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Extension of VIKOR method in intuitionistic fuzzy environment for robot selection

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ABSTRACT

Decision making is the process of finding the best option among the feasible alternatives. In classical multiple-criteria decision making methods, the ratings and the weights of the criteria are known precisely. However, if decision makers are not able to involve uncertainty in the defining of linguistic variables based on fuzzy sets, the intuitionistic fuzzy set theory can do this job very well. In this paper, VIKOR method is extended in intuitionistic fuzzy environment, aiming at solving multiple-criteria decision making problems in which the weights of criteria and ratings of alternatives are taken as triangular intuitionistic fuzzy set. For application and verification, this study presents a robot selection problem for material handling task to verify our proposed method.

1. Introduction

Decision making is universal in any human activity, either complex or simple. Most of the complex real life problems are with conflicting multi-criteria. A lot of work has been done on these complex structured multi-criteria problems and many methods are proposed to deal with them. MCDM methods are an extensively applied tool for determining the best solution among several alternatives with multiple criteria or attributes. The procedures for determining the best solution to a MCDM problem include computing the utilities of alternatives and ranking these utilities. The alternative solution with the largest utility is considered to be the optimal solution.

Due to the complex structure of the problem and conflicting nature of the criteria, a compromise solution for a problem can help the decision maker to reach a final decision. Recently, the VIKOR method Opricovic and Tzeng (2002) has been developed for multi-attribute optimization of complex systems. It determines the compromise ranking list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with initial given weights. The method focuses on ranking and selecting from a set of alternatives in the presence of conflicting attributes. The VIKOR method provides a maximum group utility for the majority and a minimum of an individual regret for the opponent. It introduces the multi-attribute ranking indexes based on the particular measure of closeness to the ideal solution. Opricovic and Tzeng (2004) have given a comparative analysis of VIKOR and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution, developed by Hwang & Yoon (1981)). Chatterjee, Athawale, and Chakraborty (2009) have used VIKOR with ELECTRE (ELimination and Choice Translating Reality, an outranking method) for material selection problem. Opricovic and Tzeng (2007) extended VIKOR method with stability analysis determining the weight stability intervals and with trade-offs analysis. Further they compared the extended VIKOR method with three MCDM methods: TOPSIS, PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) and ELECTRE. They have found the ranking obtained by PROMETHEE, and ELECTRE was relatively similar to VIKOR. Tong et al. (2007) have used VIKOR to optimize multi response process.

As the complexity of the problem increases, impreciseness and vagueness in the data of the corresponding problem also increases. Zadeh (1965) proposed the idea of fuzzy sets to deal with these uncertainties. As Fuzzy set theory (Zimmerman, 1983, 1987) came into existence, many extensions of fuzzy sets also have appeared over the time and traditional fuzzy decision making models have been extended to include these extended fuzzy type descriptions. One among these extensions of fuzzy sets is Intuitionistic Fuzzy Sets (IFSs) (Atanassov, 1986) playing an important role in decision making and have gained popularity in recent years. In IFS theory sum of degree of membership and degree of non-membership do not simply to one as in the conventional fuzzy sets. Such an extended definition helps more adequately to represent situations when decision maker abstain from expressing their assessments. The degree by which decision maker abstained is called intuitionistic fuzzy index (or hesitation degree). By this way, IFSs provide a richer tool to grasp imprecision than the conventional fuzzy sets. This feature of IFSs has led to extend VIKOR in intuitionistic fuzzy (IF)-environment.

In this paper, we extend VIKOR method in IF-environment to solve MCDM problems in which the performance rating values as well as the weights of criteria are linguistic terms which can be expressed in triangular intuitionistic fuzzy sets. Emphasis is given on

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use of intuitionistic fuzzy elements in decision making because they can provide a new quality that cannot be attained by using conventional fuzzy sets. The remaining of this paper is organized as follows; In Section 2, we briefly introduce the VIKOR method. Section 3 is with preliminary things about IFS, arithmetic operations of IFSs and linguistic variables. Section 4 describes developed VIKOR method to solve MCDM problems in IF-environment. Section 5 investigate a group decision making robot selection problem in which all the evaluation information provided by the decision makers are characterized by linguistic variables which are further expressed as triangular intuitionistic fuzzy sets. Finally, the paper is concluded with some observations in Section 6.

2. The VIKOR method

Opricovic and Tzeng (2002, 2004) developed VIKOR method, the Serbian name: VlseKriterijumska Optimizacija I Kompromisno Resenje, means multi-criteria optimization and compromise solution. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. Here, the compromise solution is a feasible solution which is the closest to the ideal solution, and a compromise means an agreement established by mutual concessions (Opricovic & Tzeng, 2007). The multi-criteria measure for compromise ranking is developed from the *Lp*-metric used as an aggregating function. Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The VIKOR method was developed to solve the following MADM problem:

	C_1	C_2	...	C_n
A_1	r_{11}	r_{12}	...	r_{1n}
A_2	r_{21}	r_{22}	...	r_{2n}
...	...	...	...	...
A_m	r_{m1}	r_{m2}	...	r_{mn}

$$W = [w_1, w_2, \ldots, w_n]$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose the optimal solution. $C_1, C_2, \ldots, C_n$ are the criteria with which alternative performance is measured. r_{ij} is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j. Development of the VIKOR method is started with the following form of *Lp*-metric:

$$L_{p_i} = \left\{ \sum_{j=1}^{n} \left[\left(r_j^+ - r_{ij}\right) \Big/ \left(r_j^+ - r_j^-\right) \right]^p \right\}^{1/p} 1 \leqslant p \leqslant \infty; \quad i = 1, 2, \ldots, m$$

In the VIKOR method $L_{1,i}$ (asS_i) and $L_{\infty,i}$ (asR_i) are used to formulate ranking measures. The solution obtained by min S_i is with a maximum group utility ("majority" rule), and the solution obtained by min R_i is with a minimum individual regret of the "opponent". The procedure of VIKOR for ranking alternatives can be described with the following steps:

(i) Determine the best r_j^+ and the worst r_j^- values of all criterion functions $j = 1, 2, \ldots, n$. If the jth function represents a benefit then:

$$r_j^+ = \max_i r_{ij}, \quad r_j^- = \min_i r_{ij}.$$

(ii) Compute the values S_i and R_i; $i = 1, 2, \ldots, m$, by these relations;

$$S_i = \sum_{j=1}^{n} w_j \left(r_j^+ - r_{ij}\right) \Big/ \left(r_j^+ - r_j^-\right),$$

$$R_i = \max_j \; w_j \left(r_j^+ - r_{ij}\right) \Big/ \left(r_j^+ - r_j^-\right).$$

where w_j; $j = 1, 2, \ldots, n$ are the weights of criteria, expressing their relative importance.

(iii) Compute the values Q_i; $i = 1, 2, \ldots, m$, by the following relation:

$$Q_i = \vartheta(S_i - S^+)/(S^- - S^+) + (1 - \vartheta)(R_i - R^+)/(R^- - R^+)$$

where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$ and $R^- = \max_i R_i$. Here ϑ is introduced as weight of the strategy of "the majority of criteria" (or maximum group utility), whereas $1 - \vartheta$ is the weight of individual regret. Rank the alternatives, sorting by the values S, R and Q in the decreasing order. The results are three ranking lists.

(iv) Propose as a compromise solution the alternative A' which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. *Acceptable advantage:*

$$Q(A'') - Q(A') \geqslant DQ$$

where A'' is the alternative with second position in the ranking list by Q; $DQ = 1/(m - 1)$; m is the number of alternatives.

C2. *Acceptable stability in decision making:* The alternative A' must also be the best ranked by S or/and R.This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when $\vartheta > 0.5$ is needed), or "by consensus" $\vartheta \approx 0.5$, or "with veto" ($\vartheta < 0.5$). Here, ϑ is the weight of decision making strategy "the majority of criteria" (or "the maximum group utility").

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A' and A'' if only condition C2 is not satisfied, or
- Alternatives $A', A'', \ldots, A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A') < DQ$ for maximum M (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q, is the one with minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with "average rate". VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by min S) of the "majority", and a minimum of the "individual regret" (represented by min R) of the "opponent". The compromise solutions are the basis for negotiations, involving the decision maker's preference by criteria weights.

3. Preliminaries

In this section, we briefly review the concept of IFSs and the arithmetic operations of triangular intuitionistic fuzzy sets with a small introduction to linguistic variables.

3.1. IFSs and its arithmetic operations

IFS introduced by Atanassov (1986) is an extension of the classical fuzzy set theory, which is an appropriate tool to deal with vagueness and uncertainty. IFS A in a finite set X can be written as:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

which is characterized by a membership function $\mu_A(x)$ and a non-membership function $\nu_A(x)$, where $\mu_A(x)$, $\nu_A(x)$: $X \to [0,1]$ with the condition $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$. A third parameter of IFS is $\pi_A(x)$, usually called the intuitionistic fuzzy index or hesitation degree, of whether x belongs to A or not can be defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, $0 \leqslant \pi_A(x) \leqslant 1$. Fuzzy sets are the special case of IFSs. For fuzzy sets $\nu_A(x) = 1 - \mu_A(x)$ and$\pi_A(x) = 0$. A triangular intuitionistic fuzzy set (TIFS) (Li, 2008; Shu, Cheng, & Chang, 2006) A in X is represented by $A = \langle[(x_1, x_2, x_3); \mu_A], [(x'_1, x_2, x'_3); \nu_A]\rangle$ as shown in Fig. 1.

For two TIFSs $A = \langle[(x_1, x_2, x_3); \mu_A], [(x'_1, x_2, x'_3); \nu_A]\rangle$ and $B = \langle[(y_1, y_2, y_3); \mu_B], [(y'_1, y_2, y'_3); \nu_B]\rangle$ with $\mu_A \neq \mu_B$, $\nu_A \neq \nu_B$, the arithmetic operation are defined as follows:

$$\begin{aligned} A + B &= \langle[(x_1 + y_1, x_2 + y_2, x_3 + y_3); \min(\mu_A, \mu_B)], \\ &\quad [(x'_1 + y'_1, x_2^+ y_2 x'_3 + y'_3); \max(\nu_A, \nu_B)]\rangle \\ A - B &= \big\langle[(x_1 - y_3, x_2 - y_2, x_3 - y_1); \min(\mu_A, \mu_B)], \\ &\quad \big[(x'_1 - y'_3, x_2 - y_2, x'_3 - y'_1); \max(\nu_A, \nu_B)\big]\big\rangle \end{aligned}$$

Moreover, for $A > 0$ and $B > 0$

$$\begin{aligned} A \times B &= \langle[(x_1.y_1, x_2.y_2, x_3.y_3); \min(\mu_A, \mu_B)], \\ &\quad [(x'_1.y'_1, x_2 y_2 x'_3.y'_3); \max(\nu_A, \nu_B)]\rangle \\ A/B &= \big\langle[(x_1/y_3, x_2/y_2, x_3/y_1); \min(\mu_A, \mu_B)], \\ &\quad \big[(x'_1/y'_3, x_2/y_2, x'_3/y'_1); \max(\nu_A, \nu_B)\big]\big\rangle \end{aligned}$$

$$\max(A,B) = \left\langle \begin{array}{l} [(\max(x_1,y_1), \max(x_2,y_2), \max(x_3,y_3); \min(\mu_A,\mu_B)], \\ [(\max(x'_1,y'_1), \max(x_2,y_2), \max(x'_3,y'_3); \max(\nu_A,\nu_B)] \end{array} \right\rangle$$

$$\min(A,B) = \left\langle \begin{array}{l} [(\min(x_1,y_1), \min(x_2,y_2), \min(x_3,y_3); \min(\mu_A,\mu_B)], \\ [(\min(x'_1,y'_1), \min(x_2,y_2), \min(x'_3,y'_3); \max(\nu_A,\nu_B)] \end{array} \right\rangle$$

3.2. Linguistic variables

Variables whose values are not numbers, but words or sentences in natural or artificial languages are called linguistic variables (Herrera & Herrera-Viedma, 1996). The concept of a linguistic variable appears as useful way for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms. The use of linguistic variable enables to specify both the importance associated with each of a set of criteria, and the preference with respect to a number of strategic criteria which impact the selection and justification of several alternatives. The values of linguistic variable can be quantified and extended to mathematical operators using IFS theory.

For example, the performance ratings of alternatives on qualitative attributes could be expressed using linguistic variable such as "Fair", "Very Good", etc. Such linguistic values can be represented using TIFSs. For example, "Fair" and "Very Good" can be represented by TIFSs as "$\langle[(2.5,5,6.5); 0.50], [(3.5,5,7.5); 0.45]\rangle$" and "$\langle[(8.5,10,10); 0.90], [(9.5,10,10); 0.10]\rangle$", respectively.

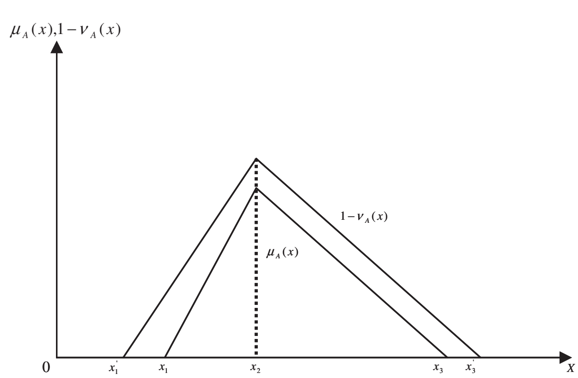

Fig. 1. A Triangular intuitionistic fuzzy set.

4. An extension of the VIKOR method in intuitionistic fuzzy environment (IF-VIKOR)

In VIKOR method, numerical measure of the relative importance of attributes and the performance of each alternative on these attributes are very important. It is difficult to precisely determine the exact data as human judgements are often vague under many situations and conditions. Fuzzy sets and other non-standard fuzzy sets (Yager, 2009) are efficient in tackling these uncertainties present in the provided data. Therefore, extension of VIKOR method to the non-standard fuzzy environment is natural. Out of these non-standard fuzzy sets, IFSs are more efficient in dealing with uncertainty. As in many situations, available information is not sufficient for the exact definition of degree of membership for certain element. There may be some hesitation degree between membership and non-membership. Thus in many real life problems, due to insufficiency in information availability, IFSs with ill known membership grades are appropriate. IFSs have been found to be particularly useful to deal with uncertainty. In this paper, criteria values as well as criteria weights are considered as linguistic variables.

Let $D = [x_{ij}]_{m \times n}$ be an IF-decision matrix for a MCDM problem in which $A_1, A_2, \ldots, A_m$ are m possible alternatives among which decision makers have to choose an optimal solution and $C_1, C_2, \ldots, C_n$ are n criteria with which alternatives performance are measured. So, x_{ij} is the rating of alternative A_i with respect to criterion C_j and w_j is the weight of criterion, which is taken as TIFS. In a group decision environment with k persons, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$w_j = \frac{1}{k}\left[w_j^1 + w_j^2 + \cdots + w_j^k\right] \tag{1}$$

$$x_{ij} = \frac{1}{k}\left[x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^k\right] \tag{2}$$

These equations represent the average values of x_{ij} and w_j given by experts, where (+) is the sum operator and is applied to the TIFSs as defined in Section 3. So the output is also TIFSs. Now, the proposed approach (IF-VKOR) to develop the VIKOR for TIFSs data can be defined as follows:

(a) Determine the best rating x_i^+ and the worst rating x_i^- for all the criteria. If i represents a benefit criterion, then:

$$x_j^+ = \max_i x_{ij}, \quad x_j^- = \min_i x_{ij}$$

$$A^+ = \{x_1^+, x_2^+, \ldots, x_n^+\}, \tag{3}$$

$$A^- = \{x_1^-, x_2^-, \ldots, x_n^-\}. \tag{4}$$

$$S_i = \sum_{j=1}^{n} w_j \times \left(\frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}\right) =$$

$$\sum_{j=1}^{n} \left\langle [(w_{1j}, w_{2j}, w_{3j}); \mu_{w_j}], \left[\left(w'_{1j}, w_{2j}, w'_{3j}\right); \nu_{w_j}\right] \right\rangle \times \left[\frac{\left\langle \begin{array}{c} \left[\left(x_{1j}^+ - x_{3ij}, x_{2j}^+ - x_{2ij}, x_{3j}^+ - x_{1ij}\right); \min(\mu_{x_j^+}, \mu_{x_{ij}})\right], \\ \left[\left(x'^+_{1j} - x'_{3ij}, x_{2j}^+ - x_{2ij}, x'^+_{3j} - x'_{1ij}\right); \max(\nu_{x_j^+}, \nu_{x_{ij}})\right] \end{array} \right\rangle}{\left\langle \begin{array}{c} \left[\left(x_{1j}^+ - x_{3j}^-, x_{2j}^+ - x_{2j}^-, x_{3j}^+ - x_{1j}^-\right); \min(\mu_{z_j^+}, \mu_{z_j^-})\right], \\ \left[\left(x'^+_{1j} - x'^-_{3j}, x_{2j}^+ - x_{2j}^-, x'^+_{3j} - x'^-_{1j}\right); \max(\nu_{x_j^+}, \nu_{x_j^-})\right] \end{array} \right\rangle}\right], \quad (5)$$

$$S_i == \left\langle [(S_{1i}, S_{2i}, S_{3i}); \mu_{S_i}], \left[(S'_{1i}, S_{2i}, S'_{3i}); \nu_{S_i}\right] \right\rangle,$$

$$R_i = \max_j \left(w_j \times \left(\frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}\right)\right) = \max_j \left(\left\langle [(w_{1j}, w_{2j}, w_{3j}); \mu_{w_j}], \left[\left(w'_{1j}, w_{2j}, w'_{3j}\right); \nu_{w_j}\right] \right\rangle \times \left[\frac{\left\langle \begin{array}{c} \left[\left(x_{1j}^+ - x_{3ij}, x_{2j}^+ - x_{2ij}, x_{3j}^+ - x_{1ij}\right); \min(\mu_{x_{ij}}, \mu_{x_j^+})\right], \\ \left[\left(x'^+_{1j} - x'_{3ij}, x_{2j}^+ - x_{2ij}, x'^+_{3j} - x'_{1ij}\right); \max(\nu_{x_{ij}}, \nu_{x_j^+})\right] \end{array} \right\rangle}{\left\langle \begin{array}{c} \left[\left(x_{1j}^+ - x_{3j}^-, x_{2j}^+ - x_{2j}^-, x_{3j}^+ - x_{1j}^-\right); \min(\mu_{x_j^+}, \mu_{x_j^-})\right], \\ \left[\left(x'^+_{1j} - x'^-_{3j}, x_{2j}^+ - x_{2j}^-, x'^+_{3j} - x'^-_{1j}\right); \max(\nu_{x_j^+}, \nu_{x_j^-})\right] \end{array} \right\rangle}\right]\right),$$

$$R_i == \left\langle [(R_{1i}, R_{2i}, R_{3i}); \mu_{R_i}], \left[(R'_{1i}, R_{2i}, R'_{3i}); \nu_{R_i}\right] \right\rangle. \quad (6)$$

A^+ and A^- are called ideal and anti-ideal scores respectively. These are imaginary score, cannot be possessed by any candidate, if so, then decision would be trivial.

(b) In this step, compute S_i and R_i values for $i = 1,2,\ldots,m$, which symbolize the average and the worst group scores for the alternative A_i respectively, with the relations as follows:

(c) Compute the ranking index Q_i; $i = 1,2,\ldots,m$ by this relation:

$$Q_i = \vartheta \left[\frac{\left\langle \begin{array}{c} \left[\left(S_{1i} - S_3^+, S_{2i} - S_2^+, S_{3i} - S_1^+\right); \min(\mu_{S_i}, \mu_{S^+})\right], \\ \left[\left(S'_{1i} - S'^+_3, S_{2i} - S_2^+, S'_{3i} - S'^+_1\right); \max(\nu_{S_i}, \nu_{S^+})\right] \end{array} \right\rangle}{\left\langle \begin{array}{c} \left[\left(S_1^- - S_3^+, S_2^- - S_2^+, S_1^- - S_3^+\right); \min(\mu_{S^-}, \mu_{S^+})\right], \\ \left[\left(S'^-_1 - S'^+_3, S_2^- - S_2^+, S'^-_1 - S'^+_3\right); \max(\nu_{S^-}, \nu_{S^+})\right] \end{array} \right\rangle}\right] + (1-\vartheta) \frac{\left\langle \begin{array}{c} \left[\left(R_{1i} - R_3^+, R_{2i} - R_2^+, R_{3i} - R_1^+\right); \min(\mu_{R_i}, \mu_{R^+})\right], \\ \left[\left(R'_{1i} - R'^+_3, R_{2i} - R_2^+, R'_{3i} - R'^+_1\right); \max(\nu_{R_i}, \nu_{R^+})\right] \end{array} \right\rangle}{\left\langle \begin{array}{c} \left[\left(R_1^- - R_3^+, R_2^- - R_2^+, R_1^- - R_3^+\right); \min(\mu_{R^-}, \mu_{R^+})\right], \\ \left[\left(R'^-_1 - R'^+_3, R_2^- - R_2^+, R'^-_1 - R'^+_3\right); \max(\nu_{R^-}, \nu_{R^+})\right] \end{array} \right\rangle},$$

$$Q_i^{=} \left\langle [(Q_{1i}, Q_{2i}, Q_{3i}); \mu_{Q_i}], \left[(Q'_{1i}, Q_{2i}, Q'_{3i}); \nu_{Q_i}\right] \right\rangle. \quad (7)$$

where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$ and $R^- = \max_i R_i$, ϑ is introduced as weight of the strategy of "the maximum group utility".

Table 1
Definitions of linguistic variables for the ratings.

Very poor (VP)	$\langle[(0,0,1);0.10],[(0,0,1.5);0.90]\rangle$
Poor (P)	$\langle[(0,1,2.5);0.20],[(0.5,1,2.5);0.75]\rangle$
Moderately poor (MP)	$\langle[(0,3,4.5);0.35],[(1.5,3,5.5);0.60]\rangle$
Fair (F)	$\langle[(2.5,5,6.5);0.50],[(3.5,5,7.5);0.45]\rangle$
Moderately good (MG)	$\langle[(4.5,7,8);0.65],[(5.5,7,9.5);0.35]\rangle$
Good (G)	$\langle[(5.5,9,9.5);0.80],[(7.5,9,10);0.15]\rangle$
Very good (VG)	$\langle[(8.5,10,10);0.90],[(9.5,10,10);0.10]\rangle$

Table 2
Definitions of linguistic variables for the importance of each criterion.

Very low (VL)	$\langle[(0,0,0.1);0.10],[(0,0,0.15);0.90]\rangle$
Low (L)	$\langle[(0,0.1,0.25);0.25],[(0.05,0.1,0.35);0.75]\rangle$
Medium low (ML)	$\langle[(0,0.3,0.45);0.40],[(0.15,0.3,0.55);0.55]\rangle$
Medium (M)	$\langle[(0.25,0.5,0.65);0.50],[(0.35,0.5,0.75);0.45]\rangle$
Medium high (MH)	$\langle[(0.45,0.7,0.8);0.60],[(0.55,0.7,0.95);0.30]\rangle$
High (H)	$\langle[(0.55,0.9,0.95);0.75],[(0.75,0.9,1);0.10]\rangle$
Very high (VH)	$\langle[(0.85,1,1);0.90],[(0.95,1,1);0.10]\rangle$

Table 3
The importance of each criterion.

	DM_1	DM_2	DM_3	DM_4
C_1	H	MH	H	VH
C_2	M	M	ML	MH
C_3	MH	H	MH	MH
C_4	H	H	H	VH
C_5	VH	VH	H	H
C_6	ML	ML	M	ML

Table 4
DM's assessments based on each criterion.

Criterion	Alternatives	Decision makers			
		DM_1	DM_2	DM_3	DM_4
C_1	(A_1)	MP	MP	M	M
	(A_2)	G	G	G	MG
	(A_3)	F	F	F	F
C_2	(A_1)	MG	G	MG	MG
	(A_2)	F	F	MG	MG
	(A_3)	MG	MG	MG	MG
C_3	(A_1)	F	F	F	MG
	(A_2)	MG	MG	MG	G
	(A_3)	MG	MG	MG	MG
C_4	(A_1)	MP	F	F	F
	(A_2)	MG	MG	G	G
	(A_3)	F	F	F	F
C_5	(A_1)	MP	F	F	MP
	(A_2)	G	G	G	MG
	(A_3)	MG	MG	MG	MG
C_6	(A_1)	F	F	MP	MP
	(A_2)	MG	G	MG	G
	(A_3)	F	F	MP	F

Table 5
The IF-decision matrix and weights

	(A_1)	(A_2)	(A_3)	Weight
C_1	$\left\langle \begin{matrix}[(1.25,4,5.5);0.35],\\ [(2.5,4,6.5);0.60]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(5.25,8.5,9.125);0.65],\\ [(7,8.5,9.875);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(2.5,5,6.5);0.50],\\ [(3.5,5,7.5);0.45]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.6,0.875,0.925);0.60],\\ [(0.75,0.875,0.9875);0.30]\end{matrix}\right\rangle$
C_2	$\left\langle \begin{matrix}[(4.75,7.5,8.375);0.65],\\ [(6,7.5,9.625);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(3.5,6,7.25);0.50],\\ [(4.5,6,8.5);0.45]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(4.5,7,8);0.65],\\ [(5.5,7,9.5);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.2375,0.5,0.6375);0.40],\\ [(0.35,0.5,0.75);0.55]\end{matrix}\right\rangle$
C_3	$\left\langle \begin{matrix}[(3,5.5,6.875);0.50],\\ [(4,5.5,8);0.45]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(4.75,7.5,8.375);0.65],\\ [(6,7.5,9.625);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(4.5,7,8);0.65],\\ [(5.5,7,9.5);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.475,0.75,0.8375);0.60],\\ [(0.6,0.75,0.9625);0.30]\end{matrix}\right\rangle$
C_4	$\left\langle \begin{matrix}[(1.875,4.5,6);0.35],\\ [(3,4.5,7);0.60]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(5,8,8.75);0.65],\\ [(6.5,8,9.75);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(2.5,5,6.5);0.50],\\ [(3.5,5,7.5);0.45]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.625,0.925,0.9625);0.75],\\ [(0.8,0.925,1);0.20]\end{matrix}\right\rangle$
C_5	$\left\langle \begin{matrix}[(1.875,4.5,6);0.35],\\ [(3,4.5,7);0.60]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(5.25,8.5,9.125);0.65],\\ [(7,8.5,9.875);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(4.5,7,8);0.65],\\ [(5.5,7,9.5);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.7,0.95,0.975);0.75],\\ [(0.85,0.95,1);0.20]\end{matrix}\right\rangle$
C_6	$\left\langle \begin{matrix}[(1.25,4,5.5);0.35],\\ [(2.5,4,6.5);0.60]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(4.75,7.5,8.375);0.65],\\ [(6,7.5,9.625);0.35]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(1.875,4.5,6);0.35],\\ [(3,4.5,7);0.60]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}[(0.0625,0.35,0.5);0.40],\\ [(0.2,0.35,0.6);0.55]\end{matrix}\right\rangle$

(d) According to the VIKOR method, the alternative that has minimum Q_i is the best alternative and it is chosen as compromise solution. But here the Q_i, $i = 1,2,\ldots,m$ are TIFSs. To choose the minimum TIFS, they are required to compare with each other. So, the final crisp value of Q_i which is shown by Q_i^* used for comparison of TIFSs can be calculated as follows:

$$Q_i^* = \frac{(Q_{1i} + Q_{2i} + Q_{3i})\mu_{Q_i} + (Q'_{1i} + Q_{2i} + Q'_{3i})\nu_{Q_i}}{6} \tag{8}$$

Ranking of the alternatives can be done by sorting each S, R, and Q^* values in an increasing order. Propose as a compromise solution the alternative A' which is the best ranked by the measure Q^* (minimum) if the following two conditions are satisfied:

C1. *Acceptable advantage:*

$$Q^*(A'') - Q^*(A') \geqslant DQ^*$$

where A'' is the alternative with second position in the ranking list by Q^*; $DQ^* = 1/(m-1)$; m is the number of alternatives.

C2. *Acceptable stability in decision making:* The alternative A' must also be the best ranked by S or/and R.This compromise solution is stable within a decision making process, which could be "voting by majority rule" (when $\vartheta > 0.5$ is needed), or "by consensus" $\vartheta \approx 0.5$, or "with veto" ($\vartheta < 0.5$). If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A' and A'' if only condition C2 is not satisfied, or
- Alternatives$A', A'', \ldots, A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q^*(A^{(M)}) - Q^*(A') < DQ^*$ for maximum M (the positions of these alternatives are "in closeness").

The best alternative, ranked by Q^*, is the one with minimum value of Q^*. The main ranking result is the compromise ranking list of alternatives and the compromise solution with "average rate". The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by min S) of the "majority", and a minimum of the "individual regret" (represented by min R) of the "opponent". The compromise solutions are the basis for negotiations, involving the decision maker's preference by criteria weights.

5. Numerical example

A robot selection problem is adopted from the literature (Chu & Lin, 2003; Liang & Wang, 1993) where parameters are hypothetically designed as TIFSs. This problem is used to demonstrate the computational procedure of the VIKOR method, proposed in previous section. A manufacturing company requires a robot to perform a material-handling task. After initial selection, three robotsA_1, A_2 and A_3 are chosen for further evaluation. To select the most suitable robot, a committee of four decision makers, DM_1, DM_2, DM_3 and DM_4 is formed. For the robot selection subjective criteria are as follows:

C_1: Man–machine interface
C_2: Programming flexibility
C_3: Vendor's service contract
C_4: Purchase cost
C_5: Load capacity
C_6: Positioning accuracy

The linguistic terms represented by TIFSs for evaluating the alternative robot under subjective criteria and the importance weights for criteria are depicted in Tables 1 and 2, respectively. Each decision maker presents his assessment based on linguistic variable for rating the performance and importance of each criterion by a linguistic variable as depicted in Tables 3 and 4, respectively. By Eqs. (1) and (2), the average weights for criteria and the average ratings for robots can be obtained, as shown in Table 5.

(a) The ideal score A^+ and anti-ideal score A^- are computed by Eqs. (3) and (4).
(b) In this step, we compute $S_i == \left\langle \left[(S_{1i}, S_{2i}, S_{3i}); \mu_{S_i}\right], [(S'_{1i}, S_{2i}, S'_{3i}); \nu_{S_i}]\right\rangle$ and $R_i == \left\langle \left[(R_{1i}, R_{2i}, R_{3i}); \mu_{R_i}\right], \left[(R'_{1i}, R_{2i}, R'_{3i}); \nu_{R_i}\right]\right\rangle$ using Eqs. (5) and (6).
(c) Q_i and Q_i^* values are computed by Eqs. (7) and (8).
(d) Thus, the ranking order of three alternatives by proposed IF-VIKOR method is $A_1 > A_3 > A_2$.

6. Conclusion

VIKOR is a helpful tool for MCDM problems, particularly in a situation where the decision maker is not able or does not know to express his preferences at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" and a minimum of the individual regret of the "opponent". Considering the fact that, in some cases, fuzzy sets get failed to tackle vagueness and uncertainty, therefore, in this paper, IF-VIKOR method is proposed to solve MCDM problems in which the performance rating values as well as the weights of criteria are linguistic terms which could be expressed by TIFSs. Utilizing the proposed VIKOR method, a robot selection problem is examined and the results are demonstrated.

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