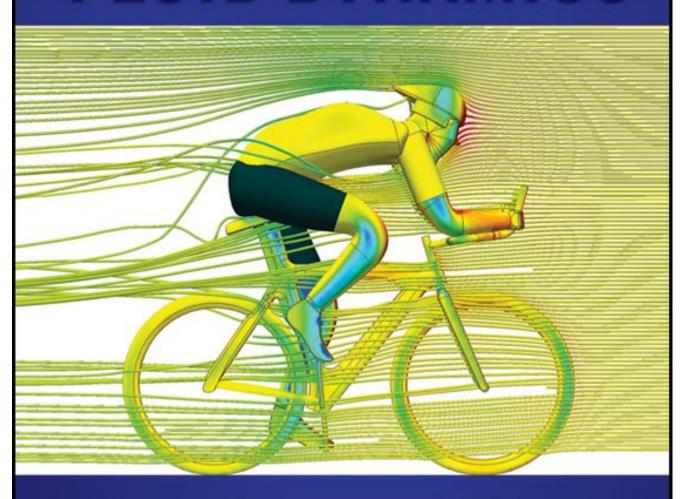
# ESSENTIAL COMPUTATIONAL FLUID DYNAMICS



OLEG ZIKANOV

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Oleg Zikanov



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