thus, the refined RC is formulated as

\[
(RC)_i^k = D_i^{k+} - D_i^{k-}, \quad i = 1, 2, \ldots, m
\]  

(14)

The higher value of \((RC)_i^k\) plausibly declares the better alternative, then the ith DM concludes his preference ranking. Actually, two conditions [15] impact the correctness of (14) when the pairwise superiority of alternatives \(a_i\) and \(a_j\) needs affirmation. If \(a_i > a_j\), then \((RC)_i^k > (RC)_j^k\), i.e. \(D_i^{k+} / (D_i^{k+} + D_i^{k-}) > D_j^{k+} / (D_j^{k+} + D_j^{k-})\), which will hold if

(i) \(D_i^{k+} < D_j^{k+}\) and \(D_i^{k-} > D_j^{k-}\); or

(ii) \(D_i^{k+} > D_j^{k+}\) and \(D_i^{k-} < D_j^{k-}\), but \(D_i^{k+} < D_j^{k+} / D_j^{k-}\)

(15)

Condition (i) of (15) shows the “regular” situation, where \(a_i\) is superior to \(a_j\) because \(a_i\) is closer to PIS with a longer distance to NIS than \(a_j\). However, condition (ii) in (15) claims an apparent defect of the original TOPSIS, namely mistaking \(a_i\) that is more distant from the PIS in fact as the better one instead of \(a_j\).

Next, we would like to prove that condition (ii) could be mathematically excluded from our model. Suppose \(A\) and \(A'\) refer to the PIS and NIS respectively, with the aforementioned \(a_i\) and \(a_j\) (\(a_i > a_j\)), the coordinate graph is demonstrated in Fig. 2.

Whereas \(a_i > a_j\) implies \(D_i^{k+} < D_j^{k+}\), we focus on whether \(D_i^{k+} > D_j^{k+}\) or not.

**Proposition 2.** \(D_i^{k+} > D_j^{k+}\)

**Proof.** Since the origin point depicts the midpoint of the distance from \(A\) to \(A'\) (termed as \(\|AA'\|\)) in Fig. 2, \(0 < ||\beta|| < ||\pi / 4||\) holds due to \(D_i^{k+} > D_j^{k+}\) and \(D_i^{k+} > D_j^{k+}\).

Based on the Law of Cosines,

\[
(D_i^{k+})^2 = \|AA'\|^2 + (D_j^{k+})^2 - 2\|AA'\|D_j^{k+}\cos\alpha
\]

\[
(D_i^{k+})^2 = \|AA'\|^2 + (D_j^{k+})^2 - 2\|AA'\|D_j^{k+}\cos\beta
\]

**Fig. 1.** The global process for IS selection.
Then
\[
\begin{align*}
D_k^r/C_0/C_16/C_17^2 \quad & = D_k^s/C_0/C_16/C_17^2
\end{align*}
\]

Therefore, it is impossible that the global process would be trapped into condition (ii) of (15) or the relative importance of the parametric values in (14) would be omitted. It inherently states that the optimal candidate via VCST metric is definitely the closest from the PIS and farthest from the NIS simultaneously.

4.3. Group decision making

As the WBF is used to vote on the rankings from all DMs, the IS which wins the highest amount of votes is eventually regarded as the optimal choice.

5. Experimental study

To fairly assess multiple ISs and identify the best one for large institutes or enterprises, the software named “Evaluator” is developed. It is based on our model and has been deployed with good performance in China Academy of Space Technology (CAST) to support MCGDM for upper-layer applications. It is designed as “Client/Server” architecture so that it could be conveniently delivered and installed. This section demonstrates how the “Evaluator” performs to make the most suitable option from a finite number of ISs by several DMs for data integration in CAST.

The IS selection task consists of three alternatives, four criteria and five DMs. The criteria cover accuracy, flexibility, cost and complaint with a brief instruction in Table 1.

Traditional “the 1-to-9 scale strategy” [26] has some defects when describing linguistic terms with inadequate smoothness for empirical comparison. If $A$ is assumed weakly more important than $B$, for instance, the proportion $A$ over $B$ is defined as “3:1”, namely

\[
\begin{align*}
\text{Type} & \quad \text{Accuracy ($C_1$)} & \quad \text{Flexibility ($C_2$)} & \quad \text{Cost ($C_3$)} & \quad \text{Complaint ($C_4$)} \\
\text{Type} & \quad \text{Positive} & \quad \text{Positive} & \quad \text{Negative} & \quad \text{Negative} \\
\text{Quantitative} & \quad \text{Positive} & \quad \text{Qualitative} & \quad \text{Quantitative} & \quad \text{Qualitative} \\
\end{align*}
\]

The mapping relationship between linguistic terms and their corresponding coverage of fuzzified values are displayed in Fig. 4. The related initial and normalization information is described in Tables 2 and 3.

Fig. 2. The distances from two alternatives to PIS and NIS.

Fig. 3. Linguistic terms and their corresponding trapezoidal fuzzy numbers.
perception of the problem when comparing and weighing each pair of criteria. Fig. 5 exemplifies a single DM’s opinion. After all the individuals’ results have been submitted and synchronized to the server via “Evaluator”, the experts could see the details converged in a single form in Fig. 6.

As there are four criteria in the comparison, the value of the Random Consistency Index (RI) equals 0.90 [26]. The observations of Consistency Index (CI), Consistency Ratio (CR) and Consistency situation are computed by CR = CI/RI in Fig. 7.

The weight vectors of AHP are calculated by computing the eigenvector of the comparison matrices offered in Fig. 6, while the weight vectors of EW are figured by Eq. 9. Then the composite weight vectors are aggregated according to the methods detailed in Section 4.2.1. The result is consequently collected in Fig. 8.

Multiplying the combined weight vector in Fig. 8 and the corresponding fuzzy normalized matrix in Table 3 for each individual DM respectively, the VCST between the alternatives and the ideal solutions is regarded as the distance to achieve the RC in Fig. 9 and individual preference ranking in Fig. 10.

The final decision is accomplished by a committee including five members with the predefined voting power V = (10, 9, 8, 7, 6), the result by WBF is shown in Fig. 11.

Therefore, the top-ranked alternative A3 is eventually identified as the optimal one.

Table 2
Initial information for each DMs.

<table>
<thead>
<tr>
<th>DMs</th>
<th>Alternative</th>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.99</td>
<td>M</td>
<td>500</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>0.95</td>
<td>L</td>
<td>300</td>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>0.90</td>
<td>H</td>
<td>450</td>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>0.95</td>
<td>VH</td>
<td>400</td>
<td>H</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0.75; 0.25, which is apparently unreasonable. So the crisp method is modified to “1-to-9 scale strategy by trapezoidal fuzzy number” via assigning the improved values relative to importance to the elements in comparison matrices. The tactic is illustrated in Tables 4 and 5 and Figs. 5–7.

Then a single-layer AHP is used during this scenario. Firstly, each DM should carefully judge the relative importance and make his decision according to their own experiences, knowledge and

Table 3
The normalized fuzzy information matrices for each DM.

<table>
<thead>
<tr>
<th>DMs</th>
<th>Normalized fuzzy information matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>([1.11, 1.10], [0.3333, 0.5625, 0.7333, 1.0], [0.60, 0.6, 0.6, 0.6], [0.3333, 0.5625, 0.7333, 1.0])</td>
</tr>
<tr>
<td>D2</td>
<td>([0.9596, 0.9596, 0.9596, 0.9596, 0.9596, 0.8125, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667])</td>
</tr>
<tr>
<td>D3</td>
<td>([0.9596, 0.9596, 0.9596, 0.9596, 0.9596, 0.8125, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667])</td>
</tr>
<tr>
<td>D4</td>
<td>([1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01, 1.01])</td>
</tr>
<tr>
<td>D5</td>
<td>([0.9596, 0.9596, 0.9596, 0.9596, 0.9596, 0.8125, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667, 0.6667])</td>
</tr>
</tbody>
</table>

Table 4
An expert-designed “1-to-9 scale strategy by trapezoidal fuzzy number”.

<table>
<thead>
<tr>
<th>Trapezoidal fuzzy number</th>
<th>Value of membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.1, 1.2)</td>
</tr>
<tr>
<td>x</td>
<td>((x - 1, x + 1, x, x + 1, x + 1, x + 1, x + 1)), (x = 2, 3, \ldots, 8)</td>
</tr>
<tr>
<td>9</td>
<td>((8, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9))</td>
</tr>
</tbody>
</table>

Table 5
The relative importance and value assigned of pairwise comparison.

<table>
<thead>
<tr>
<th>Relative importance of (x_i) over (x_j)</th>
<th>Traditional value assigned</th>
<th>Improved value assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally Important (EI)</td>
<td>1</td>
<td>5/5 = (1.1, 1.1)</td>
</tr>
<tr>
<td>Weakly More Important (WMI)</td>
<td>3</td>
<td>6/4 = (1.11, 1.13, 1.73)</td>
</tr>
<tr>
<td>Obviously More Important (OMI)</td>
<td>5</td>
<td>7/3 = (3/2, 1.73, 3.4)</td>
</tr>
<tr>
<td>Strongly More Important (SMI)</td>
<td>7</td>
<td>8/2 = (7/3, 1.73, 17/3)</td>
</tr>
<tr>
<td>Extremely More Important (EMI)</td>
<td>9</td>
<td>9/1 = (4, 17/3, 9/9)</td>
</tr>
</tbody>
</table>

Fig. 4. The linguistic variables for each criterion.
6. Comparative study and discussions

This section firstly elaborates the essentials of some classic MCDM methods, which are the prototypes of the related massive extensions, modifications and (or) advancements for both academic research and applications. Then the important procedural steps are compared with a number of recent approaches. Finally, two categories of group decision making methods are analyzed.

6.1. Rationale

ELECTRE is based on the study of outranking relations and exploitation notions of concordance by using pairwise comparisons among alternatives under each criterion separately. Concordance, discordance indexes and threshold values are used to analyze the outranking relations among the alternatives [27]. PROMETHEE also follows the outranking concept to rank the alternatives, combined with the ease of use and decreased complexity. Two complete preorders can be obtained by ranking the alternatives according to their incoming flow and their outgoing flow. The intersection of these two preorders yields the partial preorder of PROMETHEE I where incomparabilities are allowed. The ranking of the alternatives according to their net flow yields the complete preorder of PROMETHEE II [28]. Both of the two classic families of approaches intrinsically contain many outranking relations with heterogeneous expressions. Furthermore, the correlation between concordance and discordance indexes, as well as the ambiguousness on identifying the incoming flow and outgoing flow, complicates the decision making process on preference selection for DMs.

VIKOR focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the ranking index based on the particular measure of “closeness” to the “ideal” solution [29]. Similarly, the basic principle of TOPSIS is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution. TOPSIS has been proved to perform well even when the number of alternatives and criteria is too many due to its simplicity in perception and use. However, this technique is often criticized because of its inability to deal adequately with uncertainty and imprecision inherent in the process of mapping the perceptions of decision-makers [30–32]. Bellman and Zadeh [33] first introduced the theory of fuzzy sets in problems of MCDM as an effective approach to treat vagueness, lack of knowledge and ambiguity inherent in the human decision making process. TOPSIS has been expanded to deal MCDM with an uncertain decision matrix resulting in fuzzy TOPSIS.
which has successfully been applied to solve various MCDM problems. Therefore, our model uniformly transforms the initial data into trapezoidal fuzzy numbers so that it could perform in the fuzzy environment. Meanwhile, it makes use of the generic framework of TOPSIS, which needs to calculate the “proximity” to the PIS and the “remoteness” to the NIS, in terms of the sound logic that represents the rationale of human choice.

6.2. Normalization

Most of the classic models conduct the normalization in three ways: (i) vector normalization; (ii) linear normalization and its variants; (iii) non-monotonic normalization [34,35]. The vector normalization in original TOPSIS has been censured that the normalized value could be different for different evaluation unit of a particular criterion. Therefore, VIKOR uses the linear normalization and the normalized value does not depend on the assessment unit of a criterion [15,29]. All the three modes of normalization could be easily extended into the fuzzy scenario upon all the elements of each tuple for each fuzzy number. The specific linear normalization stated in Eqs. (7) and (8) is applied to our model in order to ensure compatibility between evaluation of objective criteria and linguistic ratings of subjective criteria as well as facilitating the computational problems where the different units of the attribute values present in the decision matrix.

Table 1: The pairwise comparison in AHP.

Table 2: AHP Consistency

Fig. 6. The pairwise comparison in AHP.

Fig. 7. The consistency check in AHP.
6.3. Distance metric

Various distance metrics have been attempted in similarity measurements between alternatives in MCDM [36–39]. Literature [40] directly measured the distance between two trapezoidal fuzzy numbers by a vertex method resulting in a crisp distance value and used the ideal and anti-ideal solutions to define a crisp overall score for each alternative. Several aggregation operators have been proposed and utilized with high time complexity [41,42]. Thus, the fuzzied values have been defuzzified before the calculation of distances in our model. Minkowski’s $L_p$ metric is the most prevalent one since it intuitively reflects the positional relations in n-dimensional space, including Manhattan Distance (MD, when $p = 1$), ED (when $p = 2$) and Chebyshev Distance (CD, when $p = \infty$) [43]. Gray Theory (GT) [44,45] is introduced to express the variation of situation for data sequences, scaling the similarity of the shapes about...
the corresponding curves. If the distances between candidates and NIS/PIS are measured by $MD$, $ED$, $CD GT$ and $VCST$ respectively with other unchanged constraints in our model, the decision made by $D_1$ are exemplified in Table 6.

The result in Table 6 explicitly states that $A_1$ with the highest ranking (obtaining the maximum value of RC, 0.6404) in $ED$ is merely the closest to PIS (0.0342) but not the farthest from the NIS (0.0610, lower than $A_3$’s 0.0726) simultaneously. The phenomenon confirms the defect described in Section 4.2.2.

### 6.4. Determining of weight

In this section, $x^j_i$ and $b^j_i$ still indicate the subjective and the objective weight vector of the $i$th DM respectively as discussed earlier.

#### 6.4.1. Subjective weight

In the Simple Multiple Attribute Rating Technique (SMART), participants are required to prioritize the importance of the changes in the criteria from the worst criteria levels to the best. Then 10 points are assigned to the least important criteria, and increasing numbers of points (without explicit upper limit) are assigned to the other criteria to address their relative importance to the least important criteria. The weights are calculated by normalizing the sum of the points to one. The idea of the improved version, that is SMARTER, exploits the centroid method [46].

In the Pair-Wise Comparison (PWC) method, DMs are presented a worksheet and are asked to score the relative importance of two

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Preference ranking affected by three distance metrics about $D_1$.</th>
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<tbody>
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<td>Metric</td>
<td>Alternative</td>
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Fig. 10. Individual preference ranking.

Fig. 11. Final group decision result by WBF.
criteria at a time. The scales can be various, for example, a scale of 0 (equal importance) to 3 (absolutely more important) is commonly adopted. The results are consolidated by adding up the scores obtained by each criterion when preferred to the criteria it is compared with. The results are then normalized to a total of 1. This weighting method provides a framework for comparing each criterion against all others, and helps to reveal the difference in importance between criteria. However, the consistency of participants’ preferences, especially, their transitivity is not allowed to be checked [47].

Nevertheless, in the context, AHP is preferred, because the IS selection problem could be dissociated as criteria and alternatives and the effect of each subject is demanded for measuring. The hierarchical strategy with an improved version of “the 1-to-9 scale strategy” provides DMs more information to make more subtle and reasonable decisions rather than the normal methods based on exact numbers of points and scores.

6.4.2. Objective weight

The principle of weight determination by the Standard Deviation (SD) is that the criterion obtaining the larger value of SD weights more significantly due to the higher degree of data variation and more information revealed [17,48]. The weight is characterized as

$$\phi_j^k = \delta_j^k / \sum_{j=1}^{n} \delta_j^k$$

(16)

where $\delta_j^k$ is the SD of the jth criterion for kth DM.

In Criteria Importance Through Inter-criteria Correlation (CRITIC), the weights derived incorporate both contrast intensity and conflict which are contained in the structure of the decision problem. The developed method is based on the analytical investigation of the evaluation matrix for extracting all information contained in the criteria. The amount of the information that the jth criterion is calculated by [48]

$$C_j^k = \delta_j^k \sum_{i=1 \neq j}^{n} (1 - r_{ij})$$

(17)

where $r_{ij}$ is the relation coefficient between the rth and jth criterion.

And the weight formula is given as

$$\beta_j^k = C_j^k / \sum_{j=1}^{n} C_j^k$$

(18)

Table 7 demonstrates how different methods of objective weight assignments impact the preference rating for $D_1$ without any other changed conditions in our model.

The result of Table 7 seems to suggest that objective weights derived by the EW are more significantly different to each other. This reflects the capability of the EW in providing the average intrinsic information generated by the performance of ISs. This would help the DM discriminate the most important criterion.

6.4.3. Weighting integration methods

Weighting integration methods have been applied to the evaluation and comparison of complex systems. These methods could roughly be classified into two groups of operations: multiplicative integration and additive integration [47].

The multiplicative synthesis is expressed as

$$w_j^k = \alpha_j^k \beta_j^k / \sum_{j=1}^{n} \alpha_j^k \beta_j^k, \quad j = 1, 2, \ldots, n$$

(19)

While the additive synthesis is expressed as

$$w_j^k = q \beta_j^k + (1-q) \phi_j^k, \quad j = 1, 2, \ldots, n, \quad 0 \leq q \leq 1$$

(20)

where $q$ is the additive integration coefficient.

Then both of the patterns of integration are comparably illustrated with our method about $D_1$’s preference order, for example, in Table 8.

6.5. Group decision methods

In extending TOPSIS to a group decision environment, the methods can be categorized into two varieties: mathematical methods and voting methods.
6.5.1. Mathematical methods

Most of the mathematical works aggregate the importance of the criteria and/or the rating of alternatives with respect to each criterion from individuals of the group via some specific operators [40–42,47]. Literature [49] endeavored to design a global TOPSIS after a number of individual fuzzy TOPSIS procedure, while others compared the effects of external aggregation and internal aggregation of group preferences [30].

6.5.2. Voting methods

Social preference functions are commonly based on voting rules. According to Copeland rule, the option with the largest number (i.e. with the highest ranking) in exhaustive pairwise comparison is the most recommended. The Borda rule is to select the option that on average stands highest in the voters’ rankings [50]. Under most scenarios, the diverse knowledge or comprehensibility of each group member cause different levels of professional decisions. Thus the personal authority needs to be considered, so that the Borda function with the assigned weights which is described in Section 3.2 actually reflects the rationale of committee choice.

7. Future work

As the decision making is a dynamic process, further studies should focus on users’ feedbacks and their influence on the next turn of decision, that is, how to develop a suitable strategy to manage the variable weights or voting power. Moreover, the difficulties of incomplete information resulting from various reasons, such as the inadequate knowledge or unintentional ignorance, are needed to consolidate the current research.

8. Conclusion

This paper primarily analyzes the characteristics of IS selection and points out the drawbacks of the current researches. To properly deal with these problems, a MCGDM model is elaborated. This new model relies on the chi-square test metric in a fuzzy TOPSIS fashion. It is composed of a committee-decision process with appropriate resolutions in linguistic terms, quantitative information, weight determination and RC computation using trapezoidal fuzzy numbers, and additive amalgamation of weight assignments respectively. In addition, the problem caused by the relative importance of distances to ideal/anti-ideal solutions in the original TOPSIS has been mathematically eliminated via VCST metric. After the illustrative example and comparative studies, it finally demonstrates that the model could provide an effective framework for ranking competing alternatives of IS. And it could be easily adaptable and extended to other applications.

Acknowledgements

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References