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An algorithmic method to extend TOPSIS for decision-making problems with interval data

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Abstract

In this paper, from among multi-criteria models in making complex decisions and multiple attribute models for the most preferable choice, technique for order preference by similarity ideal solution (TOPSIS) approach has been dealt with. In some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals. Therefore, the aim of this paper is to extend the TOPSIS method for decision-making problems with interval data. By extension of TOPSIS method, an algorithm to determine the most preferable choice among all possible choices, when data is interval, is presented. Finally, an example is shown to highlight the procedure of the proposed algorithm at the end of this paper.

Keywords: MCDM; TOPSIS; Interval data; Positive ideal solution; Negative ideal solution

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1. Introduction

Decision-making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. Multiple criteria decision making may be considered as a complex and dynamic process including one managerial level and one engineering level [4]. The managerial level defines the goals, and chooses the final "optimal" alternative. The multi-criteria nature of decisions is emphasized at this managerial level, at which public officials called "decision makers" have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are "off line" from the optimization procedure done at the engineering level. A MCDM problem can be concisely expressed in matrix format as

	C_1	C_2	...	C_n
A_1	x_{11}	x_{12}	...	x_{1n}
A_2	x_{21}	x_{22}	...	x_{2n}
A_m	x_{m1}	x_{m2}	...	x_{mn}

$$W = [w_1, w_2, \ldots, w_n]$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j.

The main steps of multiple criteria decision making are the following:

(a) Establishing system evaluation criteria that relate system capabilities to goals.
(b) Developing alternative systems for attaining the goals (generating alternatives).
(c) Evaluating alternatives in terms of criteria(the values of the criterion functions).
(d) Applying a normative multi-criteria analysis method.
(e) Accepting one alternative as "optimal"(preferred).
(f) If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (d), a decision maker should express his/her preferences in terms of the relative importance of criteria, and one approach is to introduce criteria weights. This weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure).

In classical MCDM methods, the ratings and the weights of the criteria are known precisely [5,6]. A survey of the methods has been presented in Hwang and Yoon [6]. Technique for order performance by similarity to ideal solution (TOPSIS) [7], one of known classical MCDM method, was first developed by Hwang and Yoon [6] for solving a MCDM problem. It based upon the concept that the chosen alternative should have the shorter distance from the positive ideal solution and the farthest from the negative ideal solution. A similar concept has also been pointed out by Zeleny [8]. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. Recently, Abo-sinna and Amer [1] extend TOPSIS approach to solve multi-objective nonlinear programming problems. Chen [2] extends the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in fuzzy environment.

Under many conditions, exact data are inadequate to model real-life situations. For example, human judgements including preferences are often vague and cannot estimate his preference with an exact numerical data, there for these data may be have some structures such as bounded data, ordinal data, interval data, and fuzzy data. In this paper, by considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore, we extended the concept of TOPSIS to develop a methodology for solving MCDM problems with interval data.

The rest of the paper is organized as follows: next section briefly introduces the original TOPSIS method. In Section 3, first, we introduce MCDM problems with interval data, then, we present an algorithm to extend TOPSIS to deal with interval data. In Section 4 we illustrate our proposed algorithmic method with an example. The final section concludes.

2. TOPSIS method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang [3], with reference to Hwang and Yoon [6]. TOPSIS is a multiple criteria method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The procedure of TOPSIS can be expressed in a series of steps:

(1) Calculate the normalized decision matrix. The normalized value n_{ij} is calculated as

$$n_{ij} = x_{ij} \Big/ \sqrt{\sum_{j=1}^{m} x_{ij}^2}, \quad j = 1, \ldots, m, \;\; i = 1, \ldots, n.$$

(2) Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as

$$v_{ij} = w_i n_{ij}, \quad j = 1, \ldots, m, \;\; i = 1, \ldots, n,$$

where w_i is the weight of the ith attribute or criterion, and $\sum_{i=1}^{n} w_i = 1$.

(3) Determine the positive ideal and negative ideal solution.

$$A^+ = \{v_1^+, \ldots, v_n^+\} = \left\{ \left(\max_j v_{ij} | i \in I \right), \left(\min_j v_{ij} | i \in J \right) \right\},$$

$$A^- = \{v_1^-, \ldots, v_n^-\} = \left\{ \left(\min_j v_{ij} | i \in I \right), \left(\max_j v_{ij} | i \in J \right) \right\},$$

where I is associated with benefit criteria, and J is associated with cost criteria.

(4) Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$$d_j^+ = \left\{ \sum_{i=1}^{n} (v_{ij} - v_i^+)^2 \right\}^{\frac{1}{2}}, \quad j = 1, \ldots, m.$$

Similarly, the separation from the negative ideal solution is given as

$$d_j^- = \left\{ \sum_{i=1}^{n} (v_{ij} - v_i^-)^2 \right\}^{\frac{1}{2}}, \quad j = 1, \ldots, m.$$

(5) Calculate the relative closeness to the ideal solution. The relative closeness of the alternative A_j with respect to A^+ is defined as

$$R_j = d_j^- / (d_j^+ + d_j^-), \quad j = 1, \ldots, m.$$

Since $d_j^- \geqslant 0$ and $d_j^+ \geqslant 0$, then, clearly, $R_j \in [0, 1]$.

(6) Rank the preference order. For ranking DMUs using this index, we can rank DMUs in decreasing order.

The basic principle of the TOPSIS method is that the chosen alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal solution. The TOPSIS method introduces two "reference" points, but it does not consider the relative importance of the distances from these points.

3. TOPSIS method with interval data

Considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore, now we try to extend TOPSIS for these interval data. Suppose $A_1, A_2, \ldots, A_m$ are m possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j and is not known exactly and only we know $x_{ij} \in [x_{ij}^L, x_{ij}^U]$. A MCDM problem with interval data can be concisely expressed in matrix format as

	C_1	C_2	...	C_n
A_1	$[x_{11}^L, x_{11}^U]$	$[x_{12}^L, x_{12}^U]$	...	$[x_{1n}^L, x_{1n}^U]$
A_2	$[x_{21}^L, x_{21}^U]$	$[x_{22}^L, x_{22}^U]$	...	$[x_{2n}^L, x_{2n}^U]$
A_m	$[x_{m1}^L, x_{m1}^U]$	$[x_{m2}^L, x_{m2}^U]$	...	$[x_{mn}^L, x_{mn}^U]$

$$W = [w_1, w_2, \ldots, w_n]$$

where w_j is the weight of criterion C_j.

3.1. The proposed algorithmic method

A systematic approach to extend the TOPSIS to the interval data is proposed in this section.

First we calculate the normalized decision matrix as follows:

The normalized values $\bar{n}_{ij}^L$ and $\bar{n}_{ij}^U$ are calculated as

$$\bar{n}_{ij}^L = x_{ij}^L \Bigg/ \sqrt{\sum_{j=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}, \quad j = 1, \ldots, m, \ \ i = 1, \ldots, n, \tag{1}$$

$$\bar{n}_{ij}^U = x_{ij}^U \Bigg/ \sqrt{\sum_{j=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}, \quad j = 1, \ldots, m, \ \ i = 1, \ldots, n. \tag{2}$$

Now interval $[\bar{n}_{ij}^L, \bar{n}_{ij}^U]$ is normalized of interval $[x_{ij}^L, x_{ij}^U]$. The normalization method mentioned above is to preserve the property that the ranges of normalized interval numbers belong to $[0, 1]$.

Considering the different importance of each criterion, we can construct the weighted normalized interval decision matrix as

$$\bar{v}_{ij}^L = w_i \bar{n}_{ij}^L, \quad j = 1, \ldots, m, \ \ i = 1, \ldots, n, \tag{3}$$

$$\bar{v}_{ij}^U = w_i \bar{n}_{ij}^U, \quad j = 1, \ldots, m, \ \ i = 1, \ldots, n, \tag{4}$$

where w_i is the weight of the ith attribute or criterion, and $\sum_{i=1}^{n} w_i = 1$.

Then, we can identify positive ideal solution and negative ideal solution as

$$\overline{A}^{+} = \{\bar{v}_1^{+}, \ldots, \bar{v}_n^{+}\} = \left\{\left(\max_j \bar{v}_{ij}^{U} | i \in I\right), \left(\min_j \bar{v}_{ij}^{L} | i \in J\right)\right\}, \tag{5}$$

$$\overline{A}^{-} = \{\bar{v}_1^{-}, \ldots, \bar{v}_n^{-}\} = \left\{\left(\min_j \bar{v}_{ij}^{L} | i \in I\right), \left(\max_j \bar{v}_{ij}^{U} | i \in J\right)\right\}, \tag{6}$$

where I is associated with benefit criteria, and J is associated with cost criteria.

The separation of each alternative from the positive ideal solution, using the n-dimensional Euclidean distance, can be currently calculated as

$$\bar{d}_j^{+} = \left\{\sum_{i \in I}\left(\bar{v}_{ij}^{L} - \bar{v}_i^{+}\right)^2 + \sum_{i \in J}\left(\bar{v}_{ij}^{U} - \bar{v}_i^{+}\right)^2\right\}^{\frac{1}{2}}, \quad j = 1, \ldots, m. \tag{7}$$

Similarly, the separation from the negative ideal solution can be calculated as

$$\bar{d}_j^{-} = \left\{\sum_{i \in I}\left(\bar{v}_{ij}^{U} - \bar{v}_i^{-}\right)^2 + \sum_{i \in J}\left(\bar{v}_{ij}^{L} - \bar{v}_i^{-}\right)^2\right\}^{\frac{1}{2}}, \quad j = 1, \ldots, m. \tag{8}$$

A closeness coefficient is defined to determine the ranking order of all alternatives once the $\bar{d}_j^{+}$ and $\bar{d}_j^{-}$ of each alternative A_j has been calculated. The relative closeness of the alternative A_j with respect to $\overline{A}^{+}$ is defined as

$$\overline{R}_j = \bar{d}_j^{-} / (\bar{d}_j^{+} + \bar{d}_j^{-}), \quad j = 1, \ldots, m. \tag{9}$$

Obviously, an alternative A_j is closer to the $\overline{A}^{+}$ and farther from $\overline{A}^{-}$ as $\overline{R}_j$ approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

3.2. The presented algorithm

In sum, an algorithm to determine the most preferable choice among all possible choices, when data is interval, with extended TOPSIS approach is given in the following:

- *Step 1*: Establishing system evaluation criteria that relate system capabilities to goals (identification the evaluation criteria).
- *Step 2*: Developing alternative systems for attaining the goals (generating alternatives).
- *Step 3*: Evaluating alternatives in terms of criteria (the values of the criterion functions which are intervals).

Step 4: Identifying the weight of criteria.

Step 5: Construct the interval decision matrix and the interval normalized decision matrix (using the formulas (1) and (2)).

Step 6: Construct the interval weighted normalized decision matrix (using the formulas (3) and (4)).

Step 7: Determine positive ideal solution and negative ideal solution(identification of $\overline{A}^{+}$ and $\overline{A}^{-}$, using the formulas (5) and (6)).

Step 8: Calculate the separation of each alternative from positive ideal solution and negative ideal solution, respectively (identification of $\bar{d}_j^{+}$ and $\bar{d}_j^{-}$, using the formulas (7) and (8)).

Step 9: Calculate the relative closeness of each alternative to positive ideal solution (identification of $\overline{R}_j$, using the formula (9)).

Step 10: Rank the preference order of all alternatives according to the closeness coefficient.

4. Numerical example

In this section, we work out a numerical example to illustrate the TOPSIS method for decision-making problems with interval data. A case study of comparing 15 bank branches $(A_1, A_2, \ldots, A_{15})$ in Iran was conducted to examine the applicability of this TOPSIS method with interval data. Four financial ratios

Table 1
The Interval decision matrix of 15 alternatives

	C_1		C_2		C_3		C_4	
	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	x_{3j}^L	x_{3j}^U	x_{4j}^L	x_{4j}^U
A_1	500.37	961.37	2 696 995	3 126 798	26 364	38 254	965.97	6957.33
A_2	873.7	1775.5	1 027 546	1 061 260	3791	50308	2285.03	3174
A_3	95.93	196.39	1 145 235	1 213 541	22 964	26 846	207.98	510.93
A_4	848.07	1752.66	390 902	395 241	492	1213	63.32	92.3
A_5	58.69	120.47	144 906	165 818	18 053	18 061	176.58	370.81
A_6	464.39	955.61	408 163	416 416	40 539	48 643	4654.71	5882.53
A_7	155.29	342.89	335 070	410 427	33 797	44 933	560.26	2506.67
A_8	1752.31	3629.54	700 842	768 593	1437	1519	58.89	86.86
A_9	244.34	495.78	641 680	696 338	11 418	24 108	1070.81	2283.08
A_{10}	730.27	1417.11	453 170	481 943	2719	2955	375.07	559.85
A_{11}	454.75	931.24	309 670	342 598	2016	2617	936.62	1468.45
A_{12}	303.58	630.01	286 149	317 186	14 918	27 070	1203.79	4335.24
A_{13}	658.81	1345.58	321 435	347 848	6616	8045	200.36	399.8
A_{14}	420.18	860.79	618 105	835 839	24 425	40 457	2781.24	4555.42
A_{15}	144.68	292.15	119 948	120 208	1494	1749	282.73	471.22

Table 2
The Interval normalized decision matrix

	C_1		C_2		C_3		C_4	
	$\bar{n}^L_{1j}$	$\bar{n}^U_{1j}$	$\bar{n}^L_{2j}$	$\bar{n}^U_{2j}$	$\bar{n}^L_{3j}$	$\bar{n}^U_{3j}$	$\bar{n}^L_{4j}$	$\bar{n}^U_{4j}$
A_1	0.0856	0.1645	0.5176	0.6001	0.1974	0.2865	0.0706	0.5086
A_2	0.1495	0.3038	0.1972	0.2037	0.0283	0.3768	0.1670	0.2320
A_3	0.0164	0.0336	0.2198	0.2329	0.1720	0.2010	0.0152	0.0373
A_4	0.1451	0.2999	0.0750	0.0758	0.0036	0.0090	0.0046	0.0067
A_5	0.0100	0.0206	0.0278	0.0318	0.1352	0.1352	0.0129	0.0271
A_6	0.0794	0.1635	0.0783	0.0799	0.3036	0.3643	0.3403	0.4300
A_7	0.0265	0.0586	0.0643	0.0787	0.2531	0.3365	0.0409	0.1832
A_8	0.2999	0.6211	0.1345	0.1475	0.0107	0.0113	0.0043	0.0063
A_9	0.0418	0.0848	0.1231	0.1336	0.0855	0.1805	0.0782	0.1669
A_{10}	0.1249	0.2425	0.0869	0.0925	0.0203	0.0221	0.0274	0.0409
A_{11}	0.0778	0.1593	0.0594	0.0657	0.0151	0.0196	0.0684	0.1073
A_{12}	0.0519	0.1078	0.0549	0.0608	0.1117	0.2027	0.0880	0.3169
A_{13}	0.1127	0.2302	0.0616	0.0667	0.0495	0.0602	0.0146	0.0292
A_{14}	0.0719	0.1473	0.1186	0.1604	0.1829	0.3030	0.2033	0.3330
A_{15}	0.0247	0.0500	0.0230	0.0230	0.0111	0.0131	0.0206	0.0344

Table 3
The Interval weighted normalized decision matrix

	C_1		C_2		C_3		C_4	
	$\bar{v}^L_{1j}$	$\bar{v}^U_{1j}$	$\bar{v}^L_{2j}$	$\bar{v}^U_{2j}$	$\bar{v}^L_{3j}$	$\bar{v}^U_{3j}$	$\bar{v}^L_{4j}$	$\bar{v}^U_{4j}$
A_1	0.0107	0.0205	0.06471	0.07502	0.0246	0.0358	0.0088	0.0635
A_2	0.0186	0.0379	0.0246	0.0254	0.0035	0.0471	0.0208	0.0290
A_3	0.0020	0.0042	0.0274	0.0291	0.0215	0.0251	0.0019	0.0046
A_4	0.0181	0.0374	0.0093	0.0094	0.0004	0.0011	0.0005	0.0008
A_5	0.0012	0.0025	0.0034	0.0039	0.0169	0.0169	0.0016	0.0033
A_6	0.0099	0.0204	0.0097	0.0099	0.0379	0.0455	0.0425	0.0537
A_7	0.0033	0.0073	0.0080	0.0098	0.0316	0.0420	0.0051	0.0229
A_8	0.0374	0.07766	0.0168	0.0184	0.0013	0.0014	0.0005	0.0007
A_9	0.0052	0.0106	0.0153	0.0167	0.0106	0.0225	0.0097	0.0208
A_{10}	0.0156	0.0303	0.0108	0.0115	0.0025	0.0027	0.0034	0.0051
A_{11}	0.0097	0.0199	0.0074	0.0082	0.0018	0.0024	0.0085	0.0134
A_{12}	0.0064	0.0134	0.0068	0.0076	0.0139	0.0253	0.0110	0.0396
A_{13}	0.0140	0.0287	0.0077	0.0083	0.0061	0.0075	0.0018	0.0036
A_{14}	0.0089	0.0184	0.0148	0.0200	0.0228	0.0378	0.0254	0.0416
A_{15}	0.0030	0.0062	0.0028	0.0028	0.0013	0.0016	0.0025	0.0043

Table 4
Distance of each alternative from the positive ideal solution

$\bar{d}^+_1$	$\bar{d}^+_2$	$\bar{d}^+_3$	$\bar{d}^+_4$	$\bar{d}^+_5$	$\bar{d}^+_6$	$\bar{d}^+_7$	$\bar{d}^+_8$	$\bar{d}^+_9$	$\bar{d}^+_{10}$	$\bar{d}^+_{11}$	$\bar{d}^+_{12}$	$\bar{d}^+_{13}$	$\bar{d}^+_{14}$	$\bar{d}^+_{15}$
0.063	0.087	0.082	0.108	0.099	0.071	0.090	0.123	0.088	0.102	0.099	0.093	0.103	0.077	0.105

Table 5
Distance of each alternative from the negative ideal solution

$\bar{d}_1^+$	$\bar{d}_2^-$	$\bar{d}_3^-$	$\bar{d}_4^-$	$\bar{d}_5^-$	$\bar{d}_6^-$	$\bar{d}_7^-$	$\bar{d}_8^-$	$\bar{d}_9^-$	$\bar{d}_{10}^-$	$\bar{d}_{11}^-$	$\bar{d}_{12}^-$	$\bar{d}_{13}^-$	$\bar{d}_{14}^-$	$\bar{d}_{15}^-$
0.122	0.083	0.083	0.059	0.078	0.097	0.088	0.043	0.079	0.062	0.069	0.085	0.064	0.089	0.074

Table 6
Closeness coefficient and ranking

Alternatives	$\overline{R}_j$	Rank
A_1	0.659352269	1
A_2	0.48911912	6
A_3	0.505445965	4
A_4	0.355647462	14
A_5	0.440416214	9
A_6	0.57596554	2
A_7	0.494120485	5
A_8	0.258369495	15
A_9	0.473078522	8
A_{10}	0.379417215	13
A_{11}	0.409684296	11
A_{12}	0.477519948	7
A_{13}	0.38233013	12
A_{14}	0.538211999	3
A_{15}	0.415388351	10

$(C_1, C_2, \ldots, C_4)$ were identified as the evaluation criteria for these banks. (Note that Steps 1, Step 2 and Step 3 are done).

Step 4: Suppose that the vector of corresponding weight of each criteria is as follows:

$$W = [0.125, 0.125, 0.125, 0.125].$$

Step 5: The interval decision matrix and interval normalized decision matrix are shown in Tables 1 and 2, respectively.

Step 6: The interval Weighted normalized decision matrix is as Table 3.

Step 7: The positive ideal solution and the negative ideal solution are then determined as:

$$\overline{A}^+ = [0.001255569, 0.075023386, 0.047105492, 0.063583238],$$
$$\overline{A}^- = [0.077647586, 0.002877994, 0.00046068, 0.000538197].$$

Step 8: A comparison between the normalized performance ratings of each alternative A_i and $\overline{A}^+$ by Eq. (7) (that is shown in Table 4), and between that of A_i and $\overline{A}^-$ by Eq. (8) (that is shown in Table 5) would indicate how the bank is performing as compared with the best performance and the worst performance of all the bank branches with respect to each criterion.

Step 9: Calculate the relative closeness of each alternative to positive ideal solution as Table 6.

Step 10: According to the closeness coefficient, ranking the preference order of all alternatives is as Table 6.

5. Conclusion

Considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, their values are considered as intervals, therefore, in this paper TOPSIS for interval data has been extended. Also, an algorithm to determine the most preferable choice among all possible choices, when data is interval, is presented. In this algorithmic method, as well as considering the distance of a DMU from the positive ideal solution, its distance from the negative ideal solution is also considered. That is to say, the less the distance of the DMU under evaluation from the positive ideal solution and the more its distance from the negative ideal solution, the better its ranking.

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