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RESEARCH PAPER

Multiobjective Transportation Model with Fuzzy Parameters: Priority based Fuzzy Goal Programming Approach

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Abstract: This paper presents a priority based fuzzy goal programming approach for solving a multiobjective transportation problem with fuzzy coefficients. In the model formulation of the problem, first the membership functions for the fuzzy goals are defined. Subsequently, the membership functions are transformed into membership goals, by assigning the highest degree (unity) of a membership function as the aspiration level and introducing deviational variables to each of them. In the solution process, negative deviational variables are minimized to obtain the most satisficing solution. Sensitivity analysis of the solution, with a change in priorities of the fuzzy goals is performed. Next the Euclidean distance function is used to identify the appropriate priority structure of the goals, thereby obtaining the most satisficing decision for the decision-making unit, by minimizing their regrets of achieving the ideal point dependent decision in the decision-making context. A numerical example is solved to demonstrate the potential use of the proposed approach.

Key Words: fuzzy goal programming; fuzzy number; membership function; multiobjective decision making; multiobjective transportation problem

1 Introduction

In the field of multiobjective decision-making problems, the priority based goal programming (GP) is one of the most prominent and powerful techniques for solving decision problems with multiple and conflicting goals in a fuzzy environment. Ijiri[1] was credited with being the first to develop the priority based GP, in 1965. Ignizio[2], Lee[3], Steuer[4], and other researches investigated the priority based GP and successfully applied it to various types of real-life problems. In the priority based GP, the ranking of goals were ordered in accordance to their priorities for achievement of the respective aspiration levels in decision-making situations. The goals of equal importance belong to the same priority level. Owing to the conflicting nature of the goals, the aspiration levels of all the goals were rarely achieved. Therefore, differential weights were assigned to their relative importance of achieving the respective target values. Hitchcock[5] first introduced the basic transportation problem in 1941. The classical transportation problem (Hitchcock transportation problem) is considered to be one of the subclasses of linear programming problems, in which all the constraints are of the equality type. The multiobjective transportation problem (MOTP) in a crisp environment was extensively studied in refs. [6, 7]. Bit *et al*.[8] applied minimum operator to solve MOTP. Abd El-Wahed and Lee[9] presented an interactive fuzzy goal programming technique for MOTP. Gao and Liu[10] developed a two-phase fuzzy goal programming technique for MOTP. They developed two-phase fuzzy algorithms for MOTP with linear and nonlinear membership functions. Li and Lai[11] developed a fuzzy approach to MOTP. Pramanik and Roy[12] discussed the Fuzzy goal programming (FGP) approach for MOTP, with crisp and fuzzy coefficients.

The FGP approach to MOTP, with a fuzzy coefficient, has not been studied extensively in literature. In this paper, priority based FGP for MOTP, with fuzzy coefficients, is considered. For model formulation of the problem, the objective functions for the fuzzy goals are defined first.

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Subsequently, the membership functions are transformed into fuzzy membership goals by means of assigning the highest degree (unity) as the aspiration level, and introducing the negative and positive deviational variables to each of them. As positive deviation from any fuzzy goals implies the full achievement of the membership value, the authors have used only negative deviational variables in the achievement function and minimized the negative deviational variables to get a satisfying solution[13].

In the priority based FGP solution technique, first the goals at the highest priority level are taken into consideration for achievement of their aspiration levels according to their relative importance of weights at that priority level. Next, the achievement of the goals of the very next higher level is taken into consideration and the process goes on until the last priority level is considered.

In the solution process, sensitivity analysis using the variations of the priority structure of the goals is performed to show how the optimal solution is sensitive to the changes in the priority structure. To identify the appropriate priority structure under which the most satisfying decision is reached in the decision-making environment, the Euclidean distance function is applied.

2 Prerequisite mathematics

The concept of a fuzzy set was introduced by Zadeh[14] in 1965, as a mathematical way of representing impreciseness and vagueness in real world problems.

Fuzzy set: a fuzzy set A in a universe of discourse X is defined by $\tilde{A}=\{\langle x, \mu_{\tilde{A}}(x)\rangle \lfloor x\in X\}\rfloor$, where $\mu_{\tilde{A}}(x)$:$X\rightarrow[0,\ 1]$ is defined as the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x\in\tilde{A}$.

Normal fuzzy set: A fuzzy set A is said to normal if there exists a point x in X such that $\mu_{\tilde{A}}(x)=1$. Otherwise A is said to be the subnormal fuzzy set.

Convex fuzzy set: a fuzzy set $\tilde{A}$ is said to be convex if and only if for any $x_1, x_2\in X$, and $\lambda\in[0, 1]$, $\mu_{\tilde{A}}[\lambda x_1+(1-\lambda)x_2]\geq\min[\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2)]$.

Union of two fuzzy sets: union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}(x)$, and $\mu_{\tilde{B}}(x)$ is defined by a fuzzy set $\tilde{C}$ whose membership function is defined by $\mu_{\tilde{A}\cup\tilde{B}}(x)=\mu_{\tilde{C}}(x)=\max[\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(x)]$, $x\in X$.

Intersection of two fuzzy sets: intersection of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ is defined by a fuzzy set $\tilde{C}$ whose membership function is defined by $\mu_{\tilde{A}\cap\tilde{B}}(x)=\mu_{\tilde{C}}(x)=\min[\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x)]$, $x\in X$.

α-cut: the α-cut of a fuzzy set A of X is a nonfuzzy set denoted by ${}^{\alpha}A$ defined by a subset of all elements $x\in X$, such that their membership functions exceed or equal to a real number $\alpha\in[0, 1]$, that is,

$$^{\alpha}A=[x:\mu_{\tilde{A}}(x)\geq\alpha,\ \alpha\in[0,1],\ \forall x\in X]$$

A fuzzy number is a special case of fuzzy set. Different definitions and properties of fuzzy numbers are widely encountered in literature, but they capture one's intuitive conceptions of approximate numbers or intervals, such as "numbers close to a real number" or "numbers that are around a given interval of real numbers". A fuzzy number is a fuzzy set in the universe of discourse X, if it is both convex and normal. A fuzzy number $\tilde{R}$ is a fuzzy set of the real line R, whose membership function $\mu_{\tilde{R}}(r)$ must possess the following properties with $\infty<r(1)<r(2)<r(3)<r(4)<\infty$.

$$\mu_{\tilde{R}}(r)=\begin{cases}0, & r\leq r^{(1)},\\ \mu_L(r), & r^{(1)}\leq r\leq r^{(2)},\\ 1, & r^{(2)}\leq r\leq r^{(3)},\\ \mu_U(r), & r^{(3)}\leq r\leq r^{(4)},\\ 0, & r\geq r^{(4)}\end{cases} \tag{1}$$

where $\mu_L(R)$: $[r(1), r(2)]\rightarrow[0, 1]$ is continuous and strictly increasing and $\mu_U(r)$ is continuous and strictly decreasing (Fig. 1).

Trapezoidal fuzzy number: A trapezoidal fuzzy number (Fig. 2) can be completely specified by the foursome $\tilde{R}=(r(1), r(2), r(3), r(4))$ with membership function.

$$\mu_{\tilde{R}}(r)=\begin{cases}0, & r\leq r^{(1)}\\ \dfrac{r-r^{(1)}}{r^{(2)}-r^{(1)}}, & r^{(1)}\leq r\leq r^{(2)}\\ 1, & r^{(2)}\leq r\leq r^{(3)}\\ \dfrac{r^{(4)}-r}{r^{(4)}-r^{(3)}}, & r^{(3)}\leq r\leq r^{(4)}\\ 0, & r\geq r^{(4)}\end{cases} \tag{2}$$

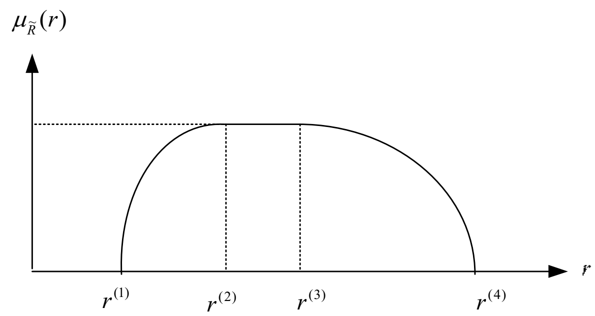

Fig. 1 General shape of a fuzzy number following the definition (1)

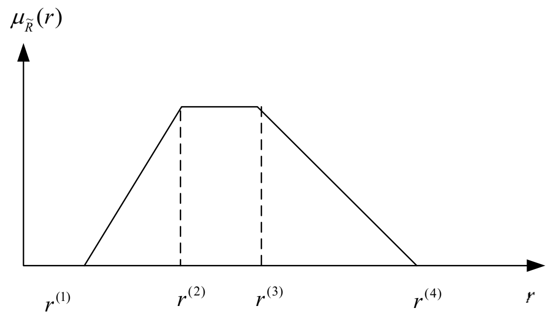

Fig. 2 Trapezoidal fuzzy number $\tilde{R}=(r(1), r(2), r(3), r(4))$

Therefore, an α-cut of $\tilde{R}$ can be represented by the following interval,

$$^{\alpha}(\tilde{R}) = [^{\alpha}(\tilde{R})^{L},\ ^{\alpha}(\tilde{R})^{U}] = [(r^{(2)} - r^{(1)})\alpha + r^{(1)} - (r^{(4)} - r^{(3)})\alpha + r^{(4)}] \tag{3}$$

It is to be noted that when $r(2)=r(3)$, $\tilde{R}$ transforms into the triangular fuzzy number, specified by ($r(1)$, $r(2)=r(3)$, $r(4)$); if $r(1)=r(2)$ and $r(3)=r(4)$ then $\tilde{R}$ is called a crisp interval; if in particular, $r(1)=r(2)=r(3)=r(4)$, then $\tilde{R}$ transforms into a real number.

A fuzzy equality constraint

$$\sum_{j=1}^{q} \tilde{A}_j X_j = \tilde{B} \tag{4}$$

is equivalent to two inequality constraints

$$\sum_{j=1}^{q} {}^{\alpha}\left(\tilde{A}_j\right)^{L} . X_j \le {}^{\alpha}\left(\tilde{B}\right)^{U} \text{ and } \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{A}_j\right)^{U} . X_j \ge {}^{\alpha}\left(\tilde{B}\right)^{L} \tag{5}$$

For proof of equivalency of (4) and (5), see Lee and Li[15].

3 Formulation of fuzzy programming with fuzzy parameters

Consider the following fuzzy optimization problem:

$$\text{Minimize} \quad Z(\overline{X}) = \{\tilde{C}_1\overline{X}, \tilde{C}_2\overline{X}, \cdots, \tilde{C}_K\overline{X}\} \tag{6}$$

$$\text{subject to} \quad \overline{X} \in S = \{\overline{X} \in R^{q} \mid \tilde{A}\overline{X} * \tilde{B},\ \overline{X} \ge \overline{0}\} \tag{7}$$

where $\tilde{C}_k (k = 1, 2, \cdots, K)$ is a q-dimensional vector, $\tilde{B}$ is a p-dimensional vector, $\tilde{A}$ is a $p \times q$ matrix, and $\tilde{C}$, $\tilde{B}$, and $\tilde{A}$ are fuzzy numbers. Here, the symbol $*$ represents ≥, =, and ≤. $\overline{X}=(X_1, X_2, \cdots, X_q)^{T}$, respectively. Suppose that the problem represented by Eq. (6) has fuzzy coefficients, which have possibilistic distributions. Assume that X is the solution of Eq. (6), where $\alpha \in [0, 1]$ represents the level of possibility at which all fuzzy coefficients are feasible.

Let $^{\alpha}(\tilde{R})$ be the α-cut of a fuzzy number $\tilde{R}$ defined by

$$^{\alpha}(\tilde{R}) = \{r \in S(\tilde{R}) \mid \mu_{\tilde{R}}(r) \ge \alpha,\ \alpha \in [0, 1]\} \tag{8}$$

where $S(\tilde{R})$ is the support of $\tilde{R}$. Let $^{\alpha}(\tilde{R})^{L}$ and $^{\alpha}(\tilde{R})^{U}$ be the lower bound and upper bound of the α-cut of R respectively, such that,

$$^{\alpha}(\tilde{R})^{L} \le r \le {}^{\alpha}(\tilde{R})^{U} \tag{9}$$

$$r \in {}^{\alpha}(\tilde{R}) = [^{\alpha}(\tilde{R})^{L},\ ^{\alpha}(\tilde{R})^{U}] \tag{10}$$

As the coefficients $\tilde{C}_{kj}$ of the objective functions are fuzzy numbers, α-cut of $\tilde{C}_{kj}$ can be defined as

$$^{\alpha}(\tilde{C}_{kj}) = \{S(\tilde{C}_{kj}) \mid \mu_{\tilde{C}_{kj}} \ge \alpha, \alpha \in [0, 1]\} \tag{11}$$

and $^{\alpha}(\tilde{C}_{kj})$ can be represented by the closed interval $[^{\alpha}(\tilde{C}_{kj})^{L},\ ^{\alpha}(\tilde{C}_{kj})^{U}]$ such that,

$$c_{kj} \in {}^{\alpha}(\tilde{C}_{kj}) = [^{\alpha}(\tilde{C}_{kj})^{L},\ ^{\alpha}(\tilde{C}_{kj})^{U}] \tag{12}$$

Then the lower bound and upper bound of the respective α-cuts of the objective functions are defined as

$$^{\alpha}\left(\tilde{Z}_k\left(\overline{X}\right)\right)^{L} = \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j \tag{13}$$

$$^{\alpha}\left(\tilde{Z}_k\left(\overline{X}\right)\right)^{U} = \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{U} X_j \tag{14}$$

Next, for a prescribed value of α, to construct a membership function for minimization-type objective function, $\tilde{Z}_k(\overline{X})$ (k=1, 2,···, K), can be replaced by the lower bound of its α-cut[15], that is,

$$^{\alpha}\left(\tilde{Z}_k\left(\overline{X}\right)\right)^{L} = \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j \tag{15}$$

Similarly, for a prescribed value of α, to construct a membership function for maximization-type objective function, $\tilde{Z}_k(\overline{X})$ (k=1, 2,···, K), can be replaced by the upper bound of its α-cut, that is,.

$$^{\alpha}\left(\tilde{Z}_k\left(\overline{X}\right)\right)^{U} = \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{U} X_j \tag{16}$$

For inequality constraints

$$\sum_{j=1}^{q} \tilde{A}_{ij} X_j \ge \tilde{B}_i,\ i = 1, 2, \cdots, p_1 \tag{17}$$

$$\sum_{j=1}^{q} \tilde{A}_{ij} X_j \le \tilde{B}_i,\ i = p_1 + 1, \cdots, p_2 \tag{18}$$

can be rewritten by the following constraints:

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{U} X_j \ge {}^{\alpha}(\tilde{B}_i)^{L},\ i = 1, 2, \cdots, p_1 \tag{19}$$

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{L} X_j \le {}^{\alpha}(\tilde{B}_i)^{U},\ i = p_1 + 1, \cdots, p_2 \tag{20}$$

For fuzzy equality constraints

$$\sum_{j=1}^{q} \tilde{A}_{ij} X_j = \tilde{B}_i,\ i = p_2 + 1, \cdots, p \tag{21}$$

can be replaced by two equivalent constraints

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{L} X_j \le {}^{\alpha}(\tilde{B}_i)^{U},\ i = p_2 + 1, \cdots, p \tag{22}$$

and

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{U} X_j \ge {}^{\alpha}(\tilde{B}_i)^{L},\ i = p_2 + 1, \cdots, p \tag{23}$$

in Section 2.2.

Therefore, for a prescribed value of α, the problem represented by Eq. (6) can be transformed to the following problem:

$$\text{Minimize} \quad ^{\alpha}\left(\tilde{Z}_k\left(\overline{X}\right)\right)^{L} = \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j,\quad k = 1, 2, \cdots, K \tag{24}$$

Subject to

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{U} X_j \geq {}^{\alpha}(\tilde{B}_i)^{L},\ i = 1, 2, \cdots p_1,\ p_2+1, \cdots, p \quad (25)$$

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{L} X_j \leq {}^{\alpha}(\tilde{B}_i)^{U},\ i = p_1+1, \cdots, p_2,\ p_2+1, \cdots, p \quad (26)$$

$$X_j \geq 0,\ j = 1, 2, \cdots, q \quad (27)$$

For simplicity, denote the system constraints (25), (26), and (27) as S.

For a prescribed value of α, problem (24) reduces to a deterministic multiobjective linear programming problem, which can be solved by applying the FGP techniques.

The resulting membership functions for minimization-type objective functions are defined as:

$${}^{\alpha}\mu_k\left(Z_k\left(\overline{X}\right)\right) = \left[{}^{\alpha}(Z_k)^{-} - \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} . X_j\right] \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O}\right]$$

$$k = 1, 2, \cdots, K \quad (28)$$

where the aspired level ${}^{\alpha}(Z_k)^{O}$ and the highest acceptable level ${}^{\alpha}(Z_k)^{-}$ are ideal and anti-ideal solutions, respectively, which can be obtained by solving each of the following problems independently:

$${}^{\alpha}(Z_k)^{O} = \min_{\overline{X}\in S} \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j, \quad k = 1, 2, \cdots, K \quad (29)$$

$${}^{\alpha}(Z_k)^{-} = \max_{\overline{X}\in S} \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{U} X_j, \quad k = 1, 2, \cdots, K \quad (30)$$

For maximization-type objective function, the ideal and anti-ideal solutions can be similarly defined.

Assume that all the fuzzy coefficients are trapezoidal fuzzy numbers. Trapezoidal fuzzy number R (Fig. 2) can be defined. $\widetilde{R}$=(r(1), r(2), r(3), r(4)) and the membership function of this fuzzy number can be interpreted by (2). Hence, an α-cut of $\widetilde{R}$ can be represented by the following interval

$${}^{\alpha}(\widetilde{R}) = [{}^{\alpha}(\widetilde{R})^{L},\ {}^{\alpha}(\widetilde{R})^{U}]$$
$$= [(r^{(2)} - r^{(1)})\alpha + r^{(1)} - (r^{(4)} - r^{(3)})\alpha + r^{(4)}] \quad (31)$$

For a given value of α, under the framework of GP, the FGP model of the problem under a pre-emptive priority structure can be presented as:

Find X so as to minimize

$$\overline{\xi} = [P_1(D^-), P_2(D^-), \cdots, P_s(D^-), P_S(D^-)] \quad (32)$$

subject to

$$\left[{}^{\alpha}(Z_k)^{-} - \sum_{j=1}^{q} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} . X_j\right] \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O}\right] + D_k^- \geq 1,$$
$$k = 1, 2, \cdots, K$$

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{U} X_j \geq {}^{\alpha}(\tilde{B}_i)^{L},\ i = 1, 2, \cdots p_1,\ p_2+1, \cdots, p$$

$$\sum_{j=1}^{q} {}^{\alpha}(\tilde{A}_{ij})^{L} X_j \leq {}^{\alpha}(\tilde{B}_i)^{U},\ i = p_1+1, \cdots, p_2,\ p_2+1, \cdots, p$$

$$X_j \geq 0,\ j = 1, 2, \cdots, q,\ D_k^- \geq 0,\ k = 1, 2, \cdots, K$$

Using the interval expression, Eq. (32) can be written as:

$$\overline{\xi} = [P_1(D^-), P_2(D^-), \cdots, P_s(D^-), \cdots, P_S(D^-)] \quad (33)$$

Subject to

$$\left\{{}^{\alpha}(Z_k)^{-} - \sum_{j=1}^{q}\left[C_{kj}^{(1)} + \left(C_{kj}^{(2)} - C_{kj}^{(1)}\right)\alpha\right]X_j\right\} \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O}\right] + D_k^- \geq 1$$

$$k = 1, 2, \cdots, K$$

$$\left[A_{ij}^{(4)} + (A_{ij}^{(4)} - A_{ij}^{(3)})\alpha\right].X_j \geq B_j^{(1)} - (B_j^{(2)} - B_j^{(1)}).\alpha$$

$$i = 1, 2, \cdots p_1,\ p_2+1, \cdots, p$$

$$\left[A_{ij}^{(1)} + (A_{ij}^{(2)} - A_{ij}^{(1)})\alpha\right].X_j \leq B_j^{(4)} - (B_j^{(4)} - B_j^{(3)}).\alpha,$$

$$i = 1, 2, \cdots p_1,\ p_2+1, \cdots, p$$

$$X_j \geq 0,\ j = 1, 2, \cdots, q,\ D_k^- \geq 0,\ k = 1, 2, \cdots, K$$

4 Fuzzy programming technique for MOTP

4.1 Mathematical model

Suppose that there are m origins (sources) $O_1, O_2, \cdots, O_m$ and n destinations $D_1, D_2, \cdots, D_n$. At each origin O_i ($i = 1, 2, \cdots, m$), let A_i be the amount of homogeneous product available to be transported to n destinations D_j ($j = 1, 2, \cdots, n$), to meet the demand for B_j units of the product there. There is a penalty C_{ij}^{K} associated with transporting a unit of the product from source O_i to destination D_j. In general, the penalty represents transportation cost, delivery time, deterioration amount of the product, and so on. Suppose that X_{ij} ($i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$) represents the unknown quantity to be transported from origin O_i to destination D_j, the mathematical model of the MOTP can be represented as the following vector minimization problem (VMP)

$$\text{Minimize } Z_k(\overline{X}) \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} X_{ij},\ k = 1, 2, \cdots, K \quad (34)$$

subject to

$$\sum_{j=1}^{n} X_{ij} = A_i, \quad i = 1, 2, \cdots, m \quad (35)$$

$$\sum_{i=1}^{m} X_{ij} = B_j \quad j = 1, 2, \cdots, n \quad (36)$$

Assume that A_i>0, B_j>0, $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$ and balanced condition

$$\sum_{i=1}^{m} A_i = \sum_{j=1}^{n} B_j \quad (37)$$

The balanced condition is a necessary and sufficient condition for the existence of a feasible solution.

4.2 MOTP with trapezoidal fuzzy coefficient

The MOTP with fuzzy (Trapezoidal fuzzy number) coefficients can be represented by VMP as follows:

$$\min \widetilde{Z}_k(\overline{X}) = \min \sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij}^{k(1)}, C_{ij}^{k(2)}, C_{ij}^{k(3)}, C_{ij}^{k(4)})X_{ij}, \quad k = 1, 2, \cdots, K \tag{38}$$

subject to

$$\sum_{j=1}^{n} X_{ij} = \left(A_i^{(1)}, A_i^{(2)}, A_i^{(3)}, A_i^{(4)}\right)X_{ij} \quad i = 1, 2, \cdots, m \tag{39}$$

$$\sum_{j=1}^{n} X_{ij} = \left(B_j^{(1)}, B_j^{(2)}, B_j^{(3)}, B_j^{(4)}\right)X_{ij}, \quad j = 1, 2, \cdots, n \tag{40}$$

$$X_{ij} \geq 0, \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n \tag{41}$$

where $(C_{ij}^{k(1)}, C_{ij}^{k(2)}, C_{ij}^{k(3)}, C_{ij}^{k(4)})$ represents the trapezoidal fuzzy number penalties for the transportation problem. The source and destination parameters are also trapezoidal fuzzy numbers of the form, $\left(A_i^{(1)}, A_i^{(2)}, A_i^{(3)}, A_i^{(4)}\right)$ and $\left(B_j^{(1)}, B_j^{(2)}, B_j^{(3)}, B_j^{(4)}\right)$, respectively.

Then the problem can be reduced to

$$\text{Minimize } \widetilde{Z}_k(\overline{X}) \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{ij}^{k(1)} + \left(C_{ij}^{k(2)} - C_{ij}^{k(3)}\right)\alpha\right]X_{ij}, \quad k = 1, 2, \cdots, K \tag{42}$$

subject to

$$\sum_{j=1}^{n} X_{ij} \geq A_i^{(1)} + \left(A_i^{(2)} - A_i^{(1)}\right)\alpha \quad i = 1, 2, \cdots, m \tag{43}$$

$$\sum_{j=1}^{n} X_{ij} \leq A_i^{(4)} - \left(A_i^{(4)} - A_i^{(3)}\right)\alpha \quad i = 1, 2, \cdots, m \tag{44}$$

$$\sum_{j=1}^{n} X_{ij} \geq B_i^{(1)} + \left(B_i^{(2)} - B_i^{(1)}\right)\alpha \quad j = 1, 2, \cdots, n \tag{45}$$

$$\sum_{j=1}^{n} X_{ij} \leq B_i^{(4)} - \left(B_i^{(4)} - B_i^{(3)}\right)\alpha \quad j = 1, 2, \cdots, n \tag{46}$$

Denote the system constraints (43), (44), (45), and (46) by S!

4.3 Priority based FGP formulation of MOTP

Resulting membership functions for the minimization objective function are defined as

$${}^{\alpha}\mu_k\left(Z_k\left(\overline{X}\right)\right) = \left\{{}^{\alpha}(Z_k)^{-} - \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{ij}^{k(1)} + \left(C_{ij}^{k(2)} - C_{ij}^{k(1)}\right)\alpha\right]X_{ij}\right\} \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{o}\right] \tag{47}$$

where the aspired level ${}^{\alpha}(Z_k)^{O}$ and highest acceptable level ${}^{\alpha}(Z_k)^{-}$ are ideal and anti-ideal solutions, respectively, which can be obtained by solving each of the following problems independently:

$${}^{\alpha}(Z_k)^{O} = \min_{\overline{X} \in S} \sum_{i=1}^{m}\sum_{j=1}^{n}[C_{ij}^{k(1)} + (C_{ij}^{k(2)} - C_{ij}^{k(1)})\alpha]X_{ij} \tag{48}$$

$${}^{\alpha}(Z_k)^{-} = \max_{\overline{X} \in S} \sum_{i=1}^{m}\sum_{j=1}^{n}[C_{ij}^{k(4)} - (C_{ij}^{k(4)} - C_{ij}^{k(3)})\alpha]X_{ij}, \quad k = 1, 2, \cdots, K \tag{49}$$

For a given value of α, under the framework of minsum GP, the FGP model of the problem can be explicitly formulated as follows:

Find $\overline{X}$ so as to minimize

$$\overline{\xi} = [P_1(D^-), P_2(D^-), \cdots, P_s(D^-), \cdots, P_S(D^-)] \tag{50}$$

subject to

$$\left({}^{\alpha}(Z_k)^{-} - \sum_{i=1}^{m}\sum_{j=1}^{n}\left[C_{ij}^{k(1)} + (C_{ij}^{k(2)} - C_{ij}^{k(1)})\alpha\right]X_{ij}\right) \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{o}\right] + D_k^- \geq 1$$

$$D_k^- \geq 0, \quad k = 1, 2, \cdots, K, \quad X_j \geq 0, \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n$$

where ξ represents the vector of the sth priority achievement functions. D_k^- is the negative deviational variable associated with the kth goal. $P_i(D\text{-})$ is a linear function of the weighted negative deviational variables, where $P_s(D\text{-})$ is of the form

$$P_S(D^-) = \sum_{k=1}^{K} W_{sk}^{-} D_{sk}^{-} \quad (k = 1, 2, \cdots, K, \; S \leq K) \tag{51}$$

where D_{sk}^- is renamed for D_k^-, to represent it at the sth priority level, W_{sk}^- is the numerical weight associated with D_{sk}^- and represents the weight of importance of achieving the aspired level of the kth goal, relative to other goals, which are grouped together at the sth priority level. The weights are determined as[13]:

$$W_{SK}^{-} = 1/[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{o}] \; k = 1, 2, \cdots, K \tag{52}$$

where $[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{o}]$ is the symbolic representation of $[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{o}]$, at the sth priority level.

In a pre-emptive priority FGP, the sth priority P_s is preferred to the next priority P_s+1, regardless of any weight associated with P_s+1. The relationships among the priorities are

$$P_1 >>> P_2 >>> \cdots >>> P_s >>> \cdots >>> P_S \tag{53}$$

Expression (53) indicates that the goals at the highest priority level (P_1) have been achieved to the maximum extent possible, before the set of goals at the second priority level (P_2) are taken into consideration and the process goes on until the last priority level P_S is considered.

It needs to be noted here that the question about the increase of computational burden involved under different structure of priorities to the membership goals may arise. If S be the total number of priority levels, then S! priority structures may be involved there. However, not more than two-to-five priority levels are important in any real world decision making problem and the conflict of assigning priorities arises at the most in three priority levels[2].

4.4 Selection of appropriate priority structure: use of Euclidean distance function

In the priority based FGP approach, the priorities are assigned to the goals on the basis of the importance of achieving their aspired levels in the decision context. However, it is worthy to observe that in the highly conflicting decision situation, the decision making unit feels confused with regard to assigning the appropriate priority structure for achieving their aspired goals, where the decision changes with the change of priorities to the goals. Consequently, a decision deadlock arises frequently in the decision -making situations.

To deal with such problems, in the proposed MOTP formulation, the concept of Euclidean distance function for group decision analysis introduced by Yu[16], can be applied, for measuring the ideal point dependent solution, to identify the appropriate priority structure of the goals under which the most satisficing solution is achieved.

In the present FGP formulation of the MOTP, as the highest membership value of each fuzzy goal is unity, the ideal point would be a vector with each element equal to unity. Suppose S be the total number of different possible priority structures that arise in the decision situation, the Euclidean distance function can be represented as:

$$D^s = [\sum_{k=1}^{K}(1-\mu_{Zk}^{S})^2]^{1/2} \tag{54}$$

where μ_{Zk}^{S} represents the achieved membership value of the kth goal under the sth priority structure of the goals. Now, the solution for which the Euclidean distance function is minimal would be the most satisficing solution. Here, it can be easily realized that the solution that is closest to the ideal point must correspond to $\min_{s=1,2,\dots,S} D^s$ $=DM$ where $1\le M\le S$.

Then the M-th priority structure can be identified as an appropriate priority structure to achieve the most satisficing solution.

5 Numerical example

To illustrate the FGP approach, consider the following MOTP[12] problem with fuzzy coefficient.

Minimize $\tilde{Z}_1(\overline{X})=X_{11}+\tilde{4}\ X_{12}+\tilde{5}\ X_{13}+4X_{21}+6.5X_{22}+5X_{23}$

Minimize $\tilde{Z}_2(\overline{X})=\tilde{2}\ X_{11}+\tilde{3}\ X_{12}+4X_{13}+X_{21}+9X_{22}+\tilde{6}\ X_{23}$

Minimize $\tilde{Z}_3(\overline{X})=4X_{11}+2X_{12}+10X_{13}+\tilde{3}\,X_{21}+\tilde{8}\,X_{22}+X_{23}$

Subject to $\sum_{j=1}^{3}X_{1j}=\tilde{11}$, $\sum_{j=1}^{3}X_{2j}=\tilde{15}$, $\sum_{j=1}^{2}X_{i1}=\tilde{7}$,

$\sum_{j=1}^{2}X_{i2}=\tilde{10}$, $\sum_{j=1}^{2}X_{i3}=\tilde{9}$

$$X_{ij}\ge 0,\quad i=1,2,\quad j=1,2,3 \tag{55}$$

where all the fuzzy numbers are assumed to be triangular fuzzy numbers and are given as follows:
$\tilde{4}$ =(3, 4, 5), $\tilde{5}$ =(4, 5, 6), $\tilde{2}$ =(0, 2, 3), $\tilde{3}$ =(2, 3, 4), $\tilde{6}$ =(4, 6, 8), $\tilde{3}$ =(2, 3, 4), $\tilde{8}$ =(6, 8, 10), $\tilde{11}$=(9, 11,13), $\tilde{15}$=(10, 15, 20), $\tilde{7}$ =(6, 7, 8), $\tilde{10}$=(9, 10, 11), $\tilde{9}$ =(8, 9, 10).

By replacing the fuzzy coefficients by their α-cuts, (55) can be transformed into the following problem:

Minimize $^{\alpha}(Z_1(\overline{X})^L)=X_{11}+(3+\alpha)X_{12}+(4+\alpha)X_{13}+4X_{21}+6.5X_{22}+5X_{23}$

Minimize $^{\alpha}(Z_2(\overline{X})^L)=(2\alpha)X_{11}+(2+\alpha)X_{12}+4X_{13}+X_{21}+9X_{22}+(4+2\alpha)X_{23}$

Minimize $^{\alpha}(Z_3(\overline{X})^L)=4X_{11}+2X_{12}+10X_{13}+(2+\alpha)X_{21}+(6+2\alpha)X_{22}+X_{23}$

Subject to $X_{11}+X_{12}+X_{13}\le 13-2\alpha$,
$X_{11}+X_{12}+X_{13}\ge 9+2\alpha$,
$X_{21}+X_{22}+X_{23}\le 20-5\alpha$,
$X_{21}+X_{22}+X_{23}\ge 10+5\alpha$,
$X_{11}+X_{21}\le 8-\alpha$,
$X_{11}+X_{21}\ge 6+\alpha$,
$X_{12}+X_{22}\le 11-\alpha$,
$X_{12}+X_{22}\ge 9+\alpha$,
$X_{13}+X_{23}\le 10-\alpha$,
$X_{13}+X_{23}\ge 8+\alpha$,
$X_{ij}\ge 0,\quad i=1,2,\ j=1,2,3$ (56)

For α=0.4, $^{\alpha}(Z_1)^-$ =94, $^{\alpha}(Z_1)^O$ =79, $^{\alpha}(Z_2)^-$ =199, $^{\alpha}(Z_2)^O$=52, $^{\alpha}(Z_3)^-$= 242, $^{\alpha}(Z_3)^O$=38.

Subsequently following the proposed procedure, the executable FGP model under a given priority structure can be obtained using Eq. (50). In the proposed solution process here, three priority factors P_i (i=1, 2, 3) are considered for achievement of the aspired levels of the stated fuzzy goals. These are executed under three different priority structures, applying the software LINGO (Ver.6.0). The results obtained for different priority structures are shown in Table 1. Now, observing the results in Table 1, the Euclidean distance value 0.026 334507 is found as the minimum. The results reflect that the priority structure under execution 1 is the appropriate one to obtain the most satisfactory solution. The optimal solution for the priority structure [$P_1(d_1/61.52+ d_2/110.2)$, $P_2(d_3/178.720)$] is given by

λ*=0.0001202221, $X_{11}^*=2.8$, $X_{12}^*=9.4$, $X_{13}^*=0$, $X_{21}^*=3.6$ $X_{22}^*=0$, $X_{23}^*=8.4$; $Z_1^*=91.16$, Z^2 =68.72, $Z_2^*=68.72$, $Z_3^*=47.04$. Sum of the objective functions $\sum_{i=1}^{3}Z_i^*=206.92$.

The membership functions are $\mu_1(Z_1)$, $\mu_2(Z_2)$, $\mu_3(Z_3)$=(1, 0.984 755, 0.978 5139); D_2=0.026 34507.

Table 1 Sensitivity analysis with variation of priority structure

Execution number	Priority structure	Membership values	Euclidean distance D_2
1	[$P_1(d_1/61.52+d_2/110.2)$, P_2 $(d_3/178.72)$]	1, 0.984 755, .0978 5139	0.026 345
2	[$P_1(d_2/110.2+d_3/178.72)$, P_2 $(d_1/61.52)$]	0.882 9649, 0.980 3993, 1	0.118 665
3	[$P_1(d2/110.2,)$ $P_2(d_3/178.723+ d_1/61.52)$]	0.890 7672, 1, 0.862 5783	0.175 546

6 Conclusions

This paper shows how the concept of Euclidean distance can be used for modeling MOTP with fuzzy parameters and solving them efficiently using priority based FGP under a priority structure to arrive at the most satisfactory decision in the decision making environment, on the basis of the needs and desires of the decision making unit. The proposed approach can be extended to optimization problems in different areas, such as. decentralized planning problems, agricultural planning problems, and other real world multiobjective programming problems, involved with fuzzily described different parameters in the decision making context.

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