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Goal programming approach to solving network design problem with multiple objectives and demand uncertainty

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ABSTRACT

The transportation network design problem (NDP) with multiple objectives and demand uncertainty was originally formulated as a spectrum of stochastic multi-objective programming models in a bi-level programming framework. Solving these stochastic multi-objective NDP (SMONDP) models directly requires generating a family of optimal solutions known as the Pareto-optimal set. For practical implementation, only a good solution that meets the goals of different stakeholders is required. In view of this, we adopt a goal programming (GP) approach to solve the SMONDP models. The GP approach explicitly considers the user-defined goals and priority structure among the multiple objectives in the NDP decision process. Considering different modeling purposes, we provide three stochastic GP models with different philosophies to model planners' NDP decision under demand uncertainty, i.e., the expected value GP model, chance-constrained GP model, and dependent-chance GP model. Meanwhile, a unified simulation-based genetic algorithm (SGA) solution procedure is developed to solve all three stochastic GP models. Numerical examples are also presented to illustrate the practicability of the GP approach in solving the SMONDP models as well as the robustness of the SGA solution procedure.

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1. Introduction

The network design problem (NDP) is to optimize the improvement of a transportation network such that a set of system-wide objectives are achieved. Due to its practical significance and theoretical value, modeling, algorithmic development, and application on this subject have been extensively investigated by transportation practitioners and researchers in the past few decades. For a comprehensive review, interested reader can refer to Bell and Iida (1997) and Yang and Bell (1998).

The NDP decision usually involves a benefit game among different stakeholders. Each stakeholder has their own requirement on the NDP decision. For example, the ***network planner*** aims to develop a network improvement strategy to minimize congestion or improve efficiency of the whole transportation system; the ***environmentalists*** aim to protect the environment (e.g., minimizing vehicular emission); and the ***network users*** aim to improve their travel times as a result of network improvement (e.g., minimizing spatial inequity). Thus, the NDP is inherently a multi-objective decision process. Unfortunately, some of these objectives are conflicting. That is, increasing the value of one objective may reduce the value attained for one or more of the other objectives. These conflicts are particularly obvious in the transportation NDP.

Besides the need of considering multiple objectives, the NDP decision has to be made under uncertainty where certain inputs are not known accurately. One of the primary uncertain inputs is the forecast travel demand. It is quite difficult (or nearly impossible) to accurately predict the origin-destination (O-D) trip table twenty years in the future since it is affected by many factors such as economic growth, land-use pattern, and socioeconomic characteristics. For the NDP under uncertainty, several models have recently been proposed in the literature to tackle the uncertainty issue, e.g., the expected value model (Chen & Yang, 2004), mean–variance model (Chen, Subprasom, & Ji, 2003), chance-constrained model (Lo & Tung, 2003), probability model (Chen, Chootinan, & Wong, 2006a; Chootinan, Wong, & Chen, 2005; Yim, Wong, Chen, Wong, & Lam, 2011), min–max model (Yin, Madanat, & Lu, 2009), and alpha reliable model (Chen, Kim, Zhou, & Chootinan, 2007). Some of these models also explicitly deal with multiple objectives (e.g., Chen, Subprasom, & Ji, 2006b, 2010). For a more detailed review on the NDP models under uncertainty, please refer to Chen, Kim, Lee, and Choi (2009), Chen, Kim, Lee, and Kim (2010) and Chen et al. (2011) and the references therein.

To solve this type of multi-objective optimization problems, there exist two main schemes: generating scheme and preference-based scheme (Gen & Cheng, 2000). The ***generating scheme***

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aims to determine the whole set of Pareto-optimal solutions or its approximation (e.g., vector evaluation method, Pareto ranking method, and random weighting method). Its focus is on generating the Pareto-optimal solution set rather than determining a good solution for practical implementation. Due to the complexity and intensive computation of finding the Pareto-optimal set, this scheme may not be suitable for practical applications. In addition, a selection methodology is needed to select a good solution for implementation. For problems with three or more objectives, selecting a good solution among the identified Pareto-optimal solutions is not trivial.

In contrast, the ***preference-based scheme*** attempts to determine a preferred or compromised solution within the tradeoff among the multiple objectives. Weighting method and goal programming method are two widely used methods for converting the multiple objectives into a single objective via a preference structure provided by the decision makers (DMs). The weighting method generally applies a set of weights provided by the DMs to aggregate the multiple objectives into a single objective. It is simple to implement. However, it is generally difficult to quantitatively measure the proportional importance among the objectives. The ***goal programming*** (GP) method, on the other hand, explores a good solution that can realize as many of the DMs- specified goals as possible. The GP method has several good features from the viewpoint of practical implementation: (1) it can incorporate user-defined priorities about the multiple objectives; (2) in practice, in order to facilitate implementation and evaluation, DMs usually only need to set a target level (or goal) for each objective instead of pursuing for a theoretically optimal solution. The GP method mimics this decision process with the aim to determine a good solution that best satisfies the set of user-specified goals; and (3) it gives a single solution that can be readily used for implementation.

In view of these good features, this study adopts the GP approach to solve the NDP with multiple objectives and demand uncertainty. Without loss of generality, we only consider the continuous capacity enhancement of the current road network in the NDP. The stochastic multi-objective NDP (SMONDP) is formulated as three GP models with different modeling philosophies: the expected value GP (EVGP) model, chance-constrained GP (CCGP) model, and dependent-chance GP (DCGP) model. Even with different modeling purposes, the three GP models are capable of finding a good NDP solution for implementation by explicitly considering the DMs-specified goals and priority structure among the objectives. Mathematically speaking, the three GP models belong to the stochastic bi-level programming (SBLP) problem, which has several complex and non-tractable characteristics such as the non-convexity, non-differentiability, and stochasticity. To solve all three GP models in the SBLP framework, we develop a unified simulation-based genetic algorithm (SGA) procedure.

The remainder of the paper is organized as follows. In the next section, we provide mathematical formulations for the three GP models. The SGA solution procedure is presented in Section 3. In Section 4, some numerical examples are provided to illustrate the robustness of the solution procedure. Finally, some concluding remarks and future research directions are given in Section 5.

2. Mathematical models

In this section, we present three GP models for the SMONDP. Notation is provided first, followed by three objective measures used in this study, a brief description of the SBLP framework, a spectrum of stochastic programming models, and three GP formulations.

2.1. Notation

Notation used throughout this paper is listed as follows and all boldface letters denote the corresponding vectors.

W	Set of O-D pairs
A	Set of links
$\overline{A}$	Set of candidate links for capacity enhancement, $\overline{A} \subseteq A$
R_w	Set of routes connecting O-D pair $w \in W$
v_a, t_a, t_a^0	Flow, travel time, and free-flow travel time on link $a \in A$, respectively
C_a, L_a	Capacity and length of link $a \in A$, respectively
u_a	Capacity enhancement on link $a \in \overline{A}$ (*decision variable*)
$u_a^{\max}$	Upper bound of capacity enhancement on link $a \in \overline{A}$
$g_a(\cdot)$	Construction cost for capacity enhancement on link $a \in \overline{A}$
B	Total construction budget available for capacity enhancement
$e_a(\cdot)$	Amount of carbon monoxide pollution from link $a \in A$
f_r^w	Flow on route $r \in R_w$ between O-D pair $w \in W$
Q_w	Random travel demand between O-D pair $w \in W$
π_w	Minimum travel time between O-D pair $w \in W$
δ_{ar}^w	Link-route incidence indicator. $\delta_{ar}^w = 1$, if route r between O-D pair w uses link a; and $\delta_{ar}^w = 0$, otherwise

2.2. Objective measures

Without loss of generality, we consider three objective measures that represent efficiency, environment, and equity in the SMONDP.

2.2.1. Efficiency

Total travel time has often been adopted as an efficiency measure in the NDP (Yang & Bell, 1998). In this study, we also adopt it to quantify the network efficiency.

$$F_1(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{u}) = \sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)\,v_a(\mathbf{u},\mathbf{Q}), \tag{1}$$

where u_a is the capacity enhancement on link a, and $\mathbf{u}$ is its vector form; $v_a(\mathbf{u},\mathbf{Q})$ is the flow on link a, and $\mathbf{v}(\mathbf{u},\mathbf{Q})$ is its vector form; $t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ is the travel time on link a. Since both $t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ and $v_a(\mathbf{u},\mathbf{Q})$ depend on the random travel demand vector $\mathbf{Q}$, the total travel time $F_1(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{Q})$ is thus a random variable.

2.2.2. Environment

For simplicity, we consider the emission effect only since it is the major part of the vehicle-based pollution contributing to the deterioration of the environment. However, the model is capable of accounting for other pollutants (e.g., noise). For vehicular emission, carbon monoxide (CO) is considered as an important indicator for the level of atmospheric pollution generated by vehicular traffic (e.g., Alexopoulos, Assimacopoulos, & Mitsoulis, 1993). Again for simplicity, we use CO as an illustration to model vehicular emission as an environment objective:

$$F_2(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{u}) = \sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)\,v_a(\mathbf{u},\mathbf{Q}), \tag{2}$$

where $e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ denotes the amount of CO pollution from link a. Since both $e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ and $v_a(\mathbf{u},\mathbf{Q})$ depend on the random demand **Q**, the total network emission $F_2(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{Q})$ is also a random variable. In this study, we adopt the nonlinear macroscopic model of Wallace, Courage, Hadi, and Gan (1998) to estimate the link-based vehicular CO emission:

$$e_a(v_a(\mathbf{u},\mathbf{Q}),u_a) = 0.2038 \cdot t_a(v_a(\mathbf{u},\mathbf{Q}),u_a) \cdot \exp\left(\frac{0.7962 \cdot L_a}{t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)}\right), \tag{3}$$

where L_a is the length (in kilometers) of link a; $t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ and $e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)$ are respectively measured in minutes and grams per hour.

2.2.3. Equity

Spatial equity in the NDP was first addressed by Meng and Yang (2002). In their study, spatial equity is measured by the maximum ratio of O-D travel times after and before capacity enhancement:

$$F_3(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{u}) = \max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\}, \tag{4}$$

where $\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))$ and $\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))$ are the minimum travel times between O-D pair $w \in W$ after and before capacity enhancement, respectively. The minimum O-D travel time is a random variable since it depends on the random demand **Q**. Therefore, the maximum ratio $F_3(\mathbf{v}(\mathbf{u},\mathbf{Q}),\mathbf{Q})$ among all O-D pairs is also a random variable.

Before presenting the GP formulations, we briefly describe the stochastic bi-level programming (SBLP) framework for modeling the transportation NDP.

2.3. Stochastic bi-level programming framework

Due to the consideration of travelers' route choice behavior, the NDP decision process is usually modeled as a bi-level structure or a non-corporative game between two players. The travelers (or the follower) choose the cheapest routes to travel, while the planners (or the leader) aim to make efficient use of limited resources to achieve the stated objectives while considering travelers' route choice behavior. In other words, in order to evaluate a design scheme, the planners need to know how travelers response to this scheme (i.e., the aggregation of travelers' route choice decisions in terms of traffic flow patterns). A general formulation of the SBLP can be illustrated in Fig. 1, where the upper-level and lower-level subprograms model the decisions of planners and travelers, respectively.

Since there is no explicit relationship between the upper-level and lower-level decision variables (i.e., **u** and **v**), the BLP problem is usually quite difficult to solve. Mathematically speaking, it is generally nonlinear and non-convex. In this study, how to tackle the stochastic objective measures due to demand uncertainty also adds more complexity to the NDP model.

2.4. A spectrum of stochastic programming models

When making decisions under uncertainty, there are three main modeling philosophies: expected value model (EVM), chance-constrained model (CCM), and dependent-chance model (DCM). The EVM aims to optimize the expected performance of a stochastic decision system while ignoring its variability. The CCM, originally developed by Charnes and Cooper (1959), models a stochastic decision system with the assumption that the chance constraints will hold at least α times, where α is the user-specified confidence level provided as an appropriate safety margin. This model focuses on the system's ability to meet the chance constraint with certain reliability under uncertainty. The DCM maximizes the probability (or chance) of satisfying certain events under uncertainty. Their mathematical formulations and meanings are summarized in Table 1. For more details on the above three modeling philosophies, interested readers may refer to Liu (2009). Note that in the context of transportation, Chen et al. (2009) adopted this spectrum to deal with the uncertainty issue in the single-objective NDP, while (Chen et al., 2010) extended these stochastic NDP models to consider multiple objectives to find an approximate Pareto-optimal solution set.

2.5. Goal programming formulations

Solving the SMONDP directly requires generating a Pareto-optimal solution set. This is not a trivial problem. In addition, a selection methodology based on secondary objectives or user preferences is needed to select a single good solution among the identified Pareto-optimal solutions for practical implementation. In view of these issues, we formulate the SMONDP as a goal programming (GP) problem by using the user-defined priority structure and target value (or goal) for each objective. Specifically, the GP model deter-

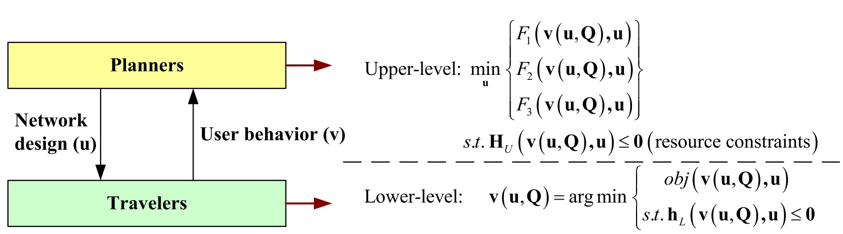

Fig. 1. Illustration of the stochastic bi-level programming framework.

Table 1
Three main modeling philosophies in stochastic programming.

Modeling philosophy	Formulation	Remark
Expected value model	$\min E[F_i(\mathbf{u},\mathbf{Q})]$	Minimize the expected value of the objective measure in consideration
Chance- constrained model	$\min \overline{F}_i$ s.t. $\Pr(F_i(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_i) \geqslant \alpha_i$ (α_i is the user-specified confidence level)	Minimize the threshold value subject to the chance constraint that guarantees the probability of the objective measure less than or equal to the threshold value is not less than the user-specified probability
Dependent-chance model	$\max \Pr(F_i(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_i)$ ($\overline{F}_i$ is the user-specified threshold value)	Maximize the probability that the objective measure does not exceed the user-specified threshold value

mines a good solution that best satisfies the set of goals. The limited budget is allocated in a sequential manner to minimize the deviations from all target values according to the priority structure. The relationship among the actual objective value $f(\mathbf{x})$, target value b, and deviations is shown in Fig. 2. The positive and negative deviations (d^+ and d^-) represent the over-achievement and under-achievement of the objective value with respect to the target value, respectively.

In the following, three GP formulations using the modeling philosophies given in Table 1 are presented, i.e., the expected value GP (EVGP) model, chance-constrained GP (CCGP) model, and dependent-chance GP (DCGP) model.

2.5.1. Expected value goal programming model

For demonstration purpose, we consider the following user-specified priority structure among the three objectives. Other priority structures can also be used in this modeling framework. In general, the priority structure should be specified according to the transportation policy of the city/region in question, and it should be updated periodically to reflect new challenges in different time periods.

Priority 1: For the efficiency objective, the expected total travel time (TTT) should not exceed its target value $\bar{F}_1$. Then, we have the following efficiency goal constraint:

$$E\left[\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)\,v_a(\mathbf{u},\mathbf{Q})\right] + d_1^- - d_1^+ = \bar{F}_1, \tag{5}$$

where the positive deviation between the expected TTT and the target value, $d_1^+ = \left[E\left[\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q})\right] - \bar{F}_1\right] \vee 0$, is to be minimized. Note that $b \vee 0$ denotes the maximum between b and 0.

Priority 2: For the environment objective, the expected total CO emission should not exceed its target value $\bar{F}_2$. Then, we have the following environment goal constraint:

$$E\left[\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)\,v_a(\mathbf{u},\mathbf{Q})\right] + d_2^- - d_2^+ = \bar{F}_2, \tag{6}$$

where $d_2^+ = \left[E\left[\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q})\right] - \bar{F}_2\right] \vee 0$ is to be minimized.

Priority 3: For the spatial equity objective, the expected maximum ratio of the minimum O-D travel times after and before capacity enhancement should not be larger than its target value $\bar{F}_3$. Then, we have the following equity goal constraint:

$$E\left[\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\}\right] + d_3^- - d_3^+ = \bar{F}_3, \tag{7}$$

where $d_3^+ = \left[E\left[\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\}\right] - \bar{F}_3\right] \vee 0$ is to be minimized.

Under the SBLP framework presented in Section 2.3, we can formulate the SMONDP as the following EVGP model:

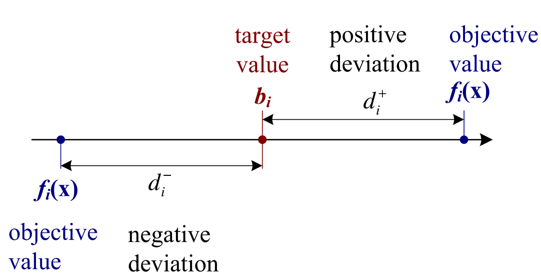

Fig. 2. Objective value, target value, and deviations in goal programming.

$$\text{EVGP}\begin{cases} \text{lexmin}_{\mathbf{u}}\ [d_1^+, d_2^+, d_3^+] \\ \text{subject to}: \\ \quad E\left[\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q})\right] + d_1^- - d_1^+ = \bar{F}_1 \\ \quad E\left[\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q})\right] + d_2^- - d_2^+ = \bar{F}_2 \\ \quad E\left[\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\}\right] + d_3^- - d_3^+ = \bar{F}_3 \\ \quad \sum_{a\in\bar{A}} g(u_a) \leqslant B \\ \quad 0 \leqslant u_a \leqslant u_a^{\max}, \quad \forall a\in\bar{A} \\ \quad d_i^+ \geqslant 0,\ d_i^- \geqslant 0, \quad i=1,2,3 \\ \quad \mathbf{v}(\mathbf{u},\mathbf{Q}) \text{ solves the lower-level subprogram for each realization of } \mathbf{Q} \end{cases}, \tag{8}$$

$$\begin{cases} \min_{\mathbf{v}} \quad \sum_{a\in A}\int_0^{v_a} t_a(\omega,u_a)\,d\omega \\ \text{subject to}: \\ \quad \sum_{r\in R_w} f_r^w = q_w, \quad \forall w\in W \\ \quad v_a = \sum_{w\in W}\sum_{r\in R_w} f_r^w \delta_{ar}^w, \quad \forall a\in A \\ \quad f_r^w \geqslant 0, \quad \forall r\in R_w, \quad w\in W \end{cases},$$

where lexmin represents lexicographically minimizing the deviation vector $\mathbf{d}^+$ between the expected objective values and the target values; $\bar{A}\subseteq A$ is the set of candidate links for capacity enhancement; $g_a(u_a)$ is the construction cost function of link a; B is the total construction budget available; $u_a^{\max}$ is the upper bound for capacity enhancement on link a. In the lower-level subprogram, q_w is a realization of the random demand Q_w between O-D pair w; f_r^w is the flow on route r between O-D pair w; δ_{ar}^w is the link-route incidence indicator: $\delta_{ar}^w = 1$ if route r of O-D pair w uses link a, and 0 otherwise.

The first three constraints are the goal constraints with respect to the efficiency, environment, and equity requirements. The fourth constraint is the total construction budgetary constraint. The fifth constraint sets the upper bound for the candidate link capacity enhancement. The sixth constraint is the non-negativity conditions on the deviations. The lower-level subprogram is to model travelers' response to a certain capacity enhancement $\mathbf{u}$. For each realization $\mathbf{q}$, $v_a(\mathbf{u},\mathbf{q})$ is the equilibrium flow on link a, which is obtained by solving the lower-level subprogram as a standard user equilibrium (UE) traffic assignment problem (Sheffi, 1985). After solving the lower-level subprogram for all realizations of the random demand, we can collect the realizations of the network-wide random objective measures. These realizations form approximate distributions of the objective measures and thus can be used to calculate the deviations.

The GP model first allocates the limited construction budget to realize the goal in Priority 1 by minimizing its corresponding deviation. If it is achieved, the remaining budget will be used to realize the goal in Priority 2 as much as possible while keeping the achievement of the first goal intact. This sequential process continues until all three goals are realized as much as possible within the budget.

2.5.2. Chance-constrained goal programming model

We consider the following priority structure in the CCGP model.

Priority 1: For the efficiency objective, the total travel time (TTT) should not exceed its target value $\bar{F}_1$ at a probability of α_1 (e.g., 0.95).

$$\Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)\,v_a(\mathbf{u},\mathbf{Q}) - \bar{F}_1 \leqslant d_1^+\right) \geqslant \alpha_1, \tag{9}$$

where the positive deviation between the α_1-percentile of TTT and the target value, $d_1^+ = \left[\min\left\{d \,|\, \Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) - \bar{F}_1 \leqslant d\right) \geqslant \alpha_1\right\}\right] \vee 0$, is to be minimized.

***Priority* 2**: For the environment objective, the total CO emission should not exceed its target value $\overline{F}_2$ at a probability of α_2 (e.g., 0.85).

$$\Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) - \overline{F}_2 \leqslant d_2^+\right) \geqslant \alpha_2, \qquad (10)$$

where $d_2^+ = \left[\min\left\{d|\Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) - \overline{F}_2 \leqslant d\right) \geqslant \alpha_2\right\}\right] \vee 0$ is to be minimized.

***Priority* 3**: For the equity objective, the maximum ratio of the minimum O-D travel times after and before capacity enhancement should not be larger than its target value $\overline{F}_3$ at a probability of α_3 (e.g., 0.75).

$$\Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} - \overline{F}_3 \leqslant d_3^+\right) \geqslant \alpha_3, \qquad (11)$$

where $d_3^+ = \left[\min\left\{d|\Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} - \overline{F}_3 \leqslant d\right) \geqslant \alpha_3\right\}\right] \vee 0$ is to be minimized.

Based on the above priority structure and goal setting, we have the following lexicographic optimization problem:

$$\text{CCGP}\begin{cases} \text{lexmin}_{\mathbf{u}} \quad [d_1^+, d_2^+, d_3^+] \\ \text{subject to :} \\ \quad \Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) - \overline{F}_1 \leqslant d_1^+\right) \geqslant \alpha_1 \\ \quad \Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) - \overline{F}_2 \leqslant d_2^+\right) \geqslant \alpha_2 \\ \quad \Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} - \overline{F}_3 \leqslant d_3^+\right) \geqslant \alpha_3 \\ \quad \sum_{a\in \overline{A}} g(u_a) \leqslant B \\ \quad 0 \leqslant u_a \leqslant u_a^{\max}, \quad \forall a \in \overline{A} \\ \quad d_i^+ \geqslant 0, \quad i=1,2,3 \\ \quad \mathbf{v}(\mathbf{u},\mathbf{Q}) \text{ solves the lower-level subprogram} \\ \qquad \text{for each realization of } \mathbf{Q} \end{cases}, \qquad (12)$$

where the lower-level subprogram is the same as that in the EVGP model (8).

2.5.3. Dependent-chance goal programming model

Similarly, we consider the following priority structure in the DCGP model:

***Priority* 1**: For the efficiency objective, the probability of the total travel time less than its threshold value $\overline{F}_1$ should achieve α_1 (e.g., 0.95).

$$\Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_1\right) + d_1^- - d_1^+ = \alpha_1, \qquad (13)$$

where the negative deviation between the target probability (α_1) and the actually achieved probability, $d_1^- = \left[\alpha_1 - \Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),\ u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_1\right)\right] \vee 0$, is to be minimized.

***Priority* 2**: For the environment objective, the probability of the total CO emission less than its threshold value $\overline{F}_2$ should achieve α_2 (e.g., 0.85).

$$\Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_2\right) + d_2^- - d_2^+ = \alpha_2, \qquad (14)$$

where $d_2^- = \left[\alpha_2 - \Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_2\right)\right] \vee 0$ is to be minimized.

***Priority* 3**: For the equity objective, the probability that the maximum ratio of the minimum O-D travel times after and before capacity enhancement is less than its threshold value $\overline{F}_3$, should achieve α_3 (e.g., 0.75).

$$\Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} \leqslant \overline{F}_3\right) + d_3^- - d_3^+ = \alpha_3, \qquad (15)$$

where $d_3^- = \left[\alpha_3 - \Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} \leqslant \overline{F}_3\right)\right] \vee 0$ is to be minimized.

The DCGP model can then be formulated as the lexicographic optimization problem below:

$$\text{DCGP}\begin{cases} \text{lexmin}_{\mathbf{u}} \ [d_1^-,\ d_2^-,\ d_3^-] \\ \text{subject to :} \\ \quad \Pr\left(\sum_{a\in A} t_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_1\right) + d_1^- - d_1^+ = \alpha_1 \\ \quad \Pr\left(\sum_{a\in A} e_a(v_a(\mathbf{u},\mathbf{Q}),u_a)v_a(\mathbf{u},\mathbf{Q}) \leqslant \overline{F}_2\right) + d_2^- - d_2^+ = \alpha_2 \\ \quad \Pr\left(\max_{w\in W}\left\{\frac{\pi_w(\mathbf{u},\mathbf{v}(\mathbf{u},\mathbf{Q}))}{\pi_w(\mathbf{0},\mathbf{v}(\mathbf{0},\mathbf{Q}))}\right\} \leqslant \overline{F}_3\right) + d_3^- - d_3^+ = \alpha_3 \\ \quad \sum_{a\in \overline{A}} g(u_a) \leqslant B \\ \quad 0 \leqslant u_a \leqslant u_a^{\max}, \quad \forall a \in \overline{A} \\ \quad d_i^+ \geqslant 0, \quad d_i^- \geqslant 0, \quad i=1,2,3 \\ \quad \mathbf{v}(\mathbf{u},\mathbf{Q}) \text{ solves the lower-level subprogram} \\ \qquad \text{for each realization of } \mathbf{Q} \end{cases}. \qquad (16)$$

where the lower-level subprogram is the same as that in the EVGP model (8).

Remark 1. The above three GP models belong to the BLP problem, which has several complex and non-tractable characteristics such as the nonlinear and non-convex constraint set. In addition, due to demand uncertainty, we have three nonlinear and non-convex goal constraints in each model, which further add complexity. To solve the spectrum of GP models with different philosophies, Section 3 presents a unified simulation-based genetic algorithm (SGA) solution procedure.

Remark 2. In each of the above three GP models, there is only one type of modeling philosophy. However, the above three modeling philosophies can be combined for different modeling purposes. For example, we may use the EVM to mimic the requirement on the efficiency, the CCM on the environment, and the DCM on the spatial equity.

3. Solution procedure

3.1. Simulation-based genetic algorithm

Solving the BLPs with multiple goals under demand uncertainty is generally a very difficult task. The complexity involves addressing three issues: (1) how to solve the BLP problem, (2) how to compute the random objective measures, and (3) how to incorporate the user-defined priority structure and goals. Evolutionary algorithms, especially genetic algorithm (GA), have shown to be effective in solving this type of complex problems (e.g., Chen et al., 2009, 2010; Xu, Wei, & Hu, 2009). In this study, we develop a unified simulation-based genetic algorithm (SGA) procedure shown in Fig. 3 to solve all three GP models.

In this procedure, there are five main modules, i.e., simulation, traffic assignment, satisfaction evaluation, chromosome rearranging, and GA. Their functions are briefly described as follows:

- Simulation module is to generate realizations of the uncertain demand according to a given distribution specification.
- For each demand realization, the traffic assignment module is used to model travelers' route choice behavior under a given capacity enhancement. For simplicity, the well-known Frank–Wolfe algorithm (Sheffi, 1985) is adopted here.
- Evaluation module is to evaluate the satisfaction degree of each capacity enhancement (i.e., each chromosome) with respect to the goals and priority structure.
- Chromosome rearranging module uses the priority structure to rank the chromosomes from good to bad for the subsequent GA operations.
- GA module (including the reproduction, crossover, and mutation operations) is to obtain better capacity enhancement solutions.

In this study, the chromosomes are represented as a string of real numbers with a length equal to the number of design variables (i.e., candidate links for capacity enhancement). For the GA operations, we adopt the commonly used roulette wheel reproduction operator, arithmetical crossover and mutation operators. In the following, we only highlight the special satisfaction function used in Step 3, and the chromosome rearranging module in Step 4. We should point out these two steps are different from those in the single-objective (e.g., Chen & Yang, 2004; Chootinan et al., 2005) and multi-objective (e.g., Chen et al., 2006b, 2010) problems. For more details of the other steps in the SGA procedure, interested readers may refer to Chen et al. (2006b, 2010) and Chootinan et al. (2005).

3.2. Strategies of handling the priority structure

When solving the GP models, how to deal with the user-defined priority structure (i.e., how to optimize the deviations in the order of the priority structure) is a critical issue. There are two main handling strategies in the GP literature: weighting (Taguchi, Ida, & Gen, 1997) and ranking (Gen & Cheng, 2000). The ***weighting strategy*** assigns a weight for each deviation according to the relative importance among the multiple objectives and then converts the multi-deviation optimization problem into a single-objective (i.e., a combined deviation) optimization problem. This strategy is easy to implement and can also decrease the computational burden. However, it does not directly reflect the satisfaction degree of the design scheme with respect to the priority structure. On the other hand, the ***ranking strategy*** directly uses the priority structure to rank the chromosomes from good to bad. This treatment has its advantage in making preparation for the subsequent GA operations. However, this strategy does not provide a direct evaluation (i.e., the quantity of the deviations) of the chromosomes. Considering the advantages and disadvantages of the two strategies, this study combined them to provide a better strategy to handle the priority structure. Specifically, we use the weighting strategy as a satisfaction function to provide an explicit assessment of each chromosome, and then use the ranking strategy to rearrange the chromosomes in the current population before performing the GA operations.

3.2.1. Satisfaction function

We formulate the normalized satisfaction function for the three GP models as follows:

$$\text{EVGP\&CCGP} \quad sat(\mathbf{u}) = \sum_{i=1}^{3} P_i \cdot \left(1 - \frac{d_i^+}{\overline{F}_i}\right) \bigg/ \sum_{i=1}^{3} P_i, \tag{17}$$

and

$$\text{DCGP} \quad sat(\mathbf{u}) = \sum_{i=1}^{3} P_i \cdot \left(1 - \frac{d_i^-}{\alpha_i}\right) \bigg/ \sum_{i=1}^{3} P_i, \tag{18}$$

where P_i ($P_1 >> P_2 >> P_3$) is the preemptive priority factor, expressing the relative importance among various goals according to the priority structure; the relative deviation value $d_i^+/\overline{F}_i$ (or d_i^-/α_i) is used to eliminate the influence of different metrical units among

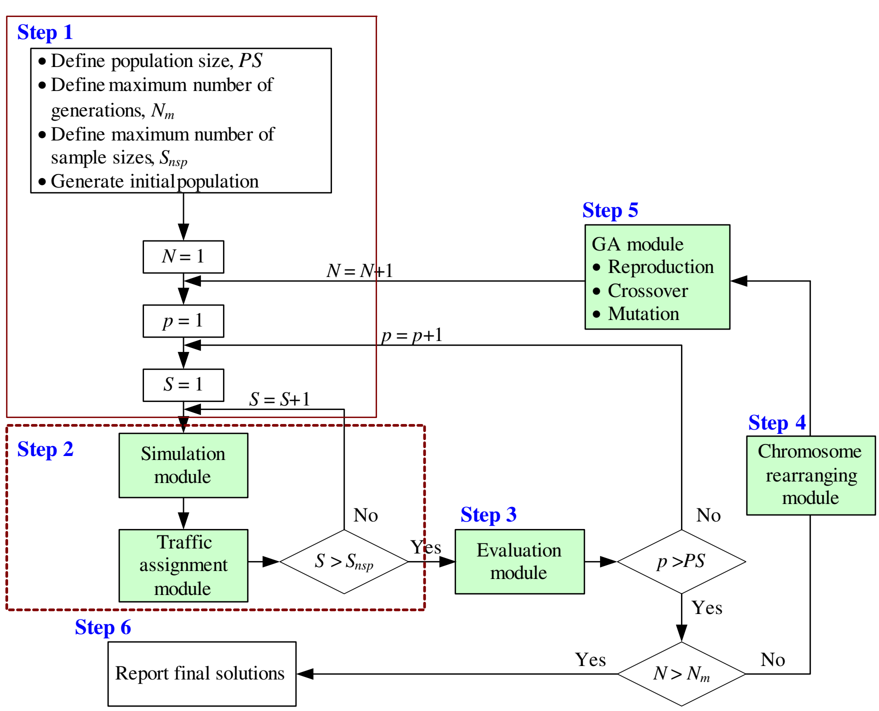

Fig. 3. Flowchart of the simulation-based genetic algorithm (SGA) procedure.

the different goals; and $1-(d_i^+/\overline{F}_i)$ (or $1-(d_i^-/\alpha_i)$) denotes the satisfaction degree with respect to the i^{th} goal. Hence, $0 \leqslant sat(\mathbf{u}) \leqslant 1$. According to Taguchi et al. (1997), P_i can be calculated in the following manner:

$$P_i = 10^{3-i}, \quad i = 1,2,3. \tag{19}$$

We should point out the satisfaction functions in Eqs. (17) and (18) are different from the weighted deviation in Taguchi et al. (1997) on two aspects: (1) Eqs. (17) and (18) are used to explicitly evaluate the normalized satisfaction degree (rather than the combined deviation) with respect to both the priority structure and goals; and (2) the influence of different metrical units among the different goals has been eliminated in Eqs. (17) and (18).

3.2.2. Ranking strategy

After the evaluation step, we have obtained the deviations for each capacity enhancement plan. Considering the special features of the GP models, we use the priority structure to rearrange the chromosomes by assigning a rank number for each chromosome in the current population (Gen & Cheng, 2000). Note that Baykasoglu (2005) also used a similar method to accept or reject a neighborhood solution in the simulated annealing algorithm. The procedure for rearranging the chromosomes is described as follows and also illustrated in Fig. 4.

Step 4 (***Chromosome rearranging procedure***)

- ***Step 4.1***: Sort the chromosomes based on the deviation value in Priority 1 and assign a rank number r_i for each chromosome in the current generation. Note that $r_i = 1$ and $r_i = PS$ correspond to the best and worst chromosomes, respectively.
- ***Step 4.2***: If there exist some chromosomes with the same deviation value in Priority 1, record them in the set Ω_2 and go to Step 4.3; else, terminate the rearranging step.
- ***Step 4.3***: Sort the chromosomes in the set Ω_2 according to the deviation value in Priority 2 and modify their rank numbersr_i.
- ***Step 4.4***: If there still exist some chromosomes with the same deviation value in Priority 2, record them in the set Ω_3 and go to Step 4.5; else, terminate the rearranging step.
- ***Step 4.5***: Sort the chromosomes in the set Ω_3 according to the deviation value in Priority 3 and modify their rank numbersr_i.
- ***Step 4.6***: If there still exist some chromosomes with the same deviation value in Priority 3, sort them randomly.

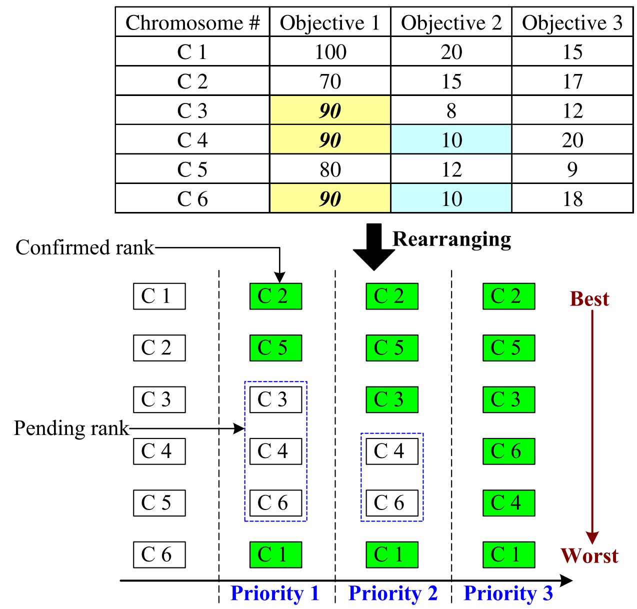

Fig. 4. Illustration of the chromosome rearranging procedure.

4. Numerical experiments

In this section, four numerical examples are provided to demonstrate the practicability of the GP approach in solving the SMONDP models as well as the robustness of the proposed solution procedure. Specifically, the effects of sample size, population size, crossover probability, and mutation probability are examined for the three GP models.

4.1. Network description and parameter setting

We use the well-known Nguyen-Dupuis (N-D) network (1984) to conduct the set of numerical experiments. The N-D network, shown in Fig. 5, consists of 13 nodes, 19 links, and 4 O-D pairs. We adopt the standard BPR (Bureau of Public Road)-type link performance function with parameters of 0.15 and 4. Link free-flow travel time, current capacity, length, and upper bound for capacity enhancement are listed in Table 2.

The link construction cost function for capacity enhancement is $g_a(u_a) = 0.30 \cdot u_a \cdot L_a, \forall a \in \overline{A}$. All 19 links are selected as candidate links for capacity enhancement and the available construction budget is 1800. The random correlated travel demands are generated according to the method by Asakura and Kashiwadani (1991) with scaling and correlation parameters of 0.60 and 0.80. The expected travel demands of O-D pairs (1,2), (1,3), (4,2), and (4,3) are 400, 800, 600, and 200, respectively. For simplicity, we use the same priority structure for all three GP models: the efficiency objective is in Priority 1, environment in Priority 2, and equity in Priority 3.

Parameters in the SGA procedure are set as follows:

Maximum number of generations (N_m)	200
Population size (PS)	16, 32
Sample size (S_{nsp})	1000
Crossover probability (P_c)	0.30, 0.50
Mutation probability (P_m)	0.10, 0.20, 0.30
Length of chromosomes	19

The maximum number of generations is selected from a trial-and-error method. Our preliminary tests for this network indicate that after 200 generations, the deviation improvement is quite marginal. Thus, we select 200 as the maximum number of generations to conduct the following experiments.

4.2. Evaluation criteria

Before presenting the numerical results for the three GP models, we describe the evaluation measures used to quantify how 'good' the SGA solution satisfies the user-defined priority structure and goals. Here, two measures are used: (1) the number of goals that are completely realized, and (2) the difference of the satisfaction value with respect to the best case, which is defined as follows:

$$\text{error}(\mathbf{u})_j = \max_j sat(\mathbf{u})_j - sat(\mathbf{u})_j, \tag{20}$$

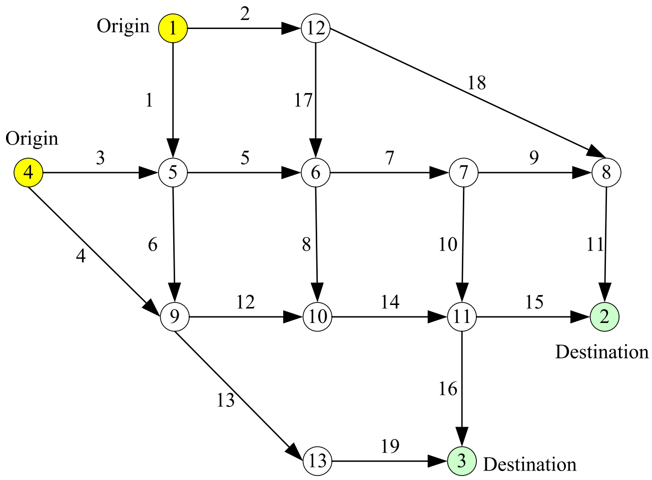

Fig. 5. Nguyen–Dupuis network.

Table 2
Link characteristics.

Link	From	To	Free-flow travel time	Current capacity	Length	Upper bound for enhancement
1	1	5	7	800	7	800
2	1	12	9	400	9	400
3	4	5	9	200	9	200
4	4	9	12	800	12	800
5	5	6	3	350	3	350
6	5	9	9	400	9	400
7	6	7	5	800	5	800
8	6	10	13	250	13	250
9	7	8	5	250	5	250
10	7	11	9	300	9	300
11	8	2	9	550	9	550
12	9	10	10	550	10	550
13	9	13	9	600	9	600
14	10	11	6	700	6	700
15	11	2	9	500	9	500
16	11	3	8	300	8	300
17	12	6	7	200	7	200
18	12	8	14	400	14	400
19	13	3	11	600	11	600

where $sat(\mathbf{u})_j$ represents the satisfaction value of the jth trial, and $\max_j sat(\mathbf{u})_j$ represents the maximal satisfaction value among all trials. The first measure provides a simple evaluation on the total number of achieved goals, while the second measure quantifies the variation of achievement degrees among various trials.

4.3. Example 1: effect of sample sizes

Recall that a simulation module is embedded in the SGA procedure to estimate the distributions of the three random objective measures. In this example, we address the question of *whether the specified number of samples is enough and appropriate for this network*. Without loss of generality, five different sample sizes are compared in Table 3. For each sample size, we use the traffic assignment results to evaluate the performance measures of the three models before capacity expansion. Specifically, we calculate the expected value of the random TTT in the EVM, the 90%-percentile of the TTT in the CCM, and the probability of the TTT less than 1.5E5 in the DCM. The relative absolute error (RAE) denotes the relative absolute difference with respect to the sample size of 1000. For example, the RAE for the sample size of 500 in the EVM is: |97820–98740|/ 98740 = 0.93%. From Table 3, we can see that the sample sizes larger than 1000 generally result in much smaller RAEs for all three models. Thus, we will use the sample size of 1000 in Examples 2–4. We should mention that, in order to solve large-scale problems, we must properly increase the sample size in the simulation module such that we have a stable network performance assessment.

4.4. Example 2: expected value goal programming model

Example 2 demonstrates the EVGP model. The target values for the efficiency, environment, and equity objectives are set as $\overline{F}_1 = 88000, \overline{F}_2 = 33400$, and $\overline{F}_3 = 0.90$, respectively. We examine twelve combinations of population size (16 and 32), crossover probability (0.3 and 0.5), and mutation probability (0.1, 0.2, and 0.3). Table 4 presents the relative deviations (i.e., the deviation divided by the corresponding target value), satisfaction value in Eq. (17), and satisfaction error in Eq. (20) used to evaluate the performance of the solution procedure for the above twelve cases. One can observe that for all cases, the solution procedure can achieve the same number of goals (i.e., efficiency and environment) and only the least important goal (i.e., equity) is not fully satisfied. In addition, the satisfaction error among the twelve cases does not exceed 0.02%. The above results indicate that the SGA solution procedure is quite robust to different GA parameter settings.

To further examine the performance of the SGA solution procedure, without loss generality, we illustrate the convergence results of Case 12 in Figs. 6 and 7. Fig. 6 shows the convergence of the satisfaction value in Eq. (17). One can see the trajectory increases steadily in the first 50 generations and stabilizes after the 90th generation. Fig. 7 shows the effect of the user-defined priority structure in the evolution process. The first goal is achieved in the 50th generation; the second goal is satisfied in the 88th generation; while the third goal is not completely realized. The best obtained relative deviation for the third goal is 7.99% in the 200th generation. These results are consistent with our pre-defined priority structure and goal setting. The first two goals can be completely satisfied, but the third goal is not achieved with a positive relative deviation of 7.99%.

4.5. Example 3: chance-constrained goal programming model

Example 3 demonstrates the CCGP model. The target values and confidence levels for the three objectives are set as follows:

Table 3
Effect of sample sizes on the network performance assessment.

Sample size		100	500	**1000**	1500	2000
EVM	Expected value	103791	97820	**98740**	99006	98795
	RAE (%)	5.12	0.93	**0.00**	0.27	0.06
CCM	90%-percentile	162271	156759	**158353**	158352	159592
	RAE (%)	2.47	1.01	**0.00**	0.00	0.78
DCM	Pr (TTT ⩽ 1.5E5)	0.870	0.884	**0.881**	0.882	0.881
	RAE (%)	1.25	0.34	**0.00**	0.11	0.00

Table 4
Effect of *PS*, P_c, and P_m on the solution quality of the EVGP model.

Case #	*PS*	P_c	P_m	Relative deviation			Satisfaction (%)	Error (%)
				1	2	3		
1	16	0.3	0.1	0	0	0.0734	99.9339	0.0042
2	16	0.3	0.2	0	0	0.0744	99.9330	0.0051
3	16	0.3	0.3	0	0	0.0830	99.9252	0.0129
4	16	0.5	0.1	0	0	0.0726	99.9346	0.0035
5	16	0.5	0.2	0	0	0.0784	99.9294	0.0087
6	16	0.5	0.3	0	0	0.0836	99.9247	0.0134
7	32	0.3	0.1	0	0	0.0725	99.9347	0.0034
8	32	0.3	0.2	0	0	0.0771	99.9305	0.0076
9	32	0.3	0.3	0	0	0.0898	99.9191	***0.0190***
10	32	0.5	0.1	0	0	0.0687	99.9381	0.0000
11	32	0.5	0.2	0	0	0.0717	99.9354	0.0027
12	32	0.5	0.3	0	0	0.0799	99.9280	0.0101

$\overline{F}_1 = 136000, \overline{F}_2 = 43800, \overline{F}_3 = 0.90, \alpha_1 = 90\%, \alpha_2 = 85\%,$ and $\alpha_3 = 80\%$. Similar to Example 2, we present the relative deviations, satisfaction value in Eq. (17), and satisfaction error in Eq. (20) for each of the twelve cases in Table 5. We can see all the cases have the same number of achieved goals (i.e., the first two goals), and the satisfaction error among the twelve cases is less 0.01%. Even though the CCGP model has a complex bi-level structure with three nonlinear and non-convex probabilistic goal constraints, the results in Table 5 suggest that the solution procedure is fairly robust to different GA parameter settings.

4.6. Example 4: dependent-chance goal programming model

Finally, Example 4 demonstrates the DCGP model. The threshold values (i.e., $\overline{F}_1, \overline{F}_2$, and $\overline{F}_3$) and target probabilities (i.e., α_1, α_2, and α_3) are set as follows: $\overline{F}_1 = 135000, \overline{F}_2 = 47500, \overline{F}_3 = 0.99$, $\alpha_1 = 90\%, \alpha_2 = 90\%$, and $\alpha_3 = 90\%$. Similar to Examples 2 and 3, we present the relative deviations, satisfaction value in Eq. (18), and satisfaction error in Eq. (20) for each of the twelve cases in Table 6. One can see that for all cases, the first two goals are achieved while the third goal is not fully satisfied. Even though different cases have different relative deviations for the third goal,

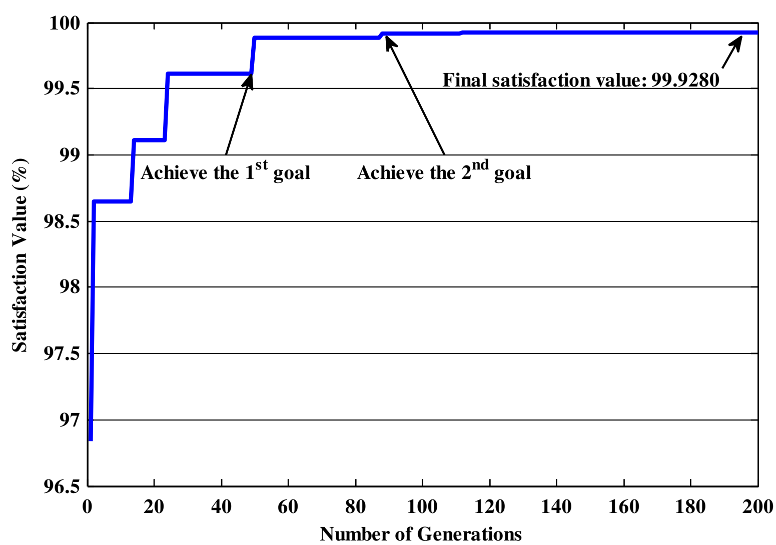

Fig. 6. Convergence curve of the satisfaction value.

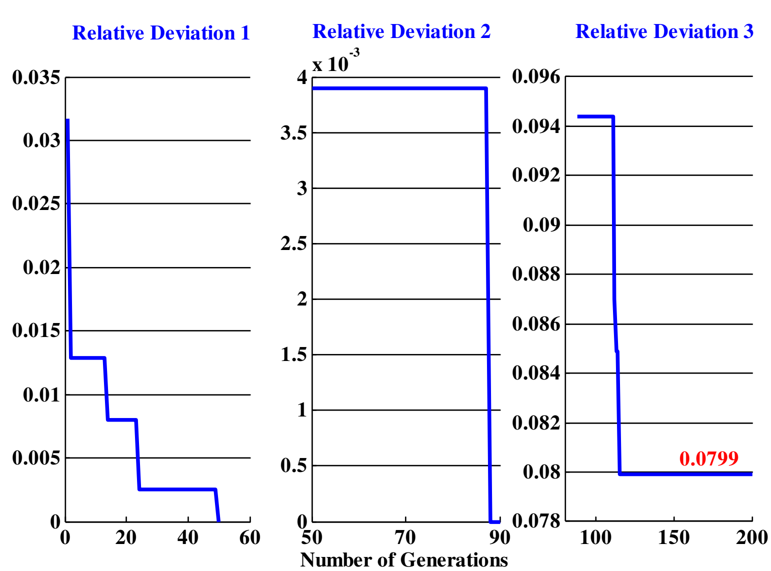

Fig. 7. Piecewise convergence curves of the three goals.

Table 5
Effect of PS, P_c, and P_m on the solution quality of the CCGP model.

Case #	PS	P_c	P_m	Relative deviation			Satisfaction (%)	Error (%)
				1	2	3		
1	16	0.3	0.1	0	0	0.1078	99.9029	0.0044
2	16	0.3	0.2	0	0	0.1055	99.9050	0.0023
3	16	0.3	0.3	0	0	0.1056	99.9049	0.0024
4	16	0.5	0.1	0	0	0.1040	99.9063	0.0010
5	16	0.5	0.2	0	0	0.1045	99.9059	0.0014
6	16	0.5	0.3	0	0	0.1066	99.9040	0.0033
7	32	0.3	0.1	0	0	0.1029	99.9073	0.0000
8	32	0.3	0.2	0	0	0.1064	99.9041	0.0032
9	32	0.3	0.3	0	0	0.1087	99.9021	***0.0052***
10	32	0.5	0.1	0	0	0.1031	99.9071	0.0002
11	32	0.5	0.2	0	0	0.1063	99.9042	0.0031
12	32	0.5	0.3	0	0	0.1075	99.9032	0.0041

Table 6
Effect of PS, P_c, and P_m on the solution quality of the DCGP model.

Case #	PS	P_c	P_m	Relative deviation			Satisfaction (%)	Error (%)
				1	2	3		
1	16	0.3	0.1	0	0	0.1506	99.8643	0.0155
2	16	0.3	0.2	0	0	0.1426	99.8715	0.0083
3	16	0.3	0.3	0	0	0.2000	99.8198	0.0600
4	16	0.5	0.1	0	0	0.1437	99.8705	0.0093
5	16	0.5	0.2	0	0	0.1862	99.8323	0.0476
6	16	0.5	0.3	0	0	0.2585	99.7671	***0.1127***
7	32	0.3	0.1	0	0	0.1489	99.8659	0.0140
8	32	0.3	0.2	0	0	0.1560	99.8595	0.0204
9	32	0.3	0.3	0	0	0.2308	99.7921	0.0877
10	32	0.5	0.1	0	0	0.1382	99.8755	0.0043
11	32	0.5	0.2	0	0	0.1334	99.8798	0.0000
12	32	0.5	0.3	0	0	0.1892	99.8295	0.0503

the satisfaction value with respect to the goals and priority structure is quite stable and the satisfaction error does not exceed 0.12%. Again, the solution procedure appears to be quite robust to different GA parameter settings and also effective in handling the nonlinear and non-convex probabilistic goal constraints.

5. Conclusion and future research

Solving the stochastic multi-objective network design problem (SMONDP) directly requires generating the Pareto-optimal set. This is not a trivial task. In order to obtain a good solution that meets the goals of different stakeholders for implementation, this study formulates the SMONDP as a goal programming (GP) problem in a stochastic bi-level programming framework. For different modeling purposes, the upper-level subprogram can use different philosophies (i.e., the EVGP, CCGP, and DCGP) to hedge against the uncertainties in the planner's NDP decision, while the lower-level subprogram models travelers' route choice decisions in responding to a certain design scheme. Even with different modeling philosophies, the GP models are able to find a good NDP solution for implementation by explicitly considering the user-defined goals and priority structure among the objectives. A unified simulation-based genetic algorithm (SGA) procedure is then developed to solve all three GP models. Numerical examples are also presented to illustrate the practicability of the GP approach in solving the SMONDP models as well as the robustness of the SGA solution procedure. The analysis results indicate that the SGA solution procedure is quite robust to the different GA parameter settings, and also effective in handling the nonlinear and non-convex probabilistic goal constraints in all three GP models. Several works are worthy of further investigation:

- In this study, the priority structure among the objectives and goal setting for each objective need to be specified accurately. This requirement can be relaxed by using the fuzzy logic theory to model the imprecise priority structure and goals (e.g., Chen & Su, 2010). This treatment may enhance the flexibility of the GP approach.
- So far, only demand uncertainty is considered. Further work should also consider supply uncertainty (i.e., the degradation of network capacity) and route choice uncertainty (i.e., risk-averse behavior toward travel time variability).
- This study uses the probability theory to model the uncertainties (or distributions) of the random travel demands and network-wide objective measures. To cater for different uncertainty data preparations, we can use the creditability theory and chance theory to construct the GP models.
- Testing the proposed solution procedure with other objectives (e.g., reliability, vulnerability, resiliency, and robustness) on realistic networks is needed for practical applications of the GP approach.

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