A fuzzy group Electre method for safety and health assessment in hazardous waste recycling facilities

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Abstract

Decision making in environmental problems is a complex task due to multiple and often conflicting criteria, varying measurements, qualitative and quantitative input parameters, and lack of exact data. In this paper, we propose a multi-criteria decision making (MCDM) model based on an integrated fuzzy approach in the context of Hazardous Waste Recycling (HWR). The proposed method: (a) takes into consideration judgments provided by multiple decision makers (DMs); (b) is based on a structured but yet flexible framework; (c) considers quantitative objective data and qualitative subjective judgments; (d) captures the ambiguity and imprecision in DMs’ judgments, and (e) results in a final priority ranking which is not vague. We demonstrate the application of the proposed model for safety and health assessment in HWR facilities.

1. Introduction

The growth of population and increasing industrialization have contributed to the steady growth of hazardous wastes. As a result, industry, government, and the general public have become increasingly aware of the need to respond to the hazardous waste problem. Land filling and incineration are the most common means of disposing hazardous wastes. However, the deposition and burning of hazardous waste causes a profound strain on the environment due to the potential contamination of the water resources and the carbon dioxide release from the incineration plants. Hazardous wastes that escape into the environment most often impact the public health through air and water contamination. An effective strategy for managing hazardous wastes is recycling (Orloff and Falk, 2003). Hazardous Waste Recycling (HWR) facilities are established to treat, store or dispose hazardous wastes. An improper control of hazardous wastes can result in a severe threat to site workers and to the general public. By anticipating and taking steps to prevent potential hazards to health and safety, work at the HWR facilities can proceed with minimum risk to workers and the public.

The safety and health assessment in the workplace is a complex multi-criteria decision making (MCDM) problem with multiple and often conflicting quantitative and qualitative criteria (Dagdeviren et al., 2008; Grassi et al., 2009; Mojtahedi et al., 2010; Zheng et al., 2012). Typical environmental safety and health assessment is performed using a cost-benefit analysis (Khadam and Kaluarachchi, 2003). The comparison of different decision alternatives is measured by an economic index such as the cost-benefit ratio. This economic index may also represent uncertainty by incorporating the expected cost of failure. This formulation of the risk–cost–benefit is widely used because of its simplicity and the ease of interpreting the results in monetary terms. Khadam and Kaluarachchi (2003) have presented a detailed discussion on the limitations of the existing risk–cost–benefit methods used in environmental safety and health assessment. They provided evidence for the need to improve the current methods with real-world applications and knowledge gathered from other fields.

In this study, we present a fuzzy group Electre (Elimination Et Choix Traduisant la Réalité) method for safety and health assessment in HWR facilities. The proposed method: (a) takes into consideration judgments provided by multiple decision makers (DMs), (b) is based on a structured but yet flexible framework, (c) considers quantitative objective data and qualitative subjective judgments, and (d) captures the ambiguity and imprecision in DMs’ judgments, and (e) results in a final priority ranking which is not vague. We demonstrate the application of the proposed model for safety and health assessment in HWR facilities.

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judgments, (d) captures the ambiguity and impreciseness in DMs’ judgments, and (e) results in a final priority ranking which is not vague.

The rest of the paper is organized as follows: In Section 2, we provide a review of literature on HWR and the fuzzy Electre methods. In Section 3, we review some basic definitions of fuzzy sets theory. In Section 4, we present the fuzzy group Electre method proposed in this study. In Section 5, we demonstrate the application of the proposed method for safety and health assessment in HWR Facilities. In Section 6, we conclude with our conclusions and future research direction.

2. Literature review

In this section, we present a review of literature on HWR and the fuzzy Electre method.

2.1. Hazardous waste recycling

Environmentally sound management of hazardous wastes involves taking all necessary steps to ensure that hazardous wastes are managed in a manner that will protect human health and the environment. As Berger et al. (1999) and Tanskanen (2000) point out, the first solid waste management models were quantitative and analytical models that dealt with specific aspects of the problem (e.g., vehicle routing, Truitt et al. (1989) and transfer station sitting, Esmaili (1972)). Morrissey and Browne (2004) categorized the current waste management models into three categories – those based on cost-benefit analysis, those based on life-cycle analysis and those based on the use of multi-criteria techniques such as Electre.

The cost-benefit analysis models enable decision makers (DMs) to assess the positive and negative effects of a set of alternatives by translating all impacts into some monetary measurement (e.g., Elagroudy et al., 2011; Mutavchi, 2012; Vigso, 2004). On completion of a cost-benefit analysis, the alternative with the greatest benefit and least cost is the preferred choice. The life-cycle analysis models study the environmental aspects of a product throughout its life from raw material acquisition through production, use and final disposal (i.e. Denison, 1996; McDougall et al., 2001; Rodriguez-Iglesias et al., 2003). Most life cycle studies have been comparative assessments of substitutable products delivering similar functions. The multi-criteria decision making (MCDM) models allow DMs to analyze the alternatives from several points of view by simultaneously taking several independent and often conflicting criteria into account (i.e. Sobral et al., 1981; Takeda, 2001; Rousis et al., 2008).

The normal approach in MCDM is to identify several alternatives (i.e. different waste management scenarios) and then evaluate them in terms of multiple criteria (i.e. risk assessment or environmental impact assessment). The result is a ranking of the alternatives. Salminen et al. (1998) compared three different multi-criteria methods based on the utility theory of Keeney and Raiffa (1976) and concluded that Electre was the most suitable MCDM method for solving environmental problems.

More recently, Luria and Aspinall (2003) proposed an industrial hazard assessment approach. Their experimental model was based on a MCDM approach – the Analytic Hierarchy Process – which introduced the use of expert opinions, complementary skills and expertise from different disciplines in conjunction with traditional quantitative analysis. This permitted the generation of quantitative data on risk assessment from a series of qualitative assessments. Daniel et al. (2004) used different models in order to extend the usability of the environmental design for industrial products. Furthermore, they examined the results that were produced by applying different impact assessment methods in the cases of the recovery and disposal chains of lead–acid batteries. Cloquell-Ballester et al. (2007) argued that the decision techniques used in the process of environmental management of hazardous wastes suffer from two drawbacks, namely the problem of compensation and the problem of identification of the “exact boundary” between sub-ranges. They discussed these issues and proposed a methodology for determining the significance of environmental impacts based on comparative and sensitivity analyses using the Electre technique.

Rousis et al. (2008) proposed a MCDM model to evaluate and rank 12 electrical waste management systems in Cyprus. They found that the optimum system that can be implemented in Cyprus is that of partial disassembly and forwarding of recyclable materials to the native existing market and disposal of the residues at landfill sites. Erkut et al. (2008) presented a multi-criteria mixed-integer linear programming model to solve the location–allocation problem for municipal solid waste management at the regional level. They compared regional and prefectoral solid waste management planning in Central Macedonia. Peche and Rodríguez (2009) presented an approach based on fuzzy logic to assess the environmental impact related to the execution of activities and projects where the information related to the impacts was limited. In their environmental impact assessment model, the impact properties were described by fuzzy numbers. Dursun et al. (2011) proposed MCDM techniques for conducting an analysis based on multi-level hierarchical structure and fuzzy logic for the evaluation of healthcare waste treatment alternatives. In addition to having the capability to consider multiple attributes that were structured in a multi-level hierarchy, the proposed approaches enabled the DMs to use linguistic terms. We propose a fuzzy group Electre method for safety and health assessment in HWR facilities.

2.2. Fuzzy Electre method

The MCDM methods are frequently used to solve risk assessment problems with multiple, conflicting, and incommensurate criteria. MCDM problems are generally categorized as continuous or discrete, depending on the domain of alternatives. Hwang and Yoon (1981) have classified the MCDM methods into two categories: multi-objective decision making (MODM) and multi-attribute decision making (MADM). MODM has been widely studied by means of mathematical programming methods with well-formulated theoretical frameworks. MODM methods have decision variable values that are determined in a continuous or integer domain with either an infinitive or a large number of alternative choices, the best of which should satisfy the DM constraints and preference priorities (Hwang and Masud, 1979; Ehrgott and Wiecek, 2005). MADM methods, on the other hand, have been used to solve problems with discrete decision spaces and a predetermined or a limited number of alternative choices. The MADM solution process requires inter and intra-attribute comparisons and involves implicit or explicit tradeoffs (Hwang and Yoon, 1981).

MADM methods are used for circumstances that necessitate the consideration of different options that cannot be measured in a single dimension. Each method provides a different approach for selecting the best among several preselected alternatives (Jancic and Reggiani, 2002). The MADM methods help DMs learn about the issues they face, the value systems of their own and other parties, and the organizational values and objectives that will consequently guide them in identifying a preferred course of action. The primary goal in MADM is to provide a set of attribute-aggregation methodologies for considering the preferences and judgments of DMs (Doumpos and Zopounidis, 2002). Several methods have been proposed for solving MADM problems (i.e., Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity
to the Ideal Solution (TOPSIS), Preference Ranking Organization Method (PROMETHEE) and Electre.

The AHP was originally introduced by Saaty (1977). The AHP simplifies complex and ill-structured problems by arranging the decision attributes and alternatives in a hierarchical structure with the help of a series of pairwise comparisons. Saaty (2000) argues that a DM finds it more natural to compare two things than to compare all things together in a list. The AHP has been a controversial technique in the operations research community. Dyer (1990) has questioned the theoretical basis underlying the AHP and argues that it can lead to preference reversals based on the alternative set being analyzed. This rank reversal is likely to occur when a copy or a near copy of an existing option is added to the set of alternatives that are being evaluated (Wang and Elhag, 2006). The large number of pairwise comparisons is another weakness of the AHP (Macharis et al., 2004). Another important disadvantage of the AHP is the artificial limitation of the use of a 9-point scale. Sometimes DMs find it difficult to use this scale and determine for example whether an alternative is six or seven times more important than another. Also, the AHP cannot account for the fact that alternative A might be 20 times more important than alternative B since it is limited by its 9-point scale (Hajkowicz et al., 2000).

The TOPSIS method was initially proposed by Hwang and Yoon (1981). Its basic principle is that the chosen alternatives should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution (Lai et al., 1994; Olson, 2004) has shown that TOPSIS performs better than the AHP in matching a base prediction model but results in more rank reversals when there are a few attributes. TOPSIS also performs less accurately than the AHP on both selecting the top ranked alternative and in converting the original scales into abstract ones with an arbitrary meaning), and the ability to take into consideration the DM’s indifference (Martel and Roy, 2006). Electre is the need for precise measurement of the performance ratings and criteria weights (Figueira et al., 2005). However, in many real-world problems, importance weights and performance ratings cannot be measured precisely as some DMs may express their judgments using linguistic terms such as low, medium or high (Zadeh, 1975; Tsaur et al., 2002). The fuzzy set theory is ideally suited for handling these ambiguities encountered in solving MADM problems. Fuzzy logic – together with fuzzy arithmetic – could be used to develop procedures for treating vague and ambiguous information which is frequently expressed with linguistic variables and whose inaccuracy is not particularly due to the variability of the measures, but to the uncertainties inherent in the available information. Since Zadeh (1965) introduced fuzzy set theory, and Bellman and Zadeh (1970) described the decision making method in fuzzy environments, an increasing number of studies have dealt with uncertain fuzzy problems by applying fuzzy set theory (Yager, 1977; Zimmermann, 1996).

According to Zadeh (1975), it is very difficult for conventional quantification to reasonably express complex situations and it is necessary to use linguistic variables whose values are words or sentences in a natural or artificial language. In response, several researchers have recently proposed various fuzzy Electre methods in the literature (Monzeta et al., 2009; Sevkli, 2010; Hatami-Marbini and Tavana, 2011; Zandi et al., 2011; Daneshvar Rouyendegh and Erkan, 2012). Fuzzy set theory has also been widely used to assess environmental impacts, indicating the potential of fuzzy logic in this field (Blanco et al., 2009; Chen, 2009; De Siqueira and De Mello, 2006; Duarte et al., 2007; Enea and Saleni, 2001; Liou et al., 2006; Liu and Lai, 2009; Parashar et al., 1997; Peche and Rodriguez, 2011; Silver, 1997).

3. Fuzzy sets theory

In this section, we review some basic definitions of fuzzy sets defined by Dubois and Prade (1978), Kiler and Yuan (1995), and Zimmermann (1996):

**Definition 1.** Let X be a universe set. The fuzzy set \( A \) in the universe of discourse \( X \) is characterized by a membership function \( \mu_A(x) : [0, 1] \), where \( \mu_A(x), \forall x \in X \), indicates the degree of membership of \( A \) to \( X \).

**Definition 2.** A triangular fuzzy number \( A \) is described as the triplet \((a_l, a, a_u)\), \( a_l < a < a_u \). The membership function \( \mu_A(x) \) is defined by...
\[ \mu_A(x) = \begin{cases} 
0, & x \leq a', \\
\frac{x-a'}{a''-a'}, & a' < x < a'', \\
1, & x = a', \\
\frac{x-a''}{a''-a'}, & a'' < x < a', \\
0, & x \geq a'. 
\end{cases} \]  

(1)

**Definition 3.** For any two fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \), the Hamming distance \( (\tilde{A}, \tilde{B}) \) is defined by the following formula proposed by Hamming (1950):

\[ \int_{\mathbb{R}} |\mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)| \, dx \]  

(2)

where \( \mathbb{R} \) is the set of real numbers.

**Definition 4.** A linguistic variable is a variable whose values are expressed in linguistic terms. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions. For example, “weight is a linguistic variable whose values can be defined as very low, low, medium, high, very high, etc. Fuzzy numbers are able to represent these linguistic values with fuzzy numbers.

**Definition 5.** Assuming two fuzzy sets, \( \tilde{A} \) and \( \tilde{B} \), their standard intersection, \( A \cap B \) and their standard union, \( A \cup B \), are defined for all \( x \in X \) as:

\[ (\tilde{A} \cap \tilde{B})(x) = \min[A(x), B(x)] \]

\[ (\tilde{A} \cup \tilde{B})(x) = \max[A(x), B(x)] \]  

(3)

where \( \min \) and \( \max \) refer to the minimum and maximum operators, respectively.

**Definition 6.** Let us further consider the two fuzzy sets, \( \tilde{A} \) and \( \tilde{B} \), defined on the universal set with X continuous membership function and \( A \cap B = \emptyset \). Assume that \( x_m \in X \) is the point such that \( (A \cap B)(x_m) \geq (A \cap B)(x) \) for all \( x \in X \), and \( x_m \) is between two mean values of \( A \) and \( B \) (if the number \( x_m \) is not unique anyone point of those \( x_m \) is suitable). Then, as suggested by Chiu and Wang (2002), the operation max can be implemented as follows:

\[ \max(A, B) = \begin{cases} 
(A \cap B)(x), & z < x_m, \\
(A \cup B)(x), & z \geq x_m 
\end{cases} \]  

(4)

where \( z \in X \), and \( \cup \) and \( \cap \) denote the standard fuzzy intersection and union, respectively.

4. Proposed framework

The Electre method is quick, operates with simple logic, and has the strength of being able to detect the presence of incompatibility, it uses a systematic computational procedure, an advantage of which is an absence of strong axiomatic assumptions (Shanian and Savadogo, 2006). The fuzzy group Electre method proposed in this study is an extension of the Electre I method described next through a series of structured and successive steps depicted in Fig. 1.

**Step 1: Construct a fuzzy decision matrix:** Assume that a decision-making committee involves \( K \) decision makers (DMs) \( D_k (k = 1, 2, \ldots, K) \). The DMs are expected to determine the importance weights of \( n \) attributes \( C_j (j = 1, 2, \ldots, n) \) and the performance ratings of \( m \) possible alternatives \( A_i (i = 1, 2, \ldots, m) \) on the attributes by means of linguistic variables. These linguistic variables will be transformed into positive triangular fuzzy numbers. The fuzzy ratings of the alternatives and the fuzzy importance weights of the attributes for each DM are characterized by \( \tilde{x}_{ijk} = (x_{jk}^l, x_{jk}^m, x_{jk}^u) \) and \( \tilde{w}_i = (w_{ij}^l, w_{ij}^m, w_{ij}^u) \), respectively (\( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \), \( k = 1, 2, \ldots, K \)). For simplicity, we apply the average value method to get the consensus of the DMs’ opinions (Wang and Chang, 2007). We also consider a voting power for each DMs, \( z_k \), as the proportion of the total power (where the total power is normalized to 1) according to some pre-specifies rule(s). In contrast, the DMs can give equal weights where appropriate. Thus, the aggregated fuzzy ratings of the alternatives can be computed as follows:

\[ \tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u), \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \]  

(5)

where

\[ x_{ij}^l = \frac{1}{K} \sum_{k=1}^{K} z_k \tilde{x}_{ijk}, \quad x_{ij}^m = \frac{1}{K} \sum_{k=1}^{K} z_k \tilde{x}_{ijk}, \quad x_{ij}^u = \frac{1}{K} \sum_{k=1}^{K} z_k \tilde{x}_{ijk} \]  

(6)

and \( z_k \) is the voting power of the \( k \)th DM. Analogously, the aggregated fuzzy importance weights of the attributes can be calculated as

\[ \tilde{w}_i = (w_{ij}^l, w_{ij}^m, w_{ij}^u), \quad j = 1, 2, \ldots, n \]

(7)

where

\[ w_{ij}^l = \frac{1}{K} \sum_{k=1}^{K} z_k w_{ij}^k, \quad w_{ij}^m = \frac{1}{K} \sum_{k=1}^{K} z_k w_{ij}^k, \quad w_{ij}^u = \frac{1}{K} \sum_{k=1}^{K} z_k w_{ij}^k \]  

(8)

Therefore, the decision problem is expressed in matrix format as

\[
\bar{U} = \begin{bmatrix}
\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\
\tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn}
\end{bmatrix}, \quad \bar{w} = (w_1, w_2, \ldots, w_n)
\]

(9)

where \( \tilde{x}_{ij} \) is the fuzzy importance of the \( i \)th alternative with respect to the \( j \)th attribute and \( \tilde{w}_i \) is the fuzzy weight of the \( j \)th attribute.

**Step 2: Normalize the fuzzy decision matrix:** A linear scale normalization is applied next to ensure that all values in the decision matrix have homogeneous and comparable units. Moreover, this transformation guarantees that every triangular fuzzy number belongs to \([0, 1]\) so that the complexity of the mathematical operations can be reduced. The normalized fuzzy decision matrix is constructed as follows:

\[ \bar{R} = [r_{ij}]_{m \times n} \]  

(10)

\[ r_{ij} = \left( r_{ij}^l, r_{ij}^m, r_{ij}^u \right) = \left( \frac{x_{ij}^l}{c_j}, \frac{x_{ij}^m}{c_j}, \frac{x_{ij}^u}{c_j} \right), \quad i = 1, 2, \ldots, m, \quad j \in B. \]  

(11)

\[ c_j = \max(x_{ij}^u), j \in B \]

where \( B \) is the set of benefit attributes and

\[ r_{ij} = \left( r_{ij}^l, r_{ij}^m, r_{ij}^u \right) = \left( \frac{a_j}{x_{ij}^l}, \frac{a_j}{x_{ij}^m}, \frac{a_j}{x_{ij}^u} \right), \quad i = 1, 2, \ldots, m, \quad j \in C. \]  

(12)

\[ a_j = \min(x_{ij}^l), j \in C \]

where \( C \) is the set of cost attributes.

**Step 3: Compute the weighted normalized fuzzy decision matrix:** Assuming that the importance weights of the attributes are differ-
ent, the weighted normalized fuzzy decision matrix is obtained by multiplying the importance weights of the attributes and the values in the normalized fuzzy decision matrix.

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}$$

$$\tilde{v}_{ij} = (v_{ij}^L, v_{ij}^U, v_{ij}^m) = \tilde{w}_i(x)\tilde{r}_j = (w_{ij}^L, w_{ij}^U, w_{ij}^m)$$

Step 4: Calculate the distance between any two alternatives: The concordance and discordance matrices are constructed by utilizing the weighted normalized fuzzy decision matrix and paired comparison among the alternatives. Considering two alternatives $A_p$ and $A_q$, the concordance set is formed as $f^c = \{ j | \tilde{v}_{pj} \preceq \tilde{v}_{qj} \}$ where $f^c$ is the concordance coalition of the attributes in which $A_p \succeq A_q$, and the discordance set is defined as $f^d = \{ j | \tilde{v}_{pj} \succeq \tilde{v}_{qj} \}$ where $f^d$ is the discordance coalition and it is against the assertion $A_p \succeq A_q$. Note that $S$ is the outranking relation and $A_p \succeq A_q$ means that “$A_p$ is at least as good as $A_q$.”

In order to compare any two alternatives $A_p$ and $A_q$ with respect to each attribute, and to define the concordance and discordance sets, we specify the least upper bound of the alternatives, $\max(\tilde{v}_{pj}, \tilde{v}_{qj})$, and then, the Hamming distance method is used to which assumes that

$$\tilde{v}_{pj} \preceq \tilde{v}_{qj} \iff d(\max(\tilde{v}_{pj}, \tilde{v}_{qj}), \tilde{v}_{qj}) \geq d(\max(\tilde{v}_{pj}, \tilde{v}_{qj}), \tilde{v}_{pj})$$

$$d(\max(\tilde{v}_{pj}, \tilde{v}_{qj}), \tilde{v}_{pj}) \geq d(\max(\tilde{v}_{pj}, \tilde{v}_{qj}), \tilde{v}_{qj})$$

Step 5: Construct the concordance and discordance matrices: The concordance and discordance matrices are obtained based on the Hamming distances (see Definition 3).

The following concordance matrix is formed in which the elements are the fuzzy summation of the fuzzy importance weights for all the attributes in the concordance set.

$$\tilde{C} = \begin{bmatrix}
\tilde{c}_{1q} & \cdots & \tilde{c}_{1m} \\
\vdots & \ddots & \vdots \\
\tilde{c}_{m1} & \cdots & \tilde{c}_{m1}
\end{bmatrix}$$

where

$$\tilde{c}_{pq} = (c_{pq}^L, c_{pq}^U, c_{pq}^m) = \sum_{j \in f^c} \tilde{w}_j = (\sum_{j \in f^c} \tilde{w}_j^U, \sum_{j \in f^c} \tilde{w}_j^L, \sum_{j \in f^c} \tilde{w}_j^m)$$

We then determine the concordance level as $\tilde{C} = (c^L, c^U, c^m)$, where

$$c^L = \sum_{p=1}^{m} \sum_{q=1}^{m} c_{pq}^L, c^U = \sum_{p=1}^{m} \sum_{q=1}^{m} c_{pq}^U$$

$$c^m = \sum_{p=1}^{m} \sum_{q=1}^{m} c_{pq}^m$$

The discordance matrix is structured as

| STEP 1.0 | 1.1 Assess the importance weights of the attributes |
| STEP 1.0 | 1.2 Evaluate the performance ratings of the alternatives with respect to each attribute |
| STEP 1.0 | 1.3 Express linguistic assessments as fuzzy numbers |
| STEP 1.0 | 1.4 Aggregate the importance weights and the performance ratings of the DMs |
| STEP 1.0 | 1.5 Construct a fuzzy decision matrix |
| STEP 2.0 | 2.0 Normalize the fuzzy decision matrix |
| STEP 3.0 | 3.0 Compute the weighted normalized fuzzy decision matrix |
| STEP 4.0 | 4.0 Calculate the distance between any two alternatives |
| STEP 5.0 | 5.1 Construct the concordance matrix |
| STEP 5.0 | 5.2 Construct the discordance matrix |
| STEP 6.0 | 6.0 Construct the Boolean matrices E and F |
| STEP 7.0 | 7.0 Construct the general matrix |
| STEP 8.0 | 8.0 Construct a decision graph and rank the alternatives |

Fig. 1. The proposed framework.
where
\[
d_{pq} = \max_{j \neq f} \frac{\max\{|v_{pj} - v_{qj}|, \max\{|v_{pj}, v_{qj}|\}}{\max_{j} d (\max(v_{pj}, v_{qj}), v_{qj})} \\
\text{and the discordance level is defined as } D = \sum_{p}^{m} \sum_{q}^{m} d_{pq} / m (m - 1).
\]

Step 6: Construct the Boolean matrices \( E \) and \( F \). The Boolean matrix \( E \) is determined by a minimum concordance level, \( \tau \), as follows:
\[
E = \begin{bmatrix}
- & \cdots & e_{1q} & \cdots & e_{1(m-1)} & e_{1m} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
- & \cdots & e_{pq} & \cdots & e_{p(m-1)} & e_{pm} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
- & \cdots & e_{mq} & \cdots & e_{m(m-1)} & -
\end{bmatrix}
\]

where
\[
\begin{cases}
\hat{e}_{pq} \geq \hat{\tau} \Leftrightarrow e_{pq} = 1 \\
\hat{e}_{pq} < \hat{\tau} \Leftrightarrow e_{pq} = 0
\end{cases}
\]

and similarly, the Boolean matrix \( F \) is obtained based on the minimum discordance level, \( \overline{D} \), as follows:
\[
F = \begin{bmatrix}
- & \cdots & f_{1q} & \cdots & f_{1(m-1)} & f_{1m} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
- & \cdots & f_{pq} & \cdots & f_{p(m-1)} & f_{pm} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
- & \cdots & f_{mq} & \cdots & f_{m(m-1)} & -
\end{bmatrix}
\]

where
\[
\begin{cases}
d_{pq} < \overline{D} \Leftrightarrow f_{pq} = 1 \\
d_{pq} \geq \overline{D} \Leftrightarrow f_{pq} = 0
\end{cases}
\]

The elements in matrices \( E \) and \( F \) with the value of 1 indicate the dominance relation between alternatives.

Step 7: Construct the general matrix: By peer-to-peer multiplication of the elements of the matrices \( E \) and \( F \), the general matrix \( G \) is constructed as
\[
G = E \otimes F
\]

Step 8: Construct a decision graph and rank the alternatives: Concerning the general matrix, a decision graph is constructed in order to determine the ranking order of the alternatives. Let \( A_{p} \) and \( A_{q} \) be two alternatives. There is an arc between the two alternatives from \( A_{p} \) to \( A_{q} \) if alternative \( A_{q} \) outranks \( A_{p} \), there is no arc between the two alternatives if alternatives \( A_{p} \) and \( A_{q} \) are incomparable, and there are two arcs between the two alternatives in both directions if these alternatives are indifferent.

The mathematical and theoretical contributions of the framework proposed in this study are threefold: (1) we addressed the gap in the Electre literature for problems involving conflicting systems of criteria, uncertainty and imprecise information; (2) we extended the Electre method to take into account uncertain, imprecise and linguistic assessments; and (3) we defined outrankings by pairwise comparisons and used decision graphs to determine which alternative is preferable, incomparable or indifferent in the fuzzy environment.

5. Case study

In this section, we present the results of a pilot study conducted for the Environmental Health Department (EHD) in the State of East Virginia\(^1\) to assess the safety and health in selected HWR facilities using the framework proposed in this paper. The EHD mission is to protect the environment and public health from the effects of improper disposal of hazardous waste from homes and small businesses in the State of East Virginia. The EHD receives state and federal funding to provide the state with practical pollution prevention options for the use, recycling, and disposal of products containing hazardous substances. The following eight alternative facilities were selected to participate in this study by the EHD officials: Alexandria (A1), Birmingham (A2), Chester (A3), Dover (A4), Edgewood (A5), Fairfield (A6), Gibbstown (A7), and Hamilton (A8). The EHD officials selected one of their seasoned inspectors to lead this effort. The lead inspector (DM1) invited one colleague from the United States Environmental Protection Agency (EPA) (DM2) and another colleague from the United States Department of Labor – Occupational Safety and Health Administration (OSHA) (DM3) to join him in this collaborative initiative. The three members of the assessment team agreed to give the EHD inspector a 41% voting power (\( z_{1} = 0.41 \)), the OSHA inspector a 34% voting power (\( z_{2} = 0.34 \)), and the EPA inspector a 25% voting power (\( z_{3} = 0.25 \)). The assessment team also agreed to use the linguistic variables presented in Tables 1 and 2 for describing the importance weights and the performance ratings of the attributes, respectively.

As shown in Table 1, the fuzzy numbers \((1, 1, 3), (1, 3, 5), (3, 5, 7), (5, 7, 9)\) and \((7, 9, 9)\) represent the five linguistic variables of Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH) which are used to characterize the importance weight of the attributes. Similarly, as shown in Table 2, the fuzzy numbers \((1, 1, 1.5), (1.5, 2, 2.5), (2.5, 3, 3.5), (3.5, 4, 4.5), (4.5, 5, 5.5), (5.5, 6, 6.5), (6.5, 7, 7.5), (7.5, 8, 8.5)\) and \((8.5, 9, 9.5)\) represent the nine linguistic variables of Very Low (VL), Very Low to Low (VLL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H), High to Very High (HHV) and Very High (VH) which are used to characterize the performance rating of each facility on each attribute.

The assessment team used the following six attributes (established by the EHD) to systematically assess the potential safety and health hazards in waste recycling facilities throughout the State of East Virginia: severity of occurrence (C1), frequency of occurrence (C2), time of exposure (C3), protective measures (C4), prevention measures (C5), and failure to detect the risk (C6).

The EHD in the State of East Virginia has developed these general attributes to assess the risk severity (i.e., what is the worst that can happen?) and the potential frequency of occurrence.

---

\(^{1}\) The names are changed to protect the anonymity of the State and the communities which participated in this study.
Table 3
The importance weights of the attributes represented by linguistic variables.

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Decision maker</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM1</td>
</tr>
<tr>
<td>Severity of occurrence (C1)</td>
<td>H</td>
</tr>
<tr>
<td>Frequency of occurrence (C2)</td>
<td>M</td>
</tr>
<tr>
<td>Time of exposure (C3)</td>
<td>M</td>
</tr>
<tr>
<td>Preventive measures (C4)</td>
<td>L</td>
</tr>
<tr>
<td>Protective measures (C5)</td>
<td>VH</td>
</tr>
<tr>
<td>Failure to detect the risk (C6)</td>
<td>VL</td>
</tr>
</tbody>
</table>

(i.e., how often can the accident occur?). Risk as a function of expected loss is determined by the severity of loss and how often the loss occurs. Some hazards are present all of the time, or most of the time, but do not cause losses. The attribute considering the time of the exposure is intended to minimize the time of exposure (i.e., do not bring the hazardous waste until the last minute). The protective and preventive measures are specified to effectively eliminate, prevent or mitigate hazards, or to reduce the associated risks to an acceptable level. The failure to detect early warnings of risk because of erroneous signals, misinterpretation of information or simply not enough information could also have detrimental safety and health consequences. We should note that attributes C1, C2, C3 and C6 were considered as beneficial attributes and attributes C4 and C5 were considered as cost attributes. The three DMs (DM1, DM2, and DM3) evaluated the six attributes established by the EHD (C1, C2, C3, C4, C5, and C6) and proposed the importance weights presented in Table 3 for each attribute.

As shown in Table 3, the importance weights of the severity of occurrence (C1), the frequency of occurrence (C2), the time of exposure (C3), the protective measures (C4), the preventive measures (C5) and the failure to detect the risk (C6) attributes were evaluated by the three DMs using the linguistic variables defined in Table 1. For example, DM1 assigned a High (H) rating to the severity of occurrence (C1) attribute whereas DM2 and DM3 assigned a Medium (M) rating to this attribute. Similarly, the DMs proposed the performance ratings presented in Table 4 for the eight alternative facilities (A1, A2, A3, A4, A5, A6, A7, and A8). As shown in Table 4, the performance ratings of the HWR facilities A1 through A8 were evaluated by the three DMs using the linguistic variables defined in Table 2. For example, all three DMs assigned a High (H) rating to facility A1 based on the severity of occurrence (C1) attribute. However, there was less agreement when they were evaluating A6 (High (H)); Very High (VH) and High to Very High (HVH) for DM1, DM2 and DM3, respectively) based on the severity of occurrence (C1).

The triangular fuzzy numbers presented in Table 5 are the product of the consensus of opinions among the three DMs on the importance weights and performance ratings. In Table 5, each cell of the second column represents the consensus of opinions of the three DMs on the importance weights.

For example, consider the importance weight of the severity of occurrence (C1) attribute. The opinion of DM1 with a 41% voting power is High (H) and it is characterized by (5, 7, 9) in Table 1 while the opinion of DM2 and DM3 with the 34% and 25% voting powers, respectively, is Medium (M) and is characterized by (3, 5, 7) in Table 1. The aggregated weight for the severity of occurrence (C1) attribute is thus calculated as follows:

\[
\omega_{c1}^1 = \frac{1}{3} (5(0.41) + 3(0.34) + 3(0.25)) = 1.27
\]

\[
\omega_{c1}^2 = \frac{1}{3} (7(0.41) + 5(0.34) + 5(0.25)) = 1.94
\]

\[
\omega_{c1}^3 = \frac{1}{3} (9(0.41) + 7(0.34) + 7(0.25)) = 2.61
\]

In addition, Table 5 presents the consensus of opinions of the three DMs on the performance ratings. For example, consider the performance rating of A1 associated with C1. Although the three DMs have three different voting powers in the decision process, the judgments provided by these DMs is High (H) and thus, the aggregated performance ratings of (2.17, 2.33, 2.50) are calculated as follows:

\[
x_{c1a1}^1 = \frac{1}{3} (6.5(0.41) + 6.5(0.34) + 6.5(0.25)) = 2.17
\]

\[
x_{c1a1}^2 = \frac{1}{3} (7(0.41) + 7(0.34) + 7(0.25)) = 2.33
\]

\[
x_{c1a1}^3 = \frac{1}{3} (7.5(0.41) + 7.5(0.34) + 7.5(0.25)) = 2.50
\]

Next, the performance values were normalized in order to eliminate potential computational problems associated with the different measurement units in the fuzzy decision matrix. The normalized fuzzy decision matrix is presented in Table 6.

As shown in Table 6, attributes C1, C2, C3 and C6 are beneficial attributes while attributes C4 and C5 are cost attributes. We obtained the normalized fuzzy decision matrix (see Table 6) using Eq. (11) for the benefit attributes and Eq. (12) for the cost attributes. For example, consider facility A1 associated with the benefit attribute C1. The normalized fuzzy value of this given performance value is calculated as follows:

\[
\bar{x}_{c1a1} = \frac{2.17 \cdot 2.33 \cdot 2.50}{2.81 \cdot 2.81 \cdot 2.81} = (0.77, 0.83, 0.89)
\]

where the denominator of the above equation is 2.81 and it is calculated as \( c_{max}^1 = \max(2.5, 2.25, 2.5, 2.39, 2.2, 2.81, 2.25, 1.83) = 2.81 \).

The weighted normalized decision matrix is calculated by multiplying the normalized decision matrix by its associated weights as reported in Table 7. For example, consider facility A1 associated with the severity of occurrence (C1) attribute. The corresponding weighted normalized decision matrix presented in Table 7 is obtained as follows:

\[(0.77, 0.83, 0.89)(\times)(1.27, 1.94, 2.61) = (0.98, 1.61, 2.32)\]

We use the Hamming distance as a metric to express the distance between two facilities f and g for each attribute. The result is presented in Table 8. The first and second number in each cell in Table 8 represent \( d(\max(x_f, y_f), y_g) \) and \( d(\max(x_f, y_f), x_g) \), respectively. Let us consider the fuzzy performance ratings of facilities A1 and A2 with respect to the severity of occurrence (C1) attribute presented in Table 7 as \( x_{c1a1} = (0.98, 1.61, 2.32) \) and \( x_{c1a2} = (0.87, 1.44, 2.09) \) with the following membership functions:
Table 4
The performance ratings of the HWR facilities represented by linguistic variables.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Decision maker</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>DM1</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>DM2</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>H</td>
</tr>
<tr>
<td>C2</td>
<td>DM1</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>DM2</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>ML</td>
</tr>
<tr>
<td>C3</td>
<td>DM1</td>
<td>VL</td>
</tr>
<tr>
<td></td>
<td>DM2</td>
<td>VL</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>VL</td>
</tr>
<tr>
<td>C4</td>
<td>DM1</td>
<td>ML</td>
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<td>DM2</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>ML</td>
</tr>
<tr>
<td>C5</td>
<td>DM1</td>
<td>ML</td>
</tr>
<tr>
<td></td>
<td>DM2</td>
<td>ML</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>ML</td>
</tr>
<tr>
<td>C6</td>
<td>DM1</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>DM2</td>
<td>VL</td>
</tr>
<tr>
<td></td>
<td>DM3</td>
<td>VLL</td>
</tr>
</tbody>
</table>

Table 5
The importance weights and performance ratings represented by triangular fuzzy numbers.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Importance weight</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(1.27, 1.94, 2.61)</td>
<td>(2.17, 2.33, 2.50)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.92, 1.08, 1.25)</td>
<td>(0.71, 0.83, 1.00)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.83, 0.98, 1.05)</td>
<td>(0.64, 0.75, 0.82)</td>
</tr>
<tr>
<td>A4</td>
<td>(0.50, 0.67, 0.75)</td>
<td>(0.46, 0.58, 0.68)</td>
</tr>
<tr>
<td>A5</td>
<td>(0.33, 0.50, 0.86)</td>
<td>(0.58, 0.69, 0.86)</td>
</tr>
</tbody>
</table>

Table 6
The normalized fuzzy decision matrix.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(0.77, 0.83, 0.89)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.68, 0.74, 0.80)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.77, 0.83, 0.89)</td>
</tr>
<tr>
<td>A4</td>
<td>(0.73, 0.79, 0.85)</td>
</tr>
<tr>
<td>A5</td>
<td>(0.66, 0.72, 0.78)</td>
</tr>
<tr>
<td>A6</td>
<td>(0.68, 0.70, 0.90)</td>
</tr>
<tr>
<td>A7</td>
<td>(0.88, 0.94, 1.00)</td>
</tr>
<tr>
<td>A8</td>
<td>(0.68, 0.74, 0.80)</td>
</tr>
</tbody>
</table>

Table 7
The weighted normalized fuzzy decision matrix.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(0.98, 1.61, 2.32)</td>
</tr>
<tr>
<td>A2</td>
<td>(0.87, 1.44, 2.09)</td>
</tr>
<tr>
<td>A3</td>
<td>(0.98, 1.61, 2.32)</td>
</tr>
<tr>
<td>A4</td>
<td>(0.93, 1.53, 2.21)</td>
</tr>
<tr>
<td>A5</td>
<td>(0.84, 1.40, 2.04)</td>
</tr>
<tr>
<td>A6</td>
<td>(0.94, 1.53, 2.21)</td>
</tr>
<tr>
<td>A7</td>
<td>(0.84, 1.40, 2.04)</td>
</tr>
<tr>
<td>A8</td>
<td>(0.87, 1.44, 2.09)</td>
</tr>
</tbody>
</table>

Table 8
The distances between any two of the HWR facilities for each attribute.

<table>
<thead>
<tr>
<th>A8</th>
<th>A7</th>
<th>A6</th>
<th>A5</th>
<th>A4</th>
<th>A3</th>
<th>A2</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.93, 3.44, 5.45)</td>
<td>(1.93, 3.44, 5.45)</td>
<td>(1.16, 2.00, 3.34)</td>
<td>(4.04, 6.21, 8.45)</td>
<td>(1.93, 3.44, 5.44)</td>
<td>(1.16, 2.00, 3.34)</td>
<td>(1.16, 2.00, 3.34)</td>
<td>–</td>
</tr>
<tr>
<td>(2.37, 3.88, 5.89)</td>
<td>(4.98, 7.82, 10.72)</td>
<td>(5.31, 8.15, 11.06)</td>
<td>(4.65, 7.32, 9.55)</td>
<td>(4.98, 7.82, 10.72)</td>
<td>(5.31, 8.15, 11.06)</td>
<td>(4.65, 7.32, 9.55)</td>
<td>–</td>
</tr>
<tr>
<td>(0.59, 1.01, 1.04)</td>
<td>(0.59, 1.01, 1.04)</td>
<td>(0.12, 0.23, 0.36)</td>
<td>(0.06, 0.14, 0.23)</td>
<td>(0.99, 0.35, 0.36)</td>
<td>(0.09, 0.14, 0.23)</td>
<td>(0.05, 0.09, 0.14)</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 9
The concordance matrix.

<table>
<thead>
<tr>
<th>A10</th>
<th>A9</th>
<th>A8</th>
<th>A7</th>
<th>A6</th>
<th>A5</th>
<th>A4</th>
<th>A3</th>
<th>A2</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td>(0.33, 0.50, 1.17)</td>
<td></td>
</tr>
<tr>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td>(2.87, 5.05, 7.72)</td>
<td></td>
</tr>
<tr>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td>(1.93, 3.44, 5.54)</td>
<td></td>
</tr>
</tbody>
</table>

The Hamming distance between \(\hat{p}_{A10|A1} \) and \(\hat{p}_{A9|A1} \) is 0 for interval \([-\infty, 0.87] \), 0.01 for interval [0.87, 0.98], 0.11 for the interval [0.98, 1.44], 0.02 for the interval [1.44, 1.61], 0.14 for the interval [1.61, 2.09], 0.04 for the interval [2.09, 2.32], 0 for the interval [2.32, \infty]. The sum of integrals is 0 + 0.11 + 0.02 + 0.14 + 0.04 + 0 = 0.32.

Next, we constructed the concordance matrix presented in Table 9. Considering two facilities \(A_p \) and \(A_q \), the jth attribute is put in the concordance assuming that \(A_p \succ A_q \), where \(S \) is the outranking relation, if and only if \(A_p \) is strictly preferred to \(A_q \). In the
In the fuzzy context, the concordance set is formed as \( F^c = \{ j | \tilde{v}_j \geq \tilde{v}_i \} \)
where \( F^c \) is the concordance coalition of the attributes in which \( A_q \) outranks \( A_p \). The concordance matrix is constructed by using Eq. (17) based on the weighted normalized fuzzy decision matrix.

For example, consider the facilities A1 and A2 to identify the concordance set. The Hamming distances for each attribute are (0, 0.32), (0, 0.25), (0, 0.04), (0, 0.35), (0, 0.96) and (0.11, 0), respectively (see Table 8). Apart from the last pair of values for attribute C6, the attributes C1, C2, C3, C4 and C5 are put in the concordance set. So, the sum of the fuzzy weights for these attributes presents the concordance level as follows:

\[
(1.27, 1.94, 2.61) + (0.77, 1.44, 2.11) + (0.83, 1.50, 2.17) + (0.50, 1.17, 1.83) + (2.11, 2.77, 3.00) = (5.48, 8.82, 11.72)
\]

We next constructed the discordance matrix presented in Table 10. Considering two facilities \( A_q \) and \( A_p \), the \( j \)th attribute is placed in the discordance set assuming that \( A_q \) outranks \( A_p \), where \( S \) is the outranking relation, and if only if \( A_q \) is preferred to \( A_p \). In the fuzzy environment, the discordance set is formed as \( F^d = \{ j | \tilde{v}_i \leq \tilde{v}_j \} \) where \( F^d \) is the discordance coalition of the attributes. The discordance matrix is constructed using Eq. (19). For example, let us consider facilities A1 and A2. In the evaluation of A1 and A2, the Hamming distances in terms of each attribute are (0, 0.32), (0, 0.25), (0, 0.04), (0, 0.35), (0, 0.96) and (0.11, 0), respectively (see Table 8) and the last pair values satisfies the definition of the discordance set. Therefore, the discordance set includes C6 and the corresponding discordance level by means of Eq. (19) as follows:

\[
\max(0 - 0.11) = 0.1145 \leq 0.11
\]

Next, we constructed the Boolean matrix \( E \) using the concordance approach presented in Table 11. The minimum concordance level, \( T = (3.4, 80, 6.64) \), is required to construct this matrix. Considering two facilities \( p \) and \( q \), the corresponding element of the Boolean matrix takes the value one if and only if the concordance level is bigger than or equal to \( T \); otherwise it is zero (see relation (21)). For example, consider facilities A1 and A2 in the first row of Table 11. In order to place a 1 for the element in the Boolean matrix \( E \), the corresponding value (5.48, 8.82, 11.72) from Table 9 must be compared with \( T = (3.4, 80, 6.64) \) using the Hamming distance.

We then constructed the Boolean matrix \( F \) using the discordance approach presented in Table 12. First, the minimum discordance level, \( D = 0.782 \), is required to construct this matrix. By means of relation (13), we construct the Boolean discordance matrix for any two facilities \( p \) and \( q \). For example, considering facilities A1 and A2, from the first row of Table 10, the corresponding value is 0.11 which is less than the minimum discordance level, resulting in a zero for this case in the Boolean matrix.

We then aggregated the Boolean matrices \( E \) and \( F \) by peer to peer multiplication of their elements to capture their simultaneous effects and constructed the general matrix \( G \) presented in Table 13. For example, the first row of Table 13 is obtained as follows: A1–A2: 1 × 1 = 1, A1–A3: 1 × 0 = 0, A1–A4: 1 × 1 = 1, A1–A5: 1 × 1 = 1, A1–A6: 1 × 0 = 0, A1–A7: 1 × 0 = 0, and A1–A8: 1 × 0 = 0.

In the final step of the process, we constructed the decision graph presented in Fig. 2. The decision graph enabled us to identify the ranking order of the facilities. For instance, consider A7. There are four connections between this facility and facilities A2, A4, A5 and A6. Facility A7 dominates facilities A2, A4 and A5 (represented by three outgoing arcs) while facility A7 is dominated by facility A6 (represented by one incoming arc). Note that facility A7 cannot be compared with facilities A1, A3 and A8 because there are no connections between this facility and the other three facilities. As is shown in this decision graph:

- Facility A1 dominates facilities A2, A4 and A5.
- Facility A2 is dominated by facilities A1, A4, A5, A6 and A7.
- Facility A3 dominates facility A4.
- Facility A4 dominates facility A2 while it is dominated by facilities A1, A3, A6 and A7.
- Facility A5 dominates facility A2 while it is dominated by facilities A1, A6 and A7.
- Facility A6 dominates facilities A2, A4, A5 and A7.
- Facility A7 dominates facilities A2, A4 and A5 while it is dominated by facility A6.
- Facility A8 is not comparable with the other facilities.

### Table 10

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>A8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>0.51</td>
<td>0.58</td>
<td>0.28</td>
<td>1</td>
<td>1</td>
<td>0.98</td>
</tr>
<tr>
<td>A2</td>
<td>1</td>
<td>0.11</td>
<td>0.84</td>
<td>0.58</td>
<td>0.28</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0.18</td>
<td>0.60</td>
<td>0.84</td>
<td>0.58</td>
<td>1</td>
<td>0.90</td>
<td>1</td>
</tr>
<tr>
<td>A4</td>
<td>0.53</td>
<td>1</td>
<td>0.11</td>
<td>0.81</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>0.53</td>
<td>0.53</td>
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According to the decision graph presented in Fig. 2, the HWR facility in Fairfield (A6) was ranked first because it had higher priorities than four other facilities. The facility in Alexandria (A1) was ranked second because it had higher priority than the facilities in Birmingham (A2), Dover (A4), and Edgewater (A5). The facility in Gibbstown (A7) was ranked third because it had higher priority than the facilities in Birmingham (A2), Dover (A4) and Edgewater (A5) but lower priority than the facility in Fairfield (A6). The facility in Chester (A3) was ranked fourth because it had higher priority than the facility in Dover (A4). The facility in Edgewater (A5) was ranked fifth because it had higher priority than the facility in Birmingham (A2) but lower priority than the facilities in Alexandria (A1), Fairfield (A6) and Gibbstown (A7). The facility in Dover (A4) was ranked sixth because it had higher priority than the facility in Birmingham (A2) but lower priority than the facilities in Alexandria (A1), Chester (A3), Fairfield (A6) and Gibbstown (A7). The facility in Birmingham (A2) was ranked seventh since it did not have higher priority than any other facility but the facilities in Alexandria (A1), Dover (A4), Edgewater (A5), Fairfield (A6) and Gibbstown (A7) had higher priorities than this facility. Finally, the facility in Hamilton (A8) was not able to compare with any other facility. Table 14 presents the final ranking of the eight HWR facilities in the State of East Virginia identified by the three DMs from the EHD (DM1), the EPA (DM2), and the OSHA (DM3) based on six safety and health assessment attributes established by the EHD (C1, C2, C3, C4, C5, and C6).

### Table 14
The final ranking of the HWR facilities.

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<td>Hamilton (A8)</td>
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**6. Conclusions and future research directions**

The management of waste has received increasing attention as the world has become more developed and societies have become more consumer-oriented. This industrialization and commercialization has given rise to a wide range of consumption activities which in turn generate a parallel increase in waste. Waste is perceived as a potential threat to human health as the environment becomes more corrosive, toxic, and infectious. Hence, there is a vital need for holistic environmental management systems that aim at reducing and disposing wastes in an environmentally sound manner that will protect human health and the environment.

We demonstrated the application of the proposed model for safety and health assessment in HWR facilities. The method considered quantitative data and qualitative judgments provided by three DMs in a real-world case study and captured the ambiguity and imprecision in their judgments with fuzzy logic. The application involved a complex pilot study conducted for an EHD to assess the safety and health in eight HWR facilities. Three DMs from three different agencies including EHD, EPA, and OSHA with different objectives and authorities established by their voting powers participated in this study. The DMs considered six conflicting qualitative and quantitative attributes with varying precise and imprecise measurements. Four attributes were considered as beneficial attributes and two attributes were considered as cost attributes. This is a complex MADM problem involving multiple DMs with different backgrounds and expertise, conflicting systems of attributes, and imprecise information. In spite of these environmental complexities and uncertainties, our three DMs were able to determine the importance weights of the beneficial and cost attributes and develop the performance ratings of the eight HWR facilities on these attributes. The structured and systematic framework proposed in this study helped the three DMs evaluate and rank the eight HWR facilities. Some of the motivations for the development of the fuzzy outranking model proposed in this study are:

1. To provide a mathematical framework for modeling vagueness and imprecision in outranking since the subjectivity, imprecision and vagueness in the estimates of the performance ratings are often encountered in the HWR decisions.
2. In contrast to the valued outranking methods that are well-documented and have been extensively used in practice, the fuzzy outranking methods are recent and are not well-documented in the literature (Bufardi et al., 2008).
3. The Electre methods with participatory approaches have not fully emerged in the MADM literature as many outranking methods and applications assume a single DM for simplicity (Munda, 2004).

When compared with the other fuzzy outranking methods, some of the features of the fuzzy outranking method proposed in this study are: (1) the fuzzy outranking procedure used here is simple yet structured and logical. Rao (2007, p. 5) stresses the need for simple, systematic and logical outranking methods; and (2) the proposed outranking method is generalized and can be applied to a great variety of complex environmental problems.

We should note that solving MADM problems is not searching for an optimal solution, but rather helping DMs master the complex judgments and the data involved in their problems and advance towards an acceptable solution. The model proposed in this study does not imply a deterministic approach to decision making and is not an off-the-shelf recipe that can be applied to every environmental management problem and situation. While our model enables DMs crystallize their thoughts and organize their judgments, it should be used very carefully. As with any MADM model, the DMs must be aware of the limitations of subjective estimates. In future research, similar studies can be conducted based on different fuzzy MADM techniques such as fuzzy AHP, fuzzy TOPSIS or fuzzy PROMETHEE for comparative purposes.


Norse, M.F. 2006. ELECTRE III as a support for participatory decision-making on the localisation of waste-treatment plants. Land Use Policy 23 (1), 76–85.


