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Nadir compromise programming: A model for optimization of multi-objective portfolio problem

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ABSTRACT

In problem of portfolio selection, financial Decision Makers (DMs) explain objectives and investment purposes in the frame of multi-objective mathematic problems which are more consistent with decision making realities. At present, various methods have introduced to optimize such problems. One of the optimization methods is the Compromise Programming (CP) method. Considering increasing importance of investment in financial portfolios, we propose a new method, called Nadir Compromising Programming (NCP) by expanding a CP-based method for optimization of multi-objective problems. In order to illustrate NCP performance and operational capability, we implement a case study by selecting a portfolio with 35 stock indices of Iran stock market. Results of comparing the CP method and proposed method under the same conditions indicate that NCP method results are more consistent with DM purposes.
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1. Introduction

Markowitz (1952, 1959) presented the so-called 'mean–variance' model, which assumes that the total return of a portfolio can be described using the mean return of the assets and the variance of return (risk) between securities. The portfolios that offer the minimum risk for a given level of return form what it is called the efficient frontier (Fernández & Gómez, 2007). The mean–variance methodology of Markowitz (1952) for portfolio selection problem has been central to research activity and has served as a basis for the development of modern financial theory. Yet, the Markowitz mean–variance model disadvantages include:

- The Markowitz model was generally criticized as not efficient with axiomatic models of preferences for choice under risk (Bell, Raiffa, & Tversky, 1988).
- The Markowitz model was a quadratic model which finally was non-linear. Because his model was non-linear, so obtained results were often local optimum.
- The mean–variance model of Markowitz (1952) was the time needed to compute the covariance matrix from historical data especially when problems were large scale and the difficulty of solving the large scale quadratic programming problems. For example, in the literature, some algorithms such as those proposed by Konno (1990), Konno and Yamazaki (1991), Sharpe (1963, 1967), Elton, Gruber, and Padberg (1976) and Young (1998), are generated in order to linearize and improve the efficiency calculation of the Markowitz mean–variance model (Nawrocki & Carter, 1998; Shing & Nagasawa, 1999). Young (1998) established portfolio scientific management, by his researches. He introduced 'sensitivity coefficient β', which measures stock volatility relative to the benchmark index or the capital market.

By considering Sharpe (1964)'s work and other present researches in the literature, it has been common in recent years to use coefficient β as measurement criterion of risk. Make use of this coefficient, in order to facilitate financial portfolio management process, is based on "modern portfolio theory".

By a certain classification, whole risk is divided into two controllable risk (unsystematic risk) and uncontrollable risk (systematic risk) parts. Also, according to "capital asset pricing model" (Sharpe, 1964), by making a various financial portfolio of market's selected stocks, unsystematic risk becomes capable of being minimized. The systematic risk, which is derived of economic, politic, social and environmental changes of capital market, is uncontrollable and almost has the same process for different stocks. Evident to this claim is the same process of different indices price in time.

The coefficient β is a criterion for systematic risk and can be an index for ranking different assets. If β for an asset is more than 1, the asset's return variations will be more than that of market and this asset is called 'aggressive asset'. Vice versa, an asset with β less than 1 means fewer variations than market variations. Similarly, this asset is called 'defensive asset'.

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Advantages of using coefficients β as risk coefficients of each security include:

- Making the risk objective function, linear and using it in optimizing portfolio selection problems.
- Increase of securities number has no effect on risk objective and computational volume does not increase, so that it is in contrast to the Markowitz mean–variance model.
- Using coefficients β as risk coefficients of each security, also consider market variations. Therefore in this paper, we will use coefficients β as risk coefficients in modeling of a problem of multi-objective portfolio selection.

One of the optimization methods for multi-objective problems is the CP model. In the CP model, DMs are able to establish, easily and precisely, goal values of the considered objectives. Zeleny (1973, 1974) proposed the CP. Of advantages of CP model are its simplicity and its capability to be used in conditions which it is not possible to access goal values of objectives.

The CP model was introduced by Romero, Amador, and Barco (1987) in the financial literature. Also Ballestero and Romero (1996) and Ballestero (1998) used the CP model to solve multi-objective problems for optimizing investment portfolios. Moreover Ballestero, Anton, and Bielza (2003), Gladish, Jones, Tamiz, and Terol (2007) were of persons who used the CP model to optimize some economical problems.

The CP model (Zeleny, 1974), only has been formulated on the basis of the utopia values. The aim of this paper is to propose a new model which called Nadir Compromise Programming (NCP) by developing the CP model which can be used to optimize multi-objective problems. Therefore, this paper continues as follow: in Section 2, we introduce the CP model and reformulate the CP model on the basis of the nadir values for each objective function in Section 3, which called NCP model. Then in Section 4, we illustrate the CP and NCP models on a sample of 35 stocks from the Iran stock exchange market by a multi-objective problem in portfolio selection under same conditions and then evaluate obtained results. Meanwhile we conclude this paper in Section 5.

2. The CP model

The CP model was proposed by Zeleny (1974). It consists of minimizing the distance between the achievement levels f_k and the utopia values ($f_k^{\max}$ or f_k^{min}) associated with each objective k. In the case that the more of the objective is better, the utopia values $f_k^{\max}$ can be obtained as follows:

$$\begin{aligned} f_k^{\max} &= \max \quad f_k, \quad k = 1,2,\ldots,K \\ \text{subject to}: &\quad g_i(x) \leqslant b_i, \quad i = 1,2,\ldots,m, \\ &\quad x \in S \end{aligned} \tag{1}$$

and the final model of CP by considering preference weights of objectives (w_k) can be formulated as follows:

$$\begin{aligned} \min \quad & \left\{ \sum_{k=1}^{K} w_k (\delta_k^-)^P \right\}^{\frac{1}{P}} \\ \text{subject to}: \quad & f_k + \delta_k^- = f_k^{\max}, \quad k = 1,2,\ldots,K, \\ & g_i(x) \leqslant b_i, \quad i = 1,2,\ldots,m, \\ & x \in S, \\ & \delta_k^- \geqslant 0, \quad k = 1,2,\ldots,K. \end{aligned} \tag{2}$$

The variable δ_k^- is deviational variable of slack for constraint related to kth objective. Also if the case that less of the objective is better, the utopia values f_k^{min} can be obtained as follows:

$$\begin{aligned} f_k^{\min} &= \min \, f_k, \quad k = 1,2,\ldots,K \\ \text{subject to}: &\; g_i(x) \leqslant b_i, \quad i = 1,2,\ldots,m, \\ &\; x \in S \end{aligned} \tag{3}$$

and the final model of CP by considering preference weights of objectives (w_k) can be formulated as follows:

$$\begin{aligned} \min \quad & \left\{ \sum_{k=1}^{K} w_k (\delta_k^+)^P \right\}^{\frac{1}{P}} \\ \text{subject to}: \quad & f_k - \delta_k^+ = f_k^{min}, \quad k = 1,2,\ldots,K, \\ & g_i(x) \leqslant b_i, \quad i = 1,2,\ldots,m, \\ & x \in S, \\ & \delta_k^+ \geqslant 0, \quad k = 1,2,\ldots,K. \end{aligned} \tag{4}$$

The variable δ_k^+ is deviational variable of surplus for constraint related to kth objective.

The CP model is based on the choice of the solutions that are closer to the ideal points ($f_k^{\max}$ or f_k^{min}), (Zeleny, 1976; Zeleny, 1982). In the CP model, P is parameter of final utility function which can have values of metrics $\{1,2,\ldots\} \cup \{\infty\}$. Also $w_k > 0$ (for $k = 1,2,\ldots,K$) and $\sum_{k=1}^{K} w_k = 1$.

The $L_{P=1}$,$L_{P=2}$ and $L_{P=\infty}$ are named Manhattan, Euclidean and Tchebycheff metrics, respectively. By considering $L_{P=\infty}$ metric, the CP (min–max) model related to Program (2) can be formulated as follows:

$$\begin{aligned} \min \quad & y \\ \text{subject to}: \; & y \geqslant \sum_{k=1}^{K} w_k (\delta_k^-) \\ & f_k + \delta_k^- = f_k^{\max}, \quad k = 1,2,\ldots,K, \\ & g_i(x) \leqslant b_i, \quad i = 1,2,\ldots,m, \\ & x \in S, \\ & y \geqslant 0; \quad \delta_k^- \geqslant 0, \quad k = 1,2,\ldots,K, \end{aligned} \tag{5}$$

where y is always non-negative.

3. The NCP model

The CP model (Zeleny, 1974), only minimizes the distance between the achievement levels f_k and the utopia values associated with each objective k. In here, we present a model on the base of the CP model and considering maximum distance from the nadir values.

(a) If objective is to optimize f_k on the basis of the goal value kth, then we have:

$$\text{Opt} \quad f_k \ (\text{for } k = 1,2,\ldots,K).$$

If $f_{(k)} \in \mathbb{R}$ (for $k = 1,2,\ldots,K$) is the goal value of kth objective, the constraint related to kth objective in the NCP model can be as follows:

$$f_{(k)} \geqslant f_k \geqslant f_{(k)} \approx f_k = f_{(k)}.$$

The above two-side constraint can be converted as two constraints $\geqslant$ and $\leqslant$. Therefore, it can be said:

$$\begin{aligned} & f_k \geqslant f_{(k)}, \\ & f_k \leqslant f_{(k)} \end{aligned}$$

and finally the NCP model by considering preference weights of objectives (w_k), can be formulated as follows:

$$\begin{aligned}
&\min && \left\{\sum_{k=1}^{K} w_k(\delta_k^+ + \delta_k^-)^P\right\}^{\frac{1}{P}} \\
&\text{subject to}: && f_k - \delta_k^+ = f_{(k)}, \quad k = 1,2,\ldots,K, \\
& && f_k + \delta_k^- = f_{(k)}, \quad k = 1,2,\ldots,K, \\
& && x \in S, \\
& && f_{(k)} \in \mathbb{R};\ \delta_k^-, \delta_k^+ \geqslant 0, \quad k = 1,2,\ldots,K.
\end{aligned} \tag{6}$$

In Program (6), the objective is to minimize δ_k^+ and δ_k^- in order to more closeness to the goal value of kth objective function.

(b) If objective is to minimize f_k on the basis of the nadir value kth, then we have:

Min f_k (for $k = 1,2,\ldots,K$).

If f_{k*} (for $k = 1,2,\ldots,K$) is the nadir value of kth objective, the constraint related to kth objective in the NCP model can be as follows:

$f_{k*} \geqslant f_k$

and finally the NCP model by considering preference weights of objectives (w_k), can be as follows:

$$\begin{aligned}
&\min && \left\{\sum_{k=1}^{K} w_k(-\delta_k^-)^P\right\}^{\frac{1}{P}} \\
&\text{subject to}: && f_k + \delta_k^- = f_{k*}, \quad k = 1,2,\ldots,K, \\
& && x \in S, \\
& && \delta_k^- \geqslant 0, \quad k = 1,2,\ldots,K.
\end{aligned} \tag{7}$$

In constraint $f_k + \delta_k^- = f_{k*}$ (for $k = 1,2,\ldots,K$) of Program (7), the objective is to maximize δ_k^- in order to more distance from the nadir value of kth objective function.

(c) If objective is to maximize f_k on the basis of the nadir value kth, then we have:

Max f_k (for $k = 1,2,\ldots,K$)

and the constraint related to this objective in the NCP model is:

$f_k \geqslant f_{k*}$

and finally the NCP model by considering preference weights of objectives (w_k), can be formulated as follows:

$$\begin{aligned}
&\min && \left\{\sum_{k=1}^{K} w_k(-\delta_k^+)^P\right\}^{\frac{1}{P}} \\
&\text{subject to}: && f_k - \delta_k^+ = f_{k^*}, \quad k = 1,2,\ldots,K, \\
& && x \in S, \\
& && \delta_k^+ \geqslant 0, \quad k = 1,2,\ldots,K.
\end{aligned} \tag{8}$$

Also, in constraint $f_k - \delta_k^+ = f_{k*}$ (for $k = 1,2,\ldots,K$) of Program (8), the objective is to maximize δ_k^+ in order to more distance from the nadir value of kth objective function.

Generally, if we optimize A objective functions on the basis of the their goal values and minimize B objective functions and maximize C objective functions, the final model of NCP can be written as follows:

$$\begin{aligned}
&\min && \left\{\sum_{a=1}^{A} w_a(\delta_a^+ + \delta_a^-)^P + \sum_{b=1}^{B} w_b(-\delta_b^-)^P + \sum_{c=1}^{C} w_c(-\delta_c^+)^P\right\}^{\frac{1}{P}} \\
&\text{subject to}: && \begin{cases} f_a - \delta_a^+ = f_{(a)}, & a = 1,2,\ldots,A, \\ f_a + \delta_a^- = f_{(a)}, & a = 1,2,\ldots,A, \end{cases} \\
& && f_b + \delta_b^- = f_{b*}, \quad b = 1,2,\ldots,B, \\
& && f_c - \delta_c^+ = f_{c*}, \quad c = 1,2,\ldots,C, \\
& && x \in S, \\
& && \delta_a^-,\ \delta_a^+ \geqslant 0;\ \delta_b^- \geqslant 0;\ \delta_c^+ \geqslant 0, \\
& && a = 1,2,\ldots,A;\ b = 1,2,\ldots,B;\ c = 1,2,\ldots,C, \\
& && f_{(a)} \in \mathbb{R}, \quad a = 1,2,\ldots,A,
\end{aligned} \tag{9}$$

where $\sum_{a=1}^{A} w_a + \sum_{b=1}^{B} w_b + \sum_{c=1}^{C} w_c = 1 (w_a, w_b, w_c > 0,$ for $a = 1,2,\ldots,A$ and $b = 1,2,\ldots,B$ and $c = 1,2,\ldots,C$).

4. Numerical example

We illustrate our developed model through a portfolio selection problem where several conflicting objectives are considered. We consider a sample of 35 stocks from the Iran stock exchange. The data and observations (from June 23, 2003 to June 29, 2008) of the in-sample period are used as the training set to determine the models parameters and specifications.

The evaluation process consists of the following steps:

- The definition of the objectives.
- The determination, for each objective, of the utopia and nadir values and the goal values.
- The formulation of, and subsequently the solution to the CP (min–max) and NCP (min–max) models. At the end of this section, we will present and discuss the results obtained from both models.

In portfolio selection problem, the DM can consider several conflicting objectives. The objectives adopted by Markowitz (1952) are mean and variance. Lee and Chesser (1980) and Zopounidis et al. (1999) propose a set of objectives that the DM can consider to evaluate the stocks. For the illustration purposes, we will consider the following objectives:

- The first linear objective $\left(\sum_{j=1}^{35} \beta_j x_j\right)$, is risk objective function. $\beta_j = \mathrm{Cov}(r_j, r_m)/\mathrm{Var}(r_m)$, where r_j $j = 1,2,\ldots,35$ is the rate of return of stock j and r_m is the rate of market return. This objective indicates the reliance of stock's return on market. Lower correlation with the market indicates the stock performance on its own rather than by the movements of the market. In order to select a portfolio as risky as the market, we propose as in Lee and Chesser (1980) to set a goal equal to 1 for this objective.
- The second linear objective $\left(E\left(\sum_{j=1}^{35} r_j x_j\right) = \sum_{j=1}^{35} \mu_j x_j\right)$, is expected rate of return objective function. The rate of return $(r_j{=})P_{j,t}{-}P_{j,t-1}{+}D_{j,t}(/P_{j,t-1})$ measures the profitability of the stock where the income can be in the form of random capital gain and dividend. Here $P_{j,t}$ is the price of the stock j at time t and $D_{j,t}$ is the dividend received during the period $[t-1, t]$. r_j $j = 1,2,\ldots,35$ are random and normally distributed with known mean μ_j and variance σ_j^2. This objective is to be maximized.
- In here our proposed objective is the third linear objective $\left(\sum_{j=1}^{35} P_j x_j\right)$, namely initial cost of investment objective function. In real world, many people suffer because they have not enough money for secure investments. Thus the aim this is which they spend less money while will obtain their favorite results from other objectives. Here P_j is the price of stock j (with known formal currency) in the last under study day. Let N be total number of present stocks in the optimum portfolio. Therefore, the initial cost of investment objective function can be obtained without considering the value N as follows:

$$\begin{aligned}
&Z = P_1(Nx_1) + P_2(Nx_2) + \cdots + P_{35}(Nx_{35}) \\
&\quad \Rightarrow Z = N(P_1x_1 + P_2x_2 + \cdots + P_{35}x_{35}) \\
&\quad \Rightarrow Z = N\left(\sum_{j=1}^{35} P_j x_j\right) \Rightarrow \frac{Z}{N} = \frac{N\left(\sum_{j=1}^{35} P_j x_j\right)}{N} \\
&\quad \Rightarrow \frac{Z}{N} = f_3 = \sum_{j=1}^{35} P_j x_j.
\end{aligned} \tag{10}$$

Table 1
Data under study.

j	Stock	Beta risk (β_j)	Expected rate of return (μ_j)	Stock price in the last exchanged day (P_j)
1	PARS AUTO	−1.03815	0.0000423	1275
2	MEH IRAN AUTO	1.15065	−0.0006437	700
3	SAIPA	0.17812	0.0015994	826
4	RAY SAIPA INV	0.60025	0.0027148	1027
5	PERSIAN BANK	1.05606	0.0021114	2330
6	KAR AFR BANK	−1.45407	0.0010685	3427
7	IRAN LEAS	−1.02369	0.0017717	951
8	IND & MIN LEAS	1.23007	0.0022249	967
9	PARS ALU	2.14956	−0.0001838	948
10	ALUMTAK	−0.82301	0.0016264	8385
11	IRAN BEHNUSH	−0.00125	−0.0004780	9573
12	PARS MINOO	3.67891	−0.0021901	534
13	PAS SHI IRAN	1.67921	0.0011433	1180
14	CHINI IRAN	2.12003	0.0011433	1180
15	SAIPA DIESEL	0.09782	−0.0004956	869
16	MELLI BANK INV	0.32507	−0.0012366	1136
17	BAHMAN INV	−0.21366	0.0000363	1027
18	MELLI DEV INV	−2.30051	0.0000633	900
19	SAIPA INV	4.03231	0.0007125	998
20	SEPAH INV	1.25362	0.0002052	1032
21	SAND BAZ INV	0.00263	0.0008053	2269
22	OIL IND INV	−1.12365	0.0005904	1606
23	MIN IND INV	1.23452	−0.0020231	1020
24	ARDABIL CEMENT	1.14029	−0.0004812	19576
25	SHOMAL CEMENT	0.75281	−0.0023246	4552
26	SUFIAN CEMENT	−1.06395	−0.0003489	4173
27	SHOGA	2.14363	0.0018342	1080
28	FULAD MOBARAKE	0.05023	0.0033556	796
29	ISFAHAN TILE	0.02148	0.0026258	2250
30	IRAN CARBON	0.10236	−0.0013975	1038
31	MELLI IND GROUP	1.33523	0.0005904	1606
32	LABAN MIHAN	−1.19237	−0.0009010	1038
33	GHADIR AUTO LEAS	0.82667	−0.0007060	1042
34	IRAN OIL	1.98995	−0.0021267	5800
35	NAVARD ALU	1.00542	−0.0010624	1715

Finally optimum value of cost for selection and allocation of optimum portfolio is equal tot $Z^* = f_3^* N$. We consider price of the last day in under study term (P_j) to purchase stock j. This objective is to be minimized.

The system constraints are defined in three parts:

Part (1): The sum of the proportions invested in stocks is equal to 1: $\sum_{j=1}^{35} x_j = 1$ (Markowitz, 1952).
Part (2): Allocated constraints:

- In order to diversify the selected portfolios and maximum utilization from the all existent capacities of investment, DM proposes to invest 20% in automotive industry (for stocks $j = 1,2,3,15$), banking (for stocks $j = 5,6$), leasing (for stocks $j = 7,8,33$), investment sectors (for stocks $j = 4, 16,\ldots,23$) and another sectors (for stocks $j = 9,10,\ldots,14, 24,\ldots,35$). In fact, summation of these constraints is equal to the constraint of Part (1).

Part (3): Setting a lower and an upper bound for each stock in order to diversify the portfolio, $0 \leqslant x_j \leqslant 0.1$, for $j = 1,2,\ldots,35$, where the x_j is the proportion to be invested in the stock j.

Table 2
Utopia and nadir values of objectives (for $k = 2,3$).

Objective functions	Utopia values	Nadir values
f_2	0.0017	−0.0006
f_3	1242.9	4293.4

The main portfolio selection problem can be formulated as follows:

$$
\begin{aligned}
\text{opt} \quad & f_1 = \sum_{j=1}^{35} \beta_j x_j \\
\text{max} \quad & f_2 = \sum_{j=1}^{35} \mu_j x_j \\
\text{min} \quad & f_3 = \sum_{j=1}^{35} P_j x_j \\
\text{subject to}: \quad & x_1 + x_2 + x_3 + x_{15} = 0.2, \\
& x_5 + x_6 = 0.2, \\
& x_7 + x_8 + x_{33} = 0.2, \\
& x_4 + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} = 0.2, \\
& x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{24} + x_{25} + x_{26} + x_{27} + \\
& x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{34} + x_{35} = 0.2, \\
& 0 \leqslant x_j \leqslant 0.1, \quad j = 1,2,\ldots,35.
\end{aligned}
\tag{11}
$$

The Program (11) is transformed to a NCP model. f_{2*} and f_{3*} are the nadir values associated with the expected return and the initial cost objectives in solution space of Program (11), respectively. The Beta goal is equal to 1. In order to use the NCP model, we consider $L_{P=\infty}$ metric for the NCP model. On this basis Program (11) can be reformulated as follows:

$$
\begin{aligned}
\text{min} \quad & y \\
\text{subject to}: \quad & y \geqslant w_1(\delta_1^+ + \delta_1^-), \\
& y \geqslant w_2(-\delta_2^+), \\
& y \geqslant w_3(-\delta_3^-), \\
& \begin{cases} f_1 - \delta_1^+ = 1, \\ f_1 + \delta_1^- = 1, \end{cases} \\
& f_2 - \delta_2^+ = f_{2^*}, \\
& f_3 + \delta_3^- = f_{3^*}, \\
& x_1 + x_2 + x_3 + x_{15} = 0.2, \\
& x_5 + x_6 = 0.2, \\
& x_7 + x_8 + x_{33} = 0.2, \\
& x_4 + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} = 0.2, \\
& x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{24} + x_{25} + x_{26} + x_{27} + \\
& x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{34} + x_{35} = 0.2, \\
& 0 \leqslant x_j \leqslant 0.1, \quad j = 1,2,\ldots,35, \\
& y \geqslant 0;\ \delta_1^-,\ \delta_1^+ \geqslant 0;\ \delta_2^+ \geqslant 0;\ \delta_3^- \geqslant 0.
\end{aligned}
\tag{12}
$$

The objectives considered, in this example, are the risk β, the expected return and the initial cost of investment. They are equally weighed ($w_1 = w_2 = w_3 = 1/3$).

In Table 1 we present data concerning the different securities of the Iran stock market for the years 2003 to 2008. These data are available in Table 1. The five columns of the Table 1 are the stock number, the stocks, the risk β, the expected rate of return and the stock price in the last exchanged day, respectively.

The utopia and nadir values of the expected return and the initial cost objectives have been represented in Table 2. The goal fixed for the risk β is equal to 1. The software of Lingo 8.0 has been used to solve the mathematical programs of the CP (min–max) and NCP (min–max) models under same conditions.

In Table 3, we present the optimal proportion of investment in each stock for two selected portfolios on the basis of the CP (min–max) and NCP (min–max) models. We notice that the results in portfolio of the CP (min–max) model are close to those obtained

Table 3
Optimum values of portfolios of the CP (min–max) and NCP (min–max) models.

x_j^*	Stock	Portfolio of CP (min–max) model	Portfolio of NCP (min–max) model
x_1^*	PARS AUTO	0	0
x_2^*	MEH IRAN AUTO	0.1	0.1
x_3^*	SAIPA	0.1	0.1
x_4^*	RAY SAIPA INV	0	0.1
x_5^*	PERSIAN BANK	0.1	0.1
x_6^*	KAR AFR BANK	0.1	0.1
x_7^*	IRAN LEAS	0.1	0.1
x_8^*	IND & MIN LEAS	0.1	0.1
x_9^*	PARS ALU	0	0.1
x_{10}^*	ALUMTAK	0	0
x_{11}^*	IRAN BEHNUSH	0	0
x_{12}^*	PARS MINOO	0.1	0.05596
x_{13}^*	PAS SHI IRAN	0	0
x_{14}^*	CHINI IRAN	0	0
x_{15}^*	SAIPA DIESEL	0	0
x_{16}^*	MELLI BANK INV	0	0
x_{17}^*	BAHMAN INV	0	0
x_{18}^*	MELLI DEV INV	0.09725	0
x_{19}^*	SAIPA INV	0.1	0.1
x_{20}^*	SEPAH INV	0	0
x_{21}^*	SAND BAZ INV	0	0
x_{22}^*	OIL IND INV	0	0
x_{23}^*	MIN IND INV	0.00275	0
x_{24}^*	ARDABIL CEMENT	0	0
x_{25}^*	SHOMAL CEMENT	0	0
x_{26}^*	SUFIAN CEMENT	0	0
x_{27}^*	SHOGA	0	0
x_{28}^*	FULAD MOBARAKE	0.1	0.04404
x_{29}^*	ISFAHAN TILE	0	0
x_{30}^*	IRAN CARBON	0	0
x_{31}^*	MELLI IND GROUP	0	0
x_{32}^*	LABAN MIHAN	0	0
x_{33}^*	GHADIR AUTO LEAS	0	0
x_{34}^*	IRAN OIL	0	0
x_{35}^*	NAVARD ALU	0	0

Table 4
Optimum values of objectives in the CP (min–max) and NCP (min–max) models (for $k = 1,2,3$).

Objective functions	Portfolio of CP (min–max) model	Portfolio of NCP (min–max) model
f_1	0.6695	1
f_2	0.0010	0.0012
f_3	1243.2	1282.3

in portfolio of the NCP (min–max) model. Also we present the optimum values of objective functions in Table 4. It should be said that considering the utopia value of each objective, the NCP (min–max) model shows better results for objectives f_1 and f_2. For objective f_1, the considered goal value is entirely satisfied by the NCP (min–max) model and yet in the CP (min–max), the optimal value obtained form objective f_1 is less than 0.7. Also it should be said that the CP (min–max) model with slight difference (approximately 39 units) shows better result than the NCP (min–max) model, only for objective f_3.

The all results of two models are feasible and optimal but it should be mentioned that existence of an optimal and successful investment is in the cover of more consistence with DM purposes, like the obtained results from the NCP (min–max) model. So it could be said that in here, this more consistence is product of modeling of the nadir value of each objective in the NCP method.

5. Conclusion

In spite of the fundamentality of the Markowitz mean–variance model in optimal portfolio selection problem, always there have been critics because of being quadratic and making use of covariance matrix in the risk objective of this model. So it was seen that using Beta risk coefficients can be a proper replacement in order to model investment risk objective function. In the other section of this paper, we presented the CP model and proposed the NCP model which can optimize multi-objective problems. The NCP model was formulated on the basis of the nadir values of each objective. Then we introduced a multi-objective problem to select optimal portfolio in Iran stock market and optimized the multi-objective problem by two the CP (min-max) and NCP (min-max) models under the same conditions. The obtained results confirmed that in spite of being feasible and optimal of two portfolios, the NCP (min-max) model can satisfy DM purposes better.

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