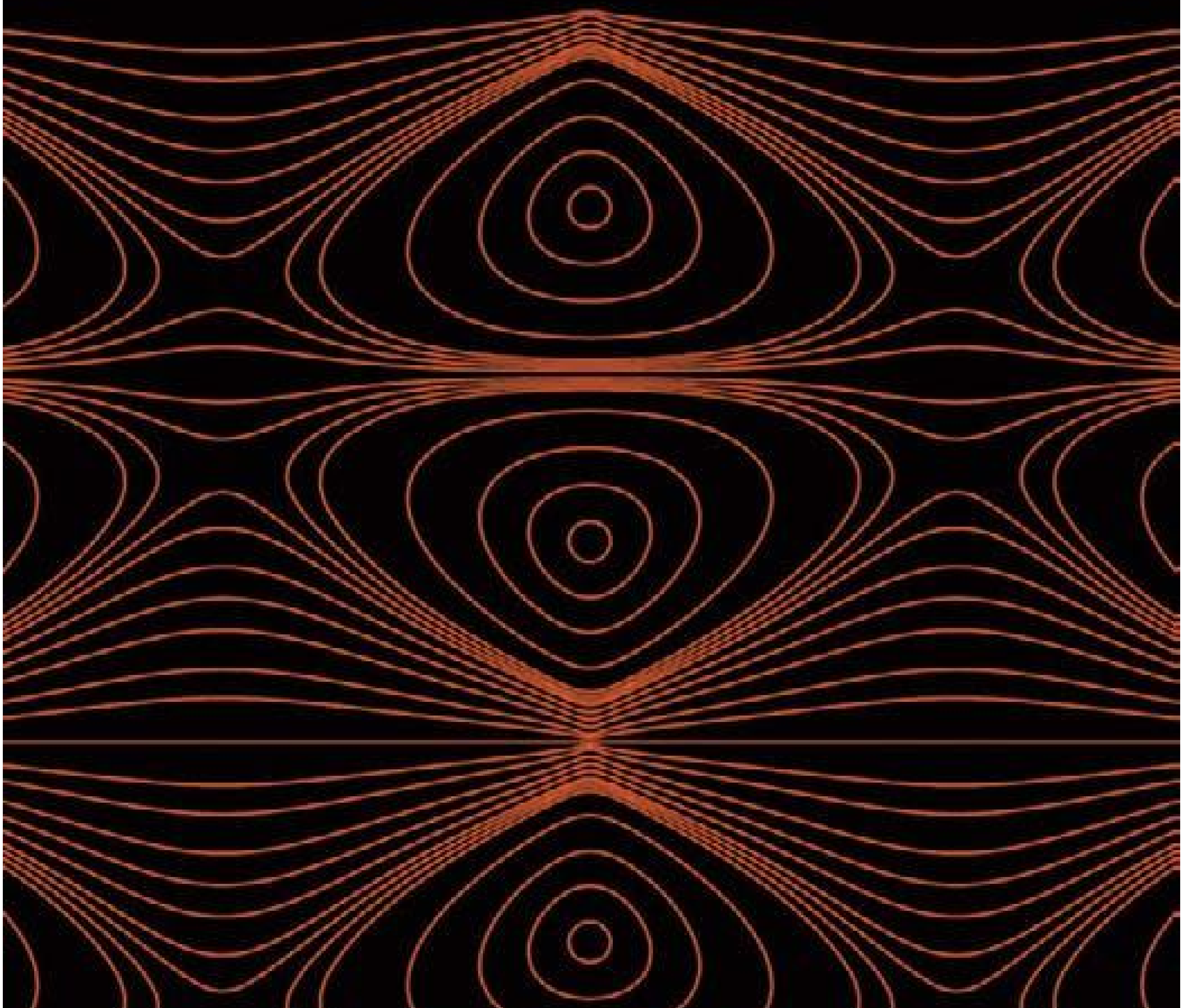
SECOND EDITION

Introduction to THEORETICAL AND COMPUTATIONAL FLUID DYNAMICS



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Second Edition

C. Pozrikidis

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Preface

My goal in this book entitled *Introduction to Theoretical and Computational Fluid Dynamics* is to provide a comprehensive and rigorous introduction to the fundamental concepts and basic equations of fluid dynamics, and simultaneously illustrate the application of numerical methods for solving a broad range of fundamental and practical problems involving incompressible Newtonian fluids. The intended audience includes advanced undergraduate students, graduate students, and researchers in most fields of science and engineering, applied mathematics, and scientific computing. Prerequisites are a basic knowledge of classical mechanics, intermediate calculus, elementary numerical methods, and some familiarity with computer programming. The chapters can be read sequentially, randomly, or in parts, according to the reader's experience, interest, and needs.

Scope

This book differs from a typical text on theoretical fluid dynamics in that the discourse is carried into the realm of numerical methods and into the discipline of computational fluid dynamics (CFD). Specific algorithms for computing incompressible flows under diverse conditions are developed, and computer codes encapsulated in the public software library FDLIB are discussed in Appendix C. This book also differs from a typical text on computational fluid dynamics in that a full discussion of the theory with minimal external references is provided, and no experience in computational fluid dynamics or knowledge of its terminology is assumed. Contemporary numerical methods and computational schemes are developed and references for specialized and advanced topics are provided.

Content

The material covered in this text has been selected according to what constitutes essential knowledge of theoretical and computational fluid dynamics. This intent explains the absence of certain specialized and advanced topics, such as turbulent motion and non-Newtonian flow. Although asymptotic and perturbation methods are discussed in several places, emphasis is placed on analytical and numerical computation. The discussion makes extensive usage of the powerful concept of Green's functions and integral representations.

Use as a text

This book is suitable as a text in an advanced undergraduate or introductory graduate course on fluid mechanics, Stokes flow, hydrodynamic stability, computational fluid dynamics, vortex dynamics, or a special topics course, as indicated in the *Note to the Instructor.* Each section is followed by a set of problems that should be solved by hand and another set of problems that should be tackled with the help of a computer. Both categories of problems are suitable for self-study, homework, and project assignment. Some computer problems are coordinated so that a function or subroutine written for one problem can be used as a module in a subsequent problem.

Preface to the Second Edition

The Second Edition considerably extends the contents of the First Edition to include contemporary topics and some new and original material. Clarifications, further explanations, detailed proofs, original derivations, and solved problems have been added in numerous places. Chapter 1 on kinematics, Chapter 8 on hydrodynamic stability, and Chapter 11 on vortex methods have been considerably expanded. Numerous schematic depictions and graphs have been included as visual guides to illustrate the results of theoretical derivations. Expanded appendices containing useful background material have been added for easy reference. These additions underscore the intended purpose of the Second Edition as a teaching, research, and reference resource.

FDLIB

The numerical methods presented in the text are implemented in computer codes contained in the software library FDLIB, as discussed in Appendix C. The directories of FDLIB include a variety of programs written in FORTRAN 77 (compatible with FORTRAN 90), Matlab, and C++. The codes are suitable for self-study, classroom instruction, and fundamental or applied research. Appendix D contains the User Guide of the eighth directory of FDLIB on hydrodynamic stability, complementing Chapter 8.

Acknowledgments

I thankfully acknowledge the support of Todd Porteous and appreciate useful comments by Jeffrey M. Davis and A. I. Hill on a draft of the Second Edition.

C. Pozrikidis

Note to the Instructor

This book is suitable for teaching several general and special-topics courses in theoretical and computational fluid dynamics, applied mathematics, and scientific computing.

Course on fluid mechanics

The first eight chapters combined with selected sections from subsequent chapters can be used in an upper-level undergraduate or entry-level introductory graduate core course on fluid mechanics. The course syllabus includes essential mathematics and numerical methods, flow kinematics, stresses, the equation of motion and flow dynamics, hydrostatics, exact solutions, Stokes flow, irrotational flow, and boundary-layer analysis. The following lecture plan is recommended:

Appendix A	Essential mathematics	Reading assignment
Appendix B	Primer of numerical methods	Reading assignment
Chapter 1	Kinematics	
Chapter 2	Kinematic description	Sections 2.2–2.8 and 2.10–2.13 are optional
Chapter 3	Equation of motion	
Chapter 4	Hydrostatics	
Chapter 5	Exact solutions	
Chapter 6	Stokes flow	Sections 6.8–6.18 are optional
Chapter 7	Irrotational flow	

Some sections can be taught as a guided reading assignment at the instructor's discretion.

Course on Stokes flow

Chapter 6 can be used in its entirety as a text in a course on theoretical and computational Stokes flow. The course syllabus includes governing equations and fundamental properties of Stokes flow, local solutions, particulate microhydrodynamics, singularity methods, boundary-integral formulations, boundary-element methods, unsteady Stokes flow, and unsteady particle motion. The students are assumed to have a basic undergraduate-level knowledge of fluid mechanics. Some topics from previous chapters can be reviewed at the beginning of the course.

Course on hydrodynamic stability

Chapter 9 combined with Appendix D can be used in a course on hydrodynamic stability. The course syllabus includes formulation of the linear stability problem, normal-mode analysis, stability of unidirectional flows, the Rayleigh equation, the Orr–Sommerfeld equation, stability of rotating flows, and stability of inviscid and viscous interfacial flows. The students are assumed to have a basic knowledge of the continuity equation, the Navier–Stokes equation, and the vorticity transport equation. These topics can be reviewed from previous chapters at the beginning of the course.

Course on computational fluid dynamics (CFD)

The following lecture plan is recommended in a course on numerical methods and computational fluid dynamics, following a graduate course on fluid mechanics:

Appendix A	Essential mathematics	Reading assignment
Appendix B	Primer of numerical methods	Reading assignment
Chapter 2	Theory of potential flow	Sections 2.1–2.5
Chapter 6	Boundary-integral methods for Stokes flow	Sections 6.5–6.10
Chapter 10	Boundary-integral methods for potential flow	
Chapter 11	Vortex motion	Selected topics
Chapter 12	Finite-difference methods	
Chapter 13	Finite-difference methods for incompressible flow	

Short course on vortex dynamics

Chapter 11 is suitable as a text in a short course on vortex dynamics. The material can be preceded or supplemented with selected sections from previous chapters to establish the necessary theoretical framework.

Special topics in fluid mechanics

Selected material from Chapters 9–13 can be used in a special-topics course in fluid mechanics, applied mathematics, computational fluids dynamics, and scientific computing. The choice of topics will depend on the students' interests and field of study.

Note to the Reader

For self-study, follow the roadmap outlined in the *Note to the Instructor,* choosing your preferred area of concentration. In the absence of a preferred area, study the text from page one onward, skipping sections that seem specialized, but keeping in mind the material contained in Appendices A and B on essential mathematics and numerical methods. Before embarking on a course of study, familiarize yourself with the entire contents of this book, including the appendices.

Notation

In the text, an italicized variable, such as a, is a scalar, and a bold-faced variable, such as $\mathbf{a}$, is a vector or matrix. Matrices are represented by upper case and bold faced symbols. Matrix–vector and matrix–matrix multiplication is indicated explicitly with a centered dot, such as $\mathbf{A}\cdot\mathbf{B}$. With this convention, a vector, $\mathbf{a}$, can be horizontal or vertical, as the need arises. It is perfectly acceptable to formulate the product $\mathbf{A}\cdot\mathbf{a}$ as well as the product $\mathbf{a}\cdot\mathbf{A}$, where $\mathbf{A}$ is an appropriate square matrix. Index notation and other conventions are defined in Appendix A.

The fluid velocity is denoted by $\mathbf{u}$ or $\mathbf{U}$. The boundary velocity is denoted by $\mathbf{v}$ or $\mathbf{V}$. Exceptions are stated, as required. Dimensionless variables are denoted by a hat (caret). We strongly advocate working with physical dimensional variables and nondimensionalizing at the end, if necessary.

Polymorphism

Occasionally in the analysis, we run out of symbols. A bold faced variable may then be used to represent a vector or a matrix with two or more indices. A mental note should be made that the variable may have different meanings, depending on the current context. This practice is consistent with the concept of polymorphism in object-oriented programming where a symbol or function may represent different entities depending on the data type supplied in the input and requested in the output. The language compiler is trained to pick up the appropriate structure.

Physical entities expressed by vectors and tensors

The velocity of an object, $\mathbf{v}$, is a physical entity characterized by magnitude and direction. In the analysis, the velocity is described by three scalar components referring to Cartesian, polar, or other orthogonal or nonorthogonal coordinates. The Cartesian components, v_x, v_y, and v_z, can be conveniently collected into a Cartesian vector,

$$\mathbf{v} = [v_x, v_y, v_z].$$

Accordingly, $\mathbf{v}$ admits a dual interpretation as a physical entity that is independent of the chosen coordinate system, and as a mathematical vector. In conceptual analysis, we refer to the physical interpretation; in practical analysis and calculations, we invoke the mathematical interpretation. To prevent confusion, the components of a vector in non-Cartesian coordinates should *never* be collected into a vector. Similar restrictions apply to matrices representing physical entities that qualify as Cartesian tensors.