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ScienceDirect
Applied Mathematics and Computation 184 (2007) 316–321
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Using AHP and L.P. for choosing the best alternatives based the gap analysis

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Abstract

Analytic Hierarchy Process is a well-known method that is only able to rank different alternatives according to their AHP weights, but it does not determine the necessary threshold for discriminate between acceptable and unacceptable alternatives. In the other words, it does not lead the DM to choose the alternatives that have higher weights than the threshold. In this paper, we introduce a new method by which we are able to accept/reject alternatives based on AHP weights. The proposed method determines the necessary threshold by comparing the anticipated status with the desired status of alternatives and using LP and AHP combination for it.

Keywords: Linear programming; AHP; Gap analysis

1. Introduction

Analytic hierarchy process is a well-known method for solving decision-making problems. In that method, pairwise comparisons are performed by the decision-maker (DM) and then the pairwise comparison matrix and the eigenvector are derived to specify the weights of each parameter in the problem. The weights guide the DM in choosing the superior alternative.

If it is possible to select more than one choice, the DM may have some difficulties. Consider the following example:

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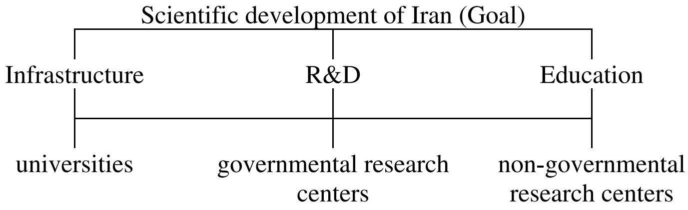
doi:10.1016/j.amc.2006.05.178

For scientific development of Iran, this hierarchy was drawn:



This example is a simplified of a real case presented in [4] that the lowest level of the hierarchy is truncated for simplification.

According to above hierarchy, the alternatives presented in the lower level are not mutually exclusive and the DM could choose more than one alternative (may, all of them). So, the DM weights the alternatives by AHP and ranks them accordingly, but he is not able to accept/reject them. This article attempts to overcome this shortcoming. This method allows the DM to choose the alternatives that require the minimum change for achievement of acceptable threshold of zero level Goal (scientific development of Iran).

2. Review of literature

In this section, we review papers dealing with application of AHP in planning, anticipating and optimizing. Then the shortcomings of the above methods are delineated and a new process is introduced to fulfill these deficiencies.

2.1. Application of AHP in planning and anticipating

Saaty not only introduced AHP [1], but also he utilized AHP in planning and anticipating for the first time [2]. He employed forward and backward process to determine logical future and then find promising control policies to attain the desire future.

In the other words, Saaty's approach attempted to reduce the gap between logical future and desired future by choosing the appropriate strategies. The most famous example in the field is the anticipating of higher education in US by Saaty [2].

Of course, saaty has anticipated price of oil and etc in that book.

2.2. Application of AHP in optimization

Application of AHP in mathematical optimization and operational research is widely practiced and the weights gained by the AHP method are frequently employed as the coefficient of the objective function in linear and integer programming. The weights have also been applied for ranking multiple objectives in goal programming.

Now a number of papers dealing with the subject are reviewed:

"Ramanathan" and "Ganesh" employ AHP for a resource allocation problem [5]. In that paper, the weight of each decision variable gained by AHP is used as a coefficient of that variable in the objective function. Also, the ratio of B/C (benefit/cost) of each decision variable may be used as the coefficient of that variable in the objective function. B/c is compute by two separate hierarchies of benefits and costs in AHP.

It's noticeable that the objective function is of maximization kind and the constraints are the same as the constraint in any other resource allocation problem.

"Ghodsypour" and "O'brien" introduce decision support systems for supplier selection in three separate papers:

In the first paper, the AHP weights are used to reduce the number of suppliers in a JIT/TQM environment [6].

In the next paper, AHP is used along with goal programming for multiple sourcing with discount prices [7].
In the last paper, AHP is used along with linear programming for supplier selection [8].
In another paper, "Stern" and "Mehrez" utilize AHP along with DEA (Date Envelopment Analysis) for ranking decision-making units [9]. The authors claim that their new methodology does not replace any of the two methods but it does eliminate the weaknesses of the named methods. That new methodology is also able to compensate for the lack of solid mathematical theories used in utility theory of the AHP method.

Many other papers regarding AHP and mathematical optimization (L.P) were reviewed by the authors of this paper and a common point was reached: In all these papers except [3], AHP weights were employed as coefficients of the objective function and they were not used as the coefficients of the decision variables in constraints (technological coefficients).

Reference [3] applies AHP weights in the constraints but the function of AHP weights for each level differs from technological coefficients. Because (as Goal programming method) operates on constraints and objectives similarly.

3. Introducing the new method

In AHP method, weights of elements of each level of hierarchy are computed respect to the element of the upper hierarchy level. This method leads to computation of the weights of elements of the lowest hierarchy level with respect to the primary zero level Goal.

For ranking alternatives, our method employs a new factor in place of weights of alternatives. This new factor can be identified as the gap between the anticipated status and the desired status.

The following steps are required in our method:

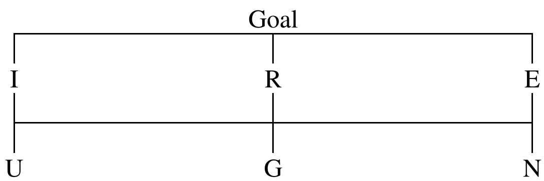
3.1. Determination the weights of different hierarchy levels

At first, it's necessary to compute the weights of each level with respect to the upper levels. To understand the issue, attention to previous example again:

Zero level of hierarchy (Goal): Scientific development of Iran

First level of hierarchy (criteria): Education (E), R&D (R), Infrastructure (I)

Second level of hierarchy (alternatives): Universities (U), Governmental research centers (G), Non-governmental research centers (N)



In the research conducted for determining scientific development policies of Iran [4], 43 experts were questioned by the pairwise comparison about the above hierarchy. The results were extracted by taking geometric average. If assume:

W_{XY} = Weight of parameter X respect to parameter Y.

The results are:

$$W_{\text{E.Goal}} = 0.44 \quad W_{\text{R.Goal}} = 0.25 \quad W_{\text{I.Goal}} = 0.31,$$
$$W_{\text{N.E}} = 0.20 \quad W_{\text{N.R}} = 0.24 \quad W_{\text{N.I}} = 0.32,$$
$$W_{\text{G.E}} = 0.12 \quad W_{\text{G.R}} = 0.35 \quad W_{\text{G.I}} = 0.45,$$
$$W_{\text{U.E}} = 0.68 \quad W_{\text{U.R}} = 0.41 \quad W_{\text{U.I}} = 0.23.$$

Therefore, the weight of alternatives respect to Goal is:

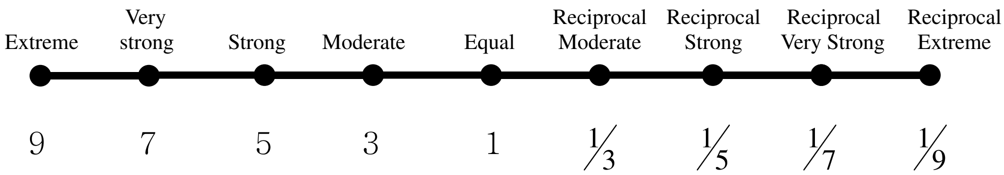
$$W_{\text{U.Goal}} = W_{\text{U.E}} \cdot W_{\text{E.Goal}} + W_{\text{U.R}} \cdot W_{\text{R.Goal}} + W_{\text{U.I}} \cdot W_{\text{I.Goal}} = (0.44)(0.68) + (0.25)(0.41) + (0.31)(0.23) = 0.47,$$
$$W_{\text{G.Goal}} = (0.44)(0.12) + (0.25)(0.35) + (0.31)(0.45) = 0.28,$$
$$W_{\text{N.Goal}} = (0.44)(0.20) + (0.25)(0.24) + (0.31)(0.32) = 0.25.$$

3.2. Evaluation of anticipated status of criteria and alternatives

Now we shall anticipate the status of lower levels of hierarchy in a specific time horizon relative to the present situation. Then we evaluate their effects on the upper levels of hierarchy, for example:

x_1 = the anticipated status of factor X with respect to its present status.

A routine pairwise comparison is conducted to compute the valve of r_1. In this case, two distinct elements are not compared to each other, but the present and future status of one single element are compared and weighted according to a scale of 1/9–9:



Now the value of u_1 and n_1 are gained by questioning the DMs. The result is compared to the aforementioned scale and the future status of U, G and N is determined give the present status continues.

Now, n_1 is computed equal 5 and g_1 is 1/3 and u_1 is 1. It means that the future status of criterion Non-governmental research centers will "strong" to the its present status and the present status of Governmental research centers will be "moderate" relative to its future status. The present and future statuses of Universities are equal.

The anticipated of criteria E, I and R and the Goal are computed accordingly:

$$e_1 = W_{\text{U.E}} \cdot u_1 + W_{\text{G.E}} \cdot g_1 + W_{\text{N.E}} \cdot n_1 = 0.68(1) + 0.12(1/3) + 0.20(5) = 1.72,$$
$$r_1 = W_{\text{U.R}} \cdot u_1 + W_{\text{G.R}} \cdot g_1 + W_{\text{N.R}} \cdot n_1 = 0.41(1) + 0.35(1/3) + 0.24(5) = 1.61,$$
$$i_1 = W_{\text{U.I}} \cdot u_1 + W_{\text{G.I}} \cdot g_1 + W_{\text{N.I}} \cdot n_1 = 0.23(1) + 0.45(1/3) + 0.32(5) = 1.98,$$
$$\text{goal}_1 = W_{\text{E.Goal}} \cdot e_1 + W_{\text{R.Goal}} \cdot r_1 + W_{\text{I.Goal}} \cdot i_1 = 0.44(1.72) + 0.25(1.61) + 0.31(1.98) = 1.77.$$

Referring to the aforementioned scale, it's obvious that if the current trend is continued, the status of the Goal will not be improved seriously in the horizon of planning.

It's important to notice that the above relations are similar to regression equations. Because the values of upper level variables (dependent variables) are functions of the values of lower level Variables (independent variables).

3.3. Determination the desired status of criteria and alternatives

In this step, we noticed that the status of our Goal in the future relative to today had a little improvement.

Now, if this degree of change satisfies the decision-maker, no further selection of alternatives and/or changes in the current alternatives will be required. But if the degree of improvement is undesirable, the improvement in lower parameters of hierarchy must be increased ever more than the anticipated amount. However the main concern here is determination of the amount of change which is required. In the other words, we need to determine the degree of change needed in each factor of lower level of hierarchy to reach the desired improvement in the upper level factors. To answer this question, we must determine the desired status of the Goal logically and realistically. Therefore we must question the DMs again to determine the value of Goal_2 according to the following definition:

$$x_2 = \text{relation of the desired status to the present status of factor } X \text{ (in a specific time horizon).}$$

It's natural to believe that the DM would determine the desired Goal status logically and realistically. Otherwise, it might be supposed that the desired status of Goal has been extremely improved that we know it is impossible. It should be noticed that the DM wants the least degree of change to gain the desired Goal status and he does not necessarily require the non-essential or impossible improvement. To reach this mean, the DM must conduct a pairwise comparison between the future Goal status and the current Goal status.

Now based on optimization theory, elements E, R and I must be changed to reach the desired goal status.

To determine the required degree of change in a lower level of hierarchy, the authors utilize linear programming technique. To do so, the authors employees the coefficients of the AHP method gained in step one (Section 3.1) as the technological coefficients of the constraints. The authors also added lower and upper- bound constraints on changes in criteria E, R and I wherever necessary.

According to result gathered from the experts, the status of Education and Infrastructure factors in the horizon of planning with respect to the current status is "strong", but for the R&D factor, it is "moderate".

Also the expert agreed that the status of the Goal (Scientific development of Iran) with respect to the current status must be at least "strong". Therefore the following L.P problem is gained:

$$\begin{aligned}
&\text{Min} \quad e_2 + r_2 + i_2,\\
&\text{s.t.}\\
&\quad 0.44\ e_2 + 0.25\ r_2 + 0.31\ i_2 \geqslant 5 \text{ (constraint for Goal improvement)}\\
&\quad 1 \leqslant e_2 \leqslant 5,\\
&\quad 1 \leqslant r_2 \leqslant 3,\\
&\quad 1 \leqslant i_2 \leqslant 5.
\end{aligned}$$

The above model is infeasible. It's meaning that Iran cannot improve the Goal (Scientific development) such its status of planning horizon (5 years) with respect to current status becomes "strong". Hence the right hand side of the first constraint of above model is reduced to 4.5. The solution of new problem is:

$$e_2 = 5 \quad i_2 = 5 \quad r_2 = 3.$$

These results are the minimum value necessary for the criteria improvement to assure achievement of the modified desired status of the Goal (4.5). Now a new problem concerning the achievement of the above values of criteria is arises. To deal with this problem, the following model determines the extent of increase necessary for the lower level:

$$\begin{aligned}
&\text{Min} \quad u_2 + g_2 + n_2,\\
&\text{s.t.}\\
&\quad 0.68\ u_2 + 0.12\ g_2 + 0.20\ n_2 \geqslant 5 \text{ (constraint for criteria E)},\\
&\quad 0.41\ u_2 + 0.35\ g_2 + 0.24\ n_2 \geqslant 3 \text{ (constraint for criteria R)},\\
&\quad 0.23\ u_2 + 0.45\ g_2 + 0.32\ n_2 \geqslant 5 \text{ (constraint for criteria I)},\\
&\quad 1 \leqslant u_2 \leqslant 5,\\
&\quad 1 \leqslant g_2 \leqslant 7,\\
&\quad 1 \leqslant n_2 \leqslant 5.
\end{aligned}$$

Solution is: $g_2 = 5\ n_2 = 5\ u_2 = 5$.

We shall notice the following points:

- These results show that if Iran desires to reach the agreed level of scientific development by minimum change in the status of universities and governmental and non-governmental research centers, it has to undertake the solution of the above model.

– We assume that values of criteria and alternatives shall not decrease since they are positive alternatives with respect to our desired Goal. (Therefore the lower-bounds of them are 1, not 1/9).
– While we are increasing criteria and alternatives, we may face resource constraints; therefore these constraints must be added to the constraints of the above L.P problem.
– Due to lack of information, the objective function coefficients are equaled to one. But we could justify the objective function coefficients according to the cost required for their improvement.

3.4. Selection of acceptable alternatives

To determine the priority of alternatives we do as following:

The ratio of the desired status to the anticipated status (x_2/x_1) for each factor of lower level is computed. Each factor (alternative) that earns a higher ratio, gains a higher priority.

This ratio for U, G and N is 5, 15, and 1 respectively. Therefore the extracted policy for scientific development of Iran can prioritized as follows: (1) *Governmental research centers* (2) *Universities.*

For the alternatives with values less than or equal to one (i.e., non-*governmental research centers),* the current trend will achieve the desired goal and no further investment is required.

4. Conclusion

The proposed Method attempts to remove deficiencies in literature with regard to combination of AHP and optimization. The following are the most significant accomplishments of that method:

– Utilization of AHP weights as technological coefficients of the L.P constraints (differ in comparison to the only available reference [3]).
– Presentation of a new model for selecting excellent alternatives. It's noticeable that this method not only chooses the proper alternatives but also ranks them respectively.
– Application of pairwise comparisons and the eigenvector. It's mentionable that the eigenvector values are regarded as the inputs of the problem and the weights of the parameters are considered as the outputs of the problem.

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